MECHANICS

THEORETICAL, APPLIED, AND EXPERIMENTAL

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BY

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PREFACE

For some years the author has felt the want of a work on Mechanics for Elementary Students which would include (1) a discussion of the principles of the subject (commonly known as Theoretical Mechanics), (2) descriptions of and instructions for carrying out experiments in the laboratory, and (3) the application of the principles to the solution of easy problems of a more or less technical nature (generally called Applied Mechanics).

The author has long felt that these cannot well be dissociated one from another, if a useful knowledge of the subject is to be imparted to junior or first-year students.

On account of the distressing ignorance of the simplest of mathematics shared by some students, the author has been careful to avoid introducing matter which is commonly named "Mixed Mathematics," and in which the part played by mechanics proper is that of a peg upon which to hang a mathematical exercise. This has no doubt assisted in the past in deterring some students from willingly devoting to Elementary Mechanics that consideration and study which its usefulness should command.

The author has been mindful of the fact that all teachers do not begin the subject at the same place, and in consequence the work has been so arranged that a start may be made with either chapters one, two, six, or eight.

In the preparation of illustrations, scale drawings have been introduced where such were necessary to give proper

ideas of an object and, which is very important, to give a sense of proportion of parts. The introduction of mechanical drawing into schools has now become so general that no excuse is needed for this type of illustration, while the technical student looks upon it as a language peculiarly his own.

In some cases mere diagrammatic illustrations have been used, and especially is this the case where a complete scale drawing would be likely to complicate the figure without adding to the information imparted.

Chapter V. may be considered rather above the capacity of the elementary student. It was introduced mainly on account of students in Building Construction who take a course in Mechanics, and, further, that it completes the course on the graphical representation of forces—a part of mechanics which is not seriously difficult because of the almost entire absence of calculation.

The properties of materials have not been touched upon, as they are quite outside of the subject of Mechanics.

Answers are given to a part only of the examples and exercises, so that teachers can please themselves in selecting questions with or without answers. Part of the examples have been selected from a number of public examination papers.

One of the main ideas in the following pages is to lead a student from the known to the unknown by easy stages. The rapid extension of laboratory instruction enables this to be done, while it gives the student a better opportunity of realizing the nature of the quantities with which he is dealing. In this way the subject will not be a mere collection of formulæ, a condition into which it has sometimes fallen in the past.

The author wishes to record his indebtedness to his colleague, Mr. Herbert Aughtie, A.M. Inst. M.E., for his kind assistance with some of the illustrations.

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MECHANICS

INTRODUCTION.

In reading this work the student will require to be familiar beforehand with but little beyond the most elementary notions of Arithmetic and Geometry, and the solution of a simple equation. Trigonometry has been dispensed with, as it has been thought that its introduction would be a bar to the study of Mechanics by those who are ignorant of the subject; at the same time any reader familiar with the elements of Trigonometry can use it to advantage in solving some problems analytically, which would otherwise have to be done geometrically.

The author is aware that there are a large number of junior and technical students who have not pursued the ordinary course in Elementary Mathematics, and for them these few lines of introduction are inserted to acquaint them of what will be required to be known by them for use in the following pages.

Of course the teacher of a class would in general supply the same information, but it is hoped that this work will be used by students to aid them in their home work and private study, and hence these pages of introduction.

Little legitimate progress can be made until a student can solve a simple equation, and hence it is here assumed that he has that knowledge.

The following results in geometry are also assumed, but the student should satisfy himself of their accuracy by drawing the figures to scale, and testing by actual measurement.

(1) The sum of the three angles of any triangle is equal to two right angles or 180°.

(2) The opposite angles between a pair of intersecting straight lines are equal. In Fig. 1 the angle A equals the

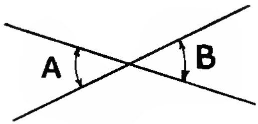


Fig. 1.

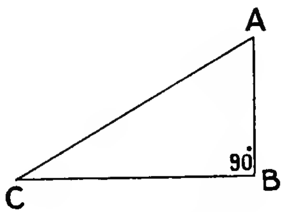


Fig. 2.

angle B. Also the angle between two lines equals the angle between two perpendiculars to the lines.

(3) In any right-angle triangle such as ABC (the right angle being at B), the square upon the hypotenuse equals the sum of the squares on the other two sides, or $AC^2 = CB^2 + BA^2$.

(4) If each angle of one triangle is equal to a corresponding angle of another triangle, the triangles are said to be similar to

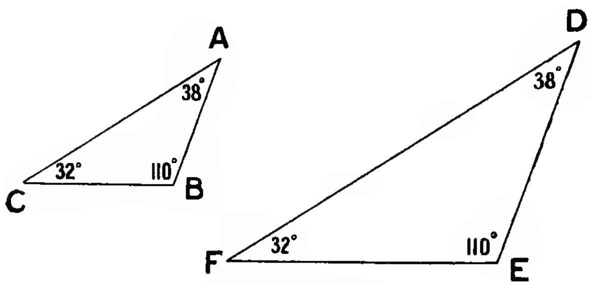


Fig. 3.—Similar triangles.

one another, and their *corresponding* sides are proportional. Thus the triangle ABC has exactly the same angles as the triangle DEF; then, if the triangles are arranged as in the figure with the angles of one triangle occupying similar positions to the angles of the other triangle, the ratio of the

side AB to the side BC is the same as the ratio of the side DE to the side EF, or, as it is sometimes written, AB is to BC as DE is to EF.

This is *generally* written as—

$$\frac{AB}{BC} = \frac{DE}{EF}$$

The side AB in one triangle corresponds to the side DE in the other triangle, and the side BC corresponds to EF. It will be noticed in the above equation that one pair of corresponding sides of the triangles is placed in the numerators, and the other pair in the denominators.

The same relation holds for *any* two sides of one triangle and the *corresponding* sides of the other; thus—

$$\frac{AB}{AC} = \frac{DE}{DF}$$

or, if we like—

$$\frac{AB}{DE} = \frac{AC}{DF}$$

The similar triangles are seldom arranged with their corresponding sides parallel as in the last figure. Suppose

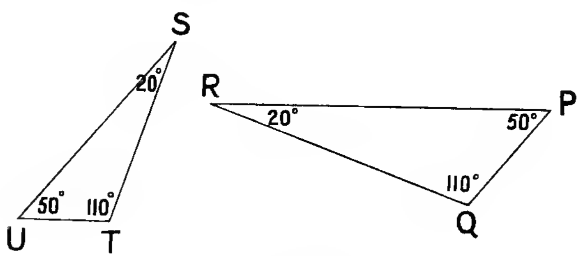


FIG. 4.—Similar triangles.

they are found in the positions shown at PQR and STU. It may puzzle the beginner to write down the proportion of two pairs of sides. Proceed as follows:—

Mark the equal angles so that they can be quickly recognized. In the figure their actual values in degrees have been written near them. Then—

$$\frac{\text{side opposite to } 110^\circ \text{ in one triangle}}{\text{side opposite to } 50^\circ \text{ in one triangle}}$$

$$= \frac{\text{side opposite to } 110^\circ \text{ in the other triangle}}{\text{side opposite to } 50^\circ \text{ in the other triangle}}$$

Here a pair of angles have been selected for illustration, and it is obvious that *any* pair might have been selected.

If we had liked, we might have written the proportion as follows :—

$$\frac{\text{side opposite to } 110^\circ \text{ in one triangle}}{\text{side opposite to } 110^\circ \text{ in the other triangle}}$$

$$= \frac{\text{side opposite to } 50^\circ \text{ in one triangle}}{\text{side opposite to } 50^\circ \text{ in the other triangle}}$$

(5) Two triangles are similar, if one angle in one is equal to one angle in the other, and any two sides of one proportional to the *corresponding* sides of the other.

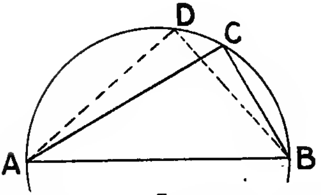


FIG. 5.

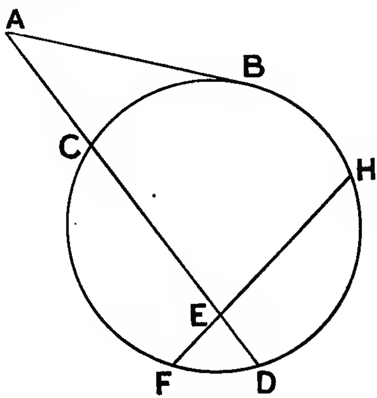


FIG. 6.

(6) The angle in a semicircle is a right angle; thus in Fig. 5, if AB is a diameter of the circle ADCB, and any point in

the circumference, such as C or D, be joined to the ends of the diameter, the angle included between the two lines is a right angle.

(7) If any two straight lines in a circle intersect such as CD and FH in Fig. 6, then the product of the two parts of one line equals the product of the parts of the other.

$$CE \times ED = FE \times EH$$

Also, if A be any point outside a circle, and AB be a tangent to the circle, while ACD cuts the circle; then the square on the tangent equals the product of the whole line cutting the circle and the part outside the circle.

$$AB^2 = AC \times AD$$

The student should also make himself familiar with the matter relating to the vernier; the equation to a straight line and the area of an irregular figure, all of which are dealt with at some length in the Appendix.

The beginner may find that the experiments on Atwood's machine and band friction, the matter relating to the pendulum, the second proof of the expression for centrifugal force, the centre of pressure, and perhaps Bernouilli's theorem, could with advantage be left till revision or second reading, these items being rather more difficult to follow than the other parts of the book.

CHAPTER I.

STRESS, STRAIN, AND FRICTION.

Force (which we may define as a push or a pull) plays a very prominent part in the study of mechanics, and hence it will be necessary at the outset to be able to measure force, though at first only approximately.

A Spring-balance is used for measuring force approximately, and is shown in Fig. 7. As the weight of a body is only the pull or force exerted on the body by the earth (due to gravity), the spring-balance will measure weight also (approximately).

The standard unit of weight is *one pound*, and is the pull of the earth on a piece of platinum called the *standard* pound deposited with the *Board of Trade in London*. Innumerable copies and multiples of the standard pound have been made, and are used daily in commerce for weighing everything.

The spring-balance (Fig. 7) consists of an outer brass shell S, containing a helical spring similar to that in Fig. 8, the upper end of which is fastened to the top of the shell, and the lower end to a smaller but similar brass tube T, which carries the pointer P. As weights are placed upon the hook H, the spring stretches and the pointer, P, moves down the scale. The balance is graduated in the following manner: with no load on the hook, the zero mark is made opposite the pointer. A one-pound weight is then put on the hook, and another mark made opposite the pointer in its new position. Successive pound weights are added, and the positions of the pointer marked on the brass shell S. Near the respective graduations are engraved the corresponding number of pound weights, and the balance is complete.

This method of graduation or calibration would produce a correct weighing balance, whatever the law of extension happened to be, provided the material of which the balance is made always behaved in the same manner, and if the pull of the earth on a body was always the same all over the surface of the earth.[1]

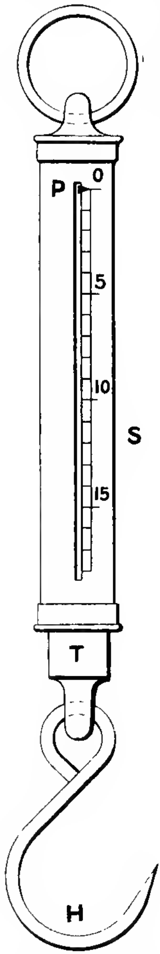


FIG. 7.—Spring-balance.

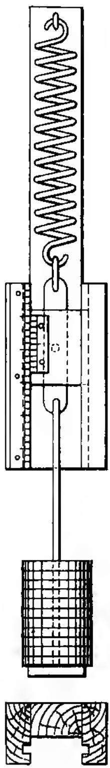


FIG. 8.

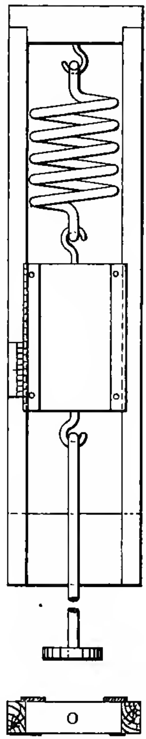


FIG. 9.

Apparatus for experiments on helical springs.

[1] The pull of the earth on a body, or what is sometimes called the attraction of gravity, is not *exactly* the same all over the earth's surface, though the difference is not

It will be interesting to discover what this law is. The two pieces of apparatus (Figs. 8 and 9) are simple and useful for the purpose. The helical spring is suspended from a stout peg in the upper end of the frame, a cross-section of which is shown at the bottom of the figure. The lower end of the spring is hooked into a link or hook which passes through or is attached to the slider. To the lower end of the slider is attached a pillar with a foot at its lower end on which weights may be placed. A scale of inches and tenths, or centimetres and millimetres, is fixed to the frame, and a vernier on the slider. For a description of the vernier and the method of using it, see the Appendix.

By putting weights on the load-pillar the spring will be stretched, and its extension can be read off on the scale. The apparatus in Fig. 9 is more suitable for springs of large diameter.

The following are some observations obtained from an experiment on a helical spring :—

RECORD OF EXPERIMENT ON HELICAL SPRING.

Date, March 10, 1902. Observer, R. R. Elliott.

Number of coils = 40.
Diameter of wire = 0·125 in.
Diameter of coils, centre to centre = 1 in.

Load.	Scale-read ing.	Load.	Scale-reading.
lbs.	inches.	lbs.	inches.
0	4·14	15	5·89
1	4·25	17	6·13
3	4·59	20	6·47
6	4·85	21	6·59
7	4·96	23	6·81
9	5·19	25	7·05
13	5·67	28	7·39
14	5·78	30	7·63

great. It is due to the different distance from the centre of the earth of different places. Some idea of the difference may be gathered from the following figures, which are the different values of the pull of the earth in lbs. on the standard pound.

At Greenwich, 1 lb. ; at Manchester, 1·00018 lb. ; at Baltimore (U.S.A.), 0·9988 lb. ; at the equator, 0·9969 lb. ; at latitude 45° and at sea-level, 0·9994 lb. ; and at the poles, 1·002 lb.

Now take a piece of squared paper and set off a scale of pounds of weight along the base, as in Fig. 10, and set off along the vertical axis a scale of inches.

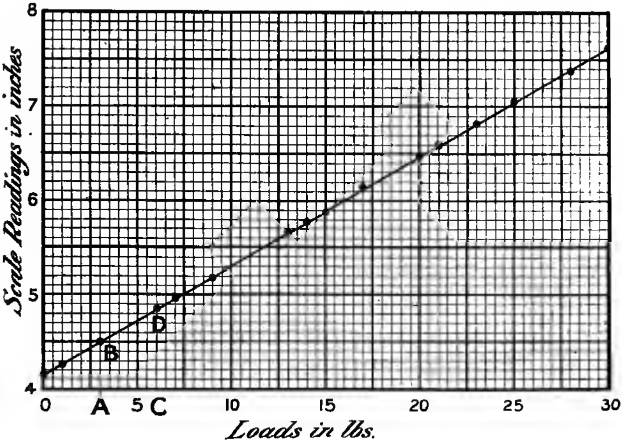


FIG. 10.—Extension of a helical spring.

Be careful to label each scale, stating what the scale represents, and always select the scale so that the side of one small square represents 1 or 0·1 or 0·5 or 10 or 100. Do not arrange that the side of a large square should represent 2, or 4 or 7.

Plot the observations in the table thus: Measure off OA = 3 lbs. along the base, and then above A mark the point B, so that its position is at 4·5 on the vertical scale. Next measure OC = 6 lbs. along the base, and mark the point D at a height 4·85 on the vertical scale. Proceeding in this way, the points shown by black dots are made to represent the observations in the table.

These points, it will be noticed, lie approximately on a straight line; hence, draw the straight line through them which will occupy the average position of the points; that is, leave as many above as below the line. The position of this line can be most easily located by stretching a piece of thread along the

points and shifting it about until it appears to occupy the best position. A piece of celluloid or tracing-cloth with a straight line drawn upon it will do as well.

Having drawn the average line through the points in Fig. 10, determine its equation as described in the Appendix. It is—

$$\text{ordinate} = \text{intercept} + \text{slope} \times \text{abscissa}$$
$$\text{or, scale reading} = 4{\cdot}14 + 0{\cdot}116 \times \text{load}$$

This equation indicates that when the load is zero, the scale reading is 4·14, and that the spring extends 0·116 in. for every 1 lb. of load put on the load-pillar.

Now consider some load not used in the experiment, say 10 lbs., and insert it in the above equation. We then find the corresponding scale-reading to be 5·3 in. Now read off the line BD, Fig. 10—the scale indication is 5·3 in.; and finally return to the apparatus and put 10 lbs. on the load-pillar, and the reading is 5·3 in. We may state the results as follows:—

Scale-reading with a load of 10 lbs.—

derived from equation . . .	5·3 inches
„ „ curve	5·3 „
„ „ experiment . . .	5·3 „

The coincidence of these numbers indicates that the experiment has probably been carried out with care, and that the results represent facts. The numbers will not always coincide *exactly* as those above do.

Stress, Strain, and Young's Modulus of Elasticity.—If a rod of metal be pulled at its two ends by a force of P lbs., and the area of its cross-section is A square inches, the force per square inch of cross-section will be—

$$\frac{P}{A} \text{ lbs.} = \frac{\text{total force}}{\text{area of section}}$$

This is called the *stress* in the material; hence—

$$\text{stress} = \frac{\text{total force}}{\text{area of section}}$$

All substances are more or less elastic; that is, they change their dimensions temporarily when acted upon by force.

If the change of length, or extension, of the metal rod be measured, together with its original length, then—

$$\frac{\text{the extension}}{\text{the original length}}$$

is called the strain of the material. This word "strain" will not be used in any other sense than this—*the amount of extension or compression per unit of original length.*

The ratio—

$$\frac{\text{stress}}{\text{strain}}$$

is called *Young's Modulus of Elasticity*, and is approximately constant for the same piece of material.

It will now be appropriate to see what information can be obtained from stretching a piece of wire and measuring the extension produced and the force producing it.

The apparatus used is shown in Fig. 11. The cross-piece at the top is made fast to an iron beam in the ceiling by two fang-bolts shown. From this cross-piece a wooden frame is suspended by two metal rods, A and B, something similar to that used with the spiral spring. In this frame is a slider S, to which is fixed a vernier V. The wire to be experimented upon is gripped between two plates on the crosspiece, and secured by a wing-nut W. The load-pillar is carefully weighed, and then attached to the wire (which is shown dotted in the fig.) just below the frame by winding the wire five or six times round the eye, and then two or three times round the vertical part of the wire or load-pillar.

There is a small spring clip, K, on the slider, beneath which the wire is placed, the slider having been previously raised *nearly* to its topmost position.

Measure with a tape the length of the wire from the underside of the clips on the crosspiece to the middle of the spring clip, K, on the slider. Obtain the diameter of the wire with a micrometer caliper, or screw gauge, in two or three places, and take the average. (For a description of the micrometer, and how to use it, see Appendix.) Read the scale and vernier,

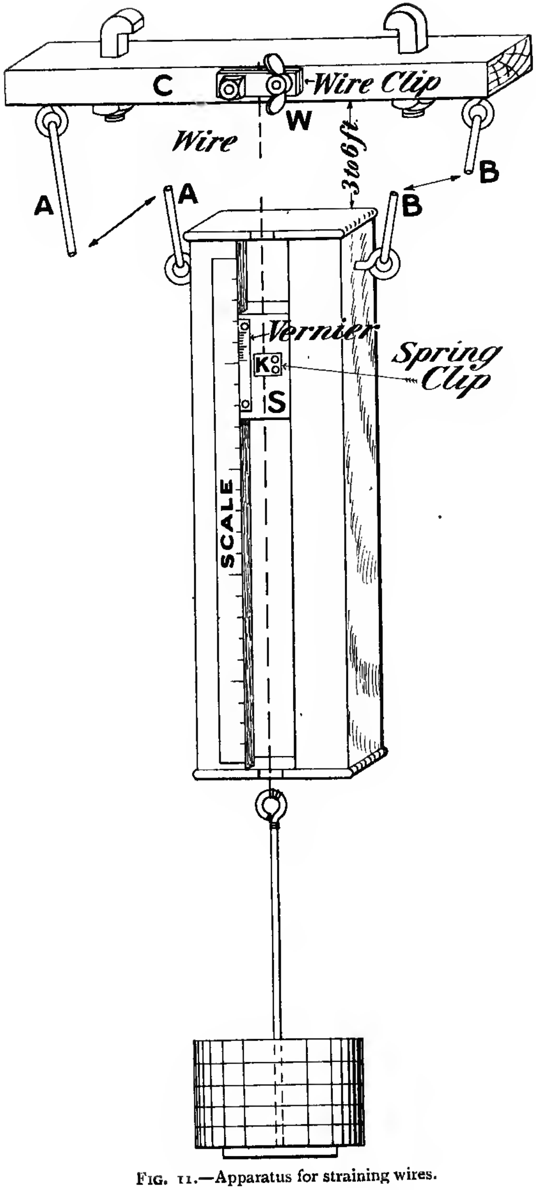


FIG. 11.—Apparatus for straining wires.

with only the load-pillar on the wire, and enter the observations in a table as below. Put a single weight *very carefully* on the load-pillar, and then read the scale again. Repeat this until the wire breaks. Sometime before this happens, the wire will stretch considerably and take some little time in doing it. It should be allowed to finish stretching before the next load is applied. A record of some observations is given below.

WIRE-STRAINING EXPERIMENT.

Date, March 15, 1902. Observer, W. Wade.
Material used—Copper wire.

Object of Experiment.—To determine the breaking stress, the stress at the elastic limit, the percentage elongation at breaking point, and Young's modulus.

Length of wire = 97·6 in.
Diameter = 0·036 in.
Sectional area of wire = 0·00102 sq. in.

Load.	Scale-reading.	Load.	Scale-reading.
lbs.	inches.	lbs.	inches.
0·0	0·15	15·0	1·33
2·0	0·17	17·0	2·17
3·5	0·18	19·0	3·21
5·0	0·195	21·0	4·39
7·0	0·21	23·5	6·4
8·0	0·22	25·5	8·0
9·0	0·23	27·5	10·05
10·5	0·24	29·5	12·64
12·0	0·35	31·5	16·0
14·0	0·91	34·5	23·4

Now plot the observations on squared paper, as shown in Fig. 12.

Between B and C the curve is straight, that is, the extension is proportional to the load, just as with the spiral spring; but beyond C the material behaves in an entirely different manner. The part BC is called the *elastic* portion of the curve, and CD the *plastic* portion. The point C, where the elastic part ends,

is called the limit of elasticity; the load at the elastic limit was 10·5 lbs., and the stress there—

$$= \frac{\text{load}}{\text{area}} = \frac{10·5}{0·00102} = 10{,}350 \text{ lbs. per square inch}$$

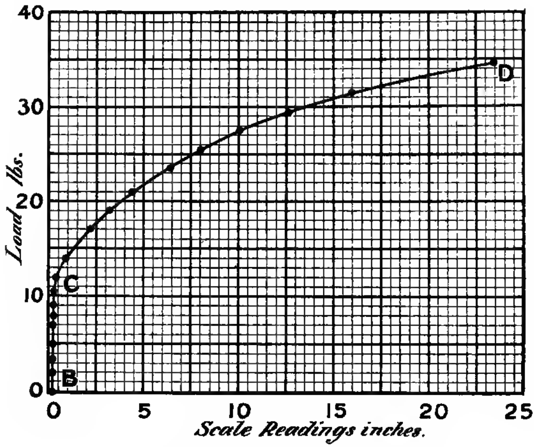


FIG. 12.—Load-extension curve, copper wire.

At the breaking-point the load was 34·5 lbs., and the stress was then—

$$\frac{\text{load}}{\text{area}} = \frac{34·5}{0·00102} = 33{,}800 \text{ lbs. per square inch}$$

The elongation at the breaking-point was 23·4 − 0·15 = 23·25 in., and the percentage this is of the original length—

$$= \frac{\text{extension}}{\text{length}} \times 100 = \frac{23·25 \times 100}{97·6} = 23·9$$

Young's modulus can best be found by replotting the observations in the elastic portion of the curve to a very greatly enlarged horizontal scale as in Fig. 13, the numbers along the base being hundredths of an inch. Take any two points, Q and R on the line, and get the slope—

$$\frac{RT}{TQ} = \frac{8 \text{ lbs.}}{0{\cdot}068 \text{ inches}} = \frac{\text{load}}{\text{extension}}$$

But—

$$\text{Young's modulus} = \frac{\text{stress}}{\text{strain}}$$

$$= \frac{\dfrac{\text{load}}{\text{area}}}{\dfrac{\text{extension}}{\text{length}}}$$

$$= \frac{\text{load}}{\text{extension}} \times \frac{\text{length}}{\text{area}}$$

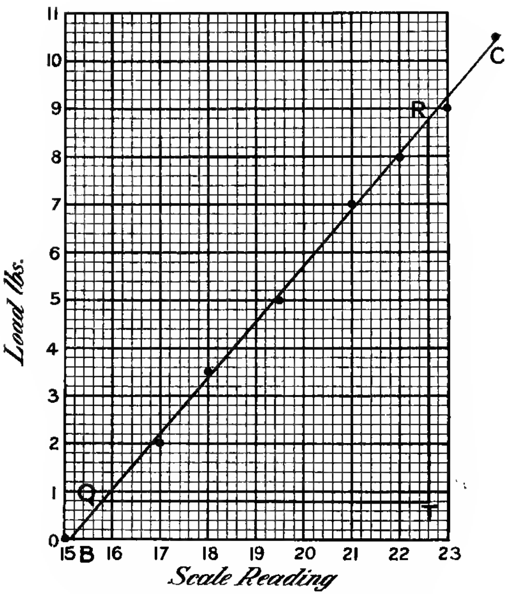


FIG. 13.—Load-extension curve below elastic limit.

Put in the value of the first fraction found above, together with the length and sectional area, and we get—

$$\frac{8}{0{\cdot}068} \times \frac{97{\cdot}6}{0{\cdot}00102} = 11{,}250{,}000 \text{ lbs. per square inch}$$

In the next figure a load-extension diagram is given for a piece of very soft annealed wrought iron. The peculiar

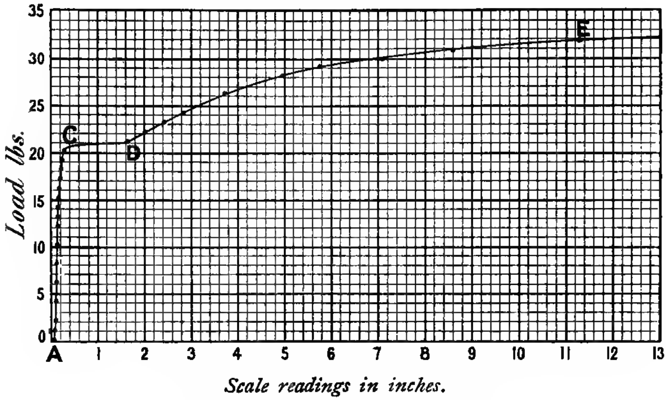


FIG. 14.—Load-extension curve, annealed wrought iron.

extension CD at the beginning of the plastic stage is generally to be met with in wrought iron, which is not brittle or hard. The elastic line is not very straight. The first three points represent a straightening of the wire in addition to stretching.

RECORD OF EXPERIMENT ON A PIECE OF CAST-STEEL WIRE.

Date, May 16, 1902. Observer, W. Wade.

Object of Experiment.—To determine the breaking stress, percentage elongation at fracture, stress at elastic limit and Young's modulus of elasticity.

Observations.

Initial length of wire = 6 ft. $1\frac{1}{2}$ in.
Mean diameter measured in five places = 0·707 mm. = 0·0278 in.
Mean sectional area of wire = 0·000604 sq. in.
Weight of load-pillar = 0·3 lb.

Load on wire.	Scale-reading.	Load on wire.	Scale-reading.
lbs.	inches.	lbs.	inches.
1·3	5·965	27·3	6·095
3·3	5·975	28·3	6·10
5·3	5·985	29·3	6·105
8·3	6·0	31·2	6·115
10·3	6·01	36·4	6·14
12·3	6·02	41·5	6·16
14·3	6·03	44·5	6·18
17·3	6·045	46·7	6·19
20·3	6·06	49·7	6·205
21·3	6·065	55·7	6·235
22·3	6·07	60·8	6·26
23·3	6·075	65·9	6·29
24·3	6·08	67·0	6·33
25·3	6·085	68·0	broke
26·3	6·09		

These observations have been plotted in Fig. 15.

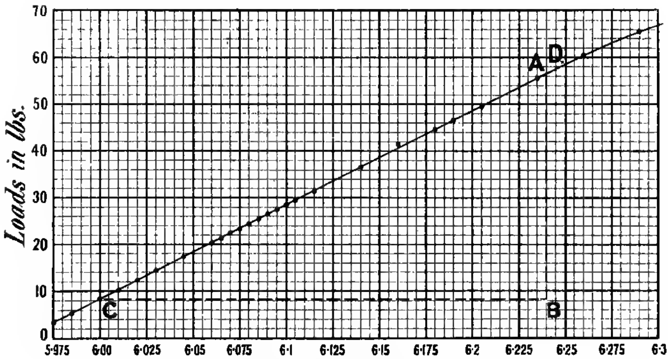


FIG. 15.—Load, extension curve, cast-steel wire.

Limit of elasticity is at D, where the load is 58 lbs. and the stress $= \dfrac{58}{0·000604} =$ 96,000 lbs. per square inch.

Breaking stress $= \dfrac{68}{0·000604} =$ 112,000 lbs. per square inch.

$$\left.\begin{array}{l}\text{Percentage extension}\\ \text{at fracture}\end{array}\right\} = \left(\frac{6{\cdot}37 - 5{\cdot}96}{73{\cdot}5}\right) \times 100$$
$$= \frac{0{\cdot}41}{73{\cdot}5} = 0{\cdot}56$$

This is obtained by taking points on the curve at no load and breaking load. These points are not shown in the diagram, Fig. 15.

The points A and C on the curve were selected for the purpose of determining Young's modulus. AB = 56·6 − 8·3 = 48·3 lbs.; CB = 6·24 − 6·0 = 0·24 inch.

$$\text{Young's Modulus} = \frac{\text{stress}}{\text{strain}} = \frac{\frac{\text{load}}{\text{area}}}{\frac{\text{extension}}{\text{length}}} = \frac{\frac{48{\cdot}3}{0{\cdot}000604}}{\frac{0{\cdot}24}{73{\cdot}5}}$$
$$= 24{,}500{,}000 \text{ lbs. per square inch.}$$

Results—

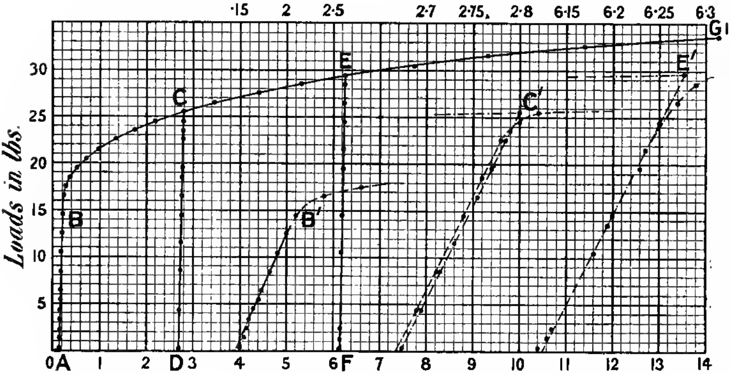
Breaking stress = 112,000 lbs. per square inch.
Stress at elastic limit = 96,000 lbs. per square inch.
Percentage elongation at fracture = 0·56
Young's modulus = 24,500,000 lbs. per square inch.

Another experiment should now be carried out on a piece of copper or iron wire; but at some little distance beyond the elastic limit, the loads should be removed one by one and the observations noted. This is shown in Fig. 16 between C and D. Now replace the loads one by one, taking readings as before. When the previous maximum load has been reached the wire goes on stretching rapidly, the curve being a continuation of ABC. After having stretched still more (up to E), remove the loads again, but this time two or three together. Replace them again a few at a time, and proceed with the experiment until the wire breaks at G.

The parts AB, CD, and EF are straight and nearly parallel, indicating that Young's modulus is approximately the same

whether the material has been previously stretched or not. But the elastic limit is raised permanently by straining the material beyond the primary limit (B).

When strained beyond the primary elastic limit, the material remains permanently stretched, or, as we say, it takes a permanent set. For example, after the load had been removed the first time the wire was permanently longer by the length



Scale-readings in inches.

FIG. 16.—Load, extension curves, soft-copper wire.

AD, and after the second removal of the load, it was permanently longer by the amount AF. The *elastic* lines AB, CD and EF, are shown by the dotted lines to a horizontal scale, twenty times as large as in the original figure, and from each of these Young's modulus has been calculated. The values obtained were 18,800,000, 16,600,000, and 16,300,000 lbs. per square inch.

The horizontal scales of these *dotted* elastic lines are shown at the top of the figure.

RECORD OF EXPERIMENT ON A PIECE OF SOFT-COPPER WIRE.

Date, May 10. Observer, W. Wade.

Object of Experiment.—To determine the breaking stress, stress at primitive elastic, percentage elongation at fracture, and Young's modulus, and to show that the elastic limit is permanently raised by straining beyond the primitive elastic limit.

Observations.

Initial length of wire = 67·6 in.
Mean initial diameter = 0·82 mm. = 0·032 in.
Mean final diameter = 0·77 mm.
Mean initial area of section = 0·00081 sq. in.
Weight of load-pillar = 0·3 lb.

Load.	Scale-readings in inches.				
	Loading.	Unloading.	Loading.	Unloading.	Loading.
lbs.					
0·3	0·15	2·675	—	6·12	6·12
1·3	0·155	—	—	—	6·13
2·3	0·157	—	—	—	6·135
3·3	0·16	—	—	—	—
4·3	0·165	2·695	2·69	—	—
5·4	0·17	—	—	—	—
6·4	0·175	—	—	—	—
8·4	0·18	2·715	2·715	6·195	—
10·5	0·19	—	—	—	6·18
11·5	—	2·73	—	—	—
12·5	0·20	—	—	—	—
14·6	0·21	—	2·74	—	6·20
16·6	0·24	2·755	—	—	—
17·6	0·28	—	—	—	—
18·6	0·36	—	2·76	—	—
19·6	0·51	2·77	—	6·23	—
20·6	0·72	—	—	—	—
21·6	0·99	—	—	—	6·235
22·6	1·34	2·75	—	—	—
23·6	1·75	—	2·79	—	—
24·6	2·20	—	2·8	6·25	—
25·6	2·80	2·8	2·82	—	—
26·6	—	—	3·45	—	6·27
27·6	—	—	4·4	—	—
28·6	—	—	5·3	—	6·29
29·6	—	—	6·26	—	6·32
30·6	—	—	—	6·28	7·75
31·6	—	—	—	—	9·30
32·6	—	—	—	—	11·4
33·6	—	—	—	—	14·3

Breaking stress $= \dfrac{33·6}{0·00081} =$ 41,500 lbs. per square inch.

Stress at primitive elastic limit $= \dfrac{12·5}{0·00081} =$ 15,400 lbs. per square inch.

Stress at second elastic limit $= \dfrac{22·5}{0·00081} =$ 27,800 lbs. per square inch.

Stress at third elastic limit $= \dfrac{22·5}{0·00081} =$ 27,800 lbs. per square inch.

Slopes of 1st, 2nd, and 3rd elastic lines respectively = 226, 200, and 196.

$$\frac{\text{Length}}{\text{sectional area}} = \frac{67 \cdot 6}{0 \cdot 00081} = 83,500$$

$$\text{Young's modulus} = \text{slope of elastic line} \times \frac{\text{length}}{\text{area}}$$

Substituting the above numbers, we get the following three values :—

1st, 18,800,000 ; 2nd, 16,600,000 ; and 3rd, 16,300,000 lbs. per square inch. Mean 17,200,000 lbs. per square inch.

NOTE.—There was a period of about ten minutes' rest between the unloading and reloading, represented by the second elastic lines.

Here the rest has allowed it to recover a very small piece of its plastic extension.

Example.—A steel tie-rod is 10 ft. long and $1\frac{1}{2}$ in. in diameter. How much will it stretch under a load of 18 tons if Young's modulus for that quality of steel is 30,000,000 lbs. per square inch?

$$\text{stress} = \frac{\text{load}}{\text{area}} = \frac{18 \times 2240 \text{ lbs.}}{\frac{\pi}{4} \times \frac{9}{4} \text{ square in.}}$$

$$= 22,850 \text{ lbs. per square inch.}$$[1]

$$\text{strain} = \frac{\text{extension}}{\text{length}} = \frac{\text{extension (in.)}}{120 \text{ (in.)}}$$

$$\text{and Young's modulus (E)} = \frac{\text{stress}}{\text{strain}}$$

$$= \frac{22,850}{\frac{\text{extension (in.)}}{120}}$$

But E = 30,000,000 lbs. per sq. in. Hence—

$$30,000,000 = \frac{22,850 \times 120}{\text{extension (in.)}}$$

$$\text{or extension} = \frac{22,850 \times 120}{30,000,000}$$

$$= 0 \cdot 0915 \text{ in.}$$

[1] This, and all other solutions, have been obtained by the aid of a pocket-calculator, which is equivalent in accuracy to a 10-in. slide-rule.

Example.—A square rod 12 ft. long and length of side $\frac{3}{4}$ in. stretches $\frac{3}{32}$ in. under a load of $4\frac{1}{2}$ tons suspended at its extremity. Determine the stress in the material and Young's modulus for the material.

$$\text{stress} = \frac{\text{load}}{\text{area}} = \frac{4{\cdot}5 \times 2240}{\frac{9}{16}}$$

$$= 18{,}000 \text{ lbs. per square inch}$$

$$\text{strain} = \frac{\text{extension}}{\text{length}} = \frac{\frac{3}{32}}{144}$$

$$= 0{\cdot}0006$$

$$\text{Young's modulus (E)} = \frac{\text{stress}}{\text{strain}} = \frac{18{,}000}{0{\cdot}0006}$$

$$= 30{,}000{,}000 \text{ lbs. per square inch}$$

Example.—A steel tie-bar is used to draw together two walls of a building which have bulged. It is 2 inches in diameter, it is placed in position, and the nuts tightened with the bar at a temperature of 120° F. What pull will it exert to draw the walls together when it cools down to 50° F. Young's modulus for the material is 12000 tons per square inch, and the linear co-efficient of expansion of steel is 0·000007.

Let F = force in tons exerted by cooled rod. Then—

$$\text{stress} = \frac{\text{force}}{\text{area}} = \frac{F}{\frac{\pi}{4} \times 4} \text{ tons per square inch}$$

The extension of the bar due to the given rise of temperature of 70° F = 70 × 0·000007 × length. Then—

$$\text{strain} = \frac{\text{extension}}{\text{length}}$$

$$= \frac{70 \times 0{\cdot}000007 \times \text{length}}{\text{length}}$$

$$= 0{\cdot}00049$$

$$12{,}000 = E = \frac{\text{stress}}{\text{strain}} = \frac{\frac{F}{\pi}}{0{\cdot}00049}$$

$$\text{therefore } F = 12{,}000 \times 0{\cdot}00049 \times \pi$$

$$= 18{\cdot}7 \text{ tons.}$$

Experiment on String.—It is interesting to subject other materials to the same treatment as described in the last experiment. A piece of hard string, very much like whipcord in appearance, was loaded like the wire, with the result indicated by the curve in Fig. 17. The diameter of the cord was 0·055 in., and the length 5 ft. 6 in.

The curve indicates that there is scarcely any *elastic* stage, as with the copper wire, and certainly there is no plastic stage,

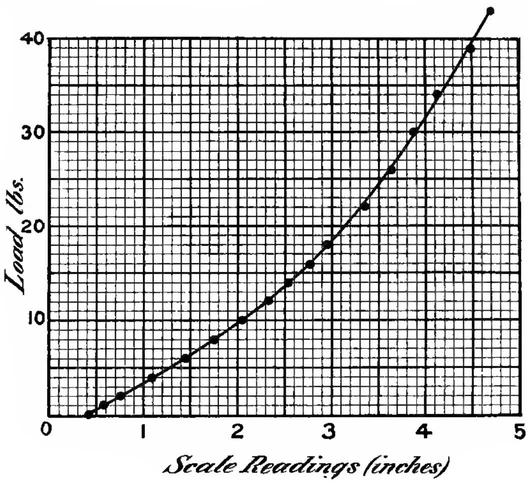


FIG. 17.—Load, extension curve, hard string.

as the curve turns *upwards* and not downwards, as is the case with the wire. Percentage elongation $= \frac{4\cdot3}{65} \times 100 = 6\cdot6$.

Breaking stress $= \frac{43}{0\cdot785 \times 0\cdot055^2} = 18{,}200$ lbs. per square inch.

Professor Barr's Autographic Wire-straining Apparatus, as made by Mr. Cussons, is shown in Fig. 18, while a detail of the recording arrangement is shown in the next figure. The wire to be strained is W, secured in a clip at C, the load being gradually applied by allowing fine shot to pass from the vessel R, through T, into the load-can V. The weight of shot

in R is proportional to the extension of the spring S. A fine thread, C, shown dotted, is attached to the end of the spring, and passes round a small pulley, round the drum D, and over another pulley to the small weight B. As the end of the spring moves, the thread C compels the drum D to rotate and thus record the increase of load in V. The pen or pencil at P is coupled to the wire W by a light spring clip, and it slides along the rods Q. In this way the extension of the wire is recorded on paper on the drum D.

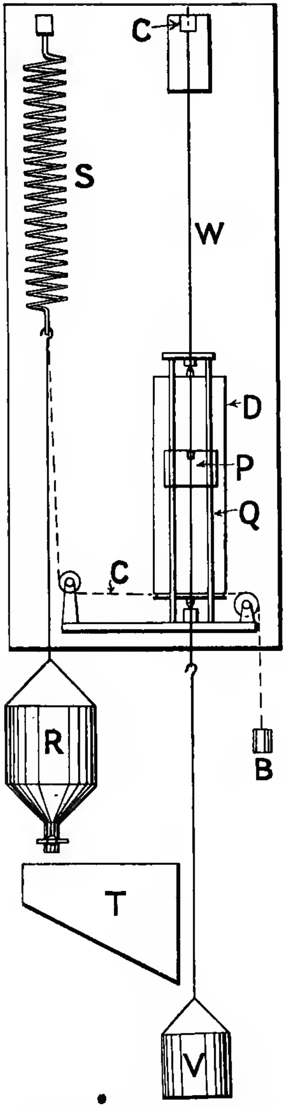


FIG. 18.—Professor Barr's apparatus for automatically recording the load —extension curve.

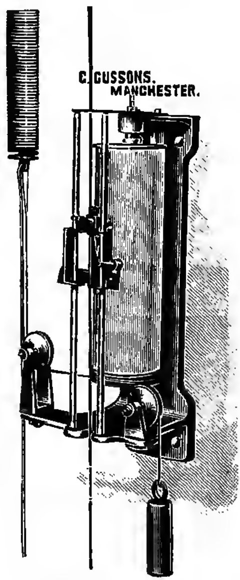


FIG. 19.—Details of pen attachment. Barr's recorder.

Friction.

The Resistance of Friction is most easily found in the following manner :—

A stiff plank, A, Fig. 20, with a fairly smooth upper surface is supported on a couple of legs or on a table. On the plank

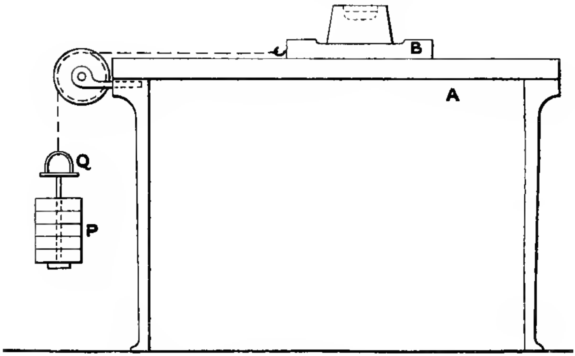


Fig. 20.—Apparatus for experiments on friction.

rests a slider, B, connected to a scale-pan or load-pillar by a strong string.

The load-pillar is weighed, as is also the slider, B.

A weight of say 7 lbs. is placed upon the middle of the slider, and smaller weights are placed on the load-pillar at P. These weights may be conveniently 1 lb. each. Smaller weights of $\frac{1}{10}$ lb., $\frac{2}{10}$ lb., etc., may be placed at Q.

It will be found at the beginning of the experiment that more weights are required on the load-pillar to start the slider than are required to keep the slider moving slowly along the plank. This suggests that there are either two different frictions, one of rest and the other of motion ; or that, while at rest, the two surfaces in contact appear to be able to adhere to one another more closely than while in motion. Both of these frictions must be experimented upon, but we will first deal with the friction of the surfaces while they are in motion, called *sliding friction.*

As the slider will not start with a force sufficient to *keep it moving*, it must be started by hand. This may be done by tapping the plank, A, with the knuckles or by pushing the slider gently with the hand. The latter method is generally preferable, but great care should be taken not to start the slider with a rush. To prevent this, it is better to rest the hand firmly on the plank A, and then gently push the slider. This prevents the hand from imparting any appreciable impetus to the slider.

The weight on the load-pillar must be carefully adjusted so that the slider moves *very slowly* and at a uniform rate along the plank A.

Now add another weight to the slider, and repeat with six or eight different weights. Tabulate the observations as below.

EXPERIMENT ON SLIDING FRICTION.

Date, May 10, 1902. Observer, W. Wade.

Object of Experiment.—To determine the relation between the resistance of friction and the pressure (between the surfaces in contact) which produces friction; when the surfaces are in actual uniform motion.

Weight of slider = 4·4 lbs.
Weight of load-pillar = 0·7 lbs.
Kind of material = oak.

Pressure between surfaces, producing friction.	Resistance of friction = force parallel to surfaces.
lbs.	lbs.
11·4	1·8
18·4	2·7
22·4	3·5
29·4	4·3
39·4	5·7
50·4	7·3
60·4	8·7
74·4	10·8
88·4	12·8
118·4	17·2

Result—Friction = 0·145 × pressure between surfaces.

Now plot the pressure between the surfaces along the base (Fig. 21), and the resistance of friction vertically upwards. We find the points lie on a straight line through the origin whose equation [1] is—

Resistance of Friction $= 0·145 \times$ pressure between surfaces.

The number 0·145 is called the *co-efficient of sliding friction.*

Static Friction, or the friction between surfaces at rest can be determined in the same manner, the only difference being that the slider is not started by hand. As the slider now starts off with a rush, it is necessary to prevent damage

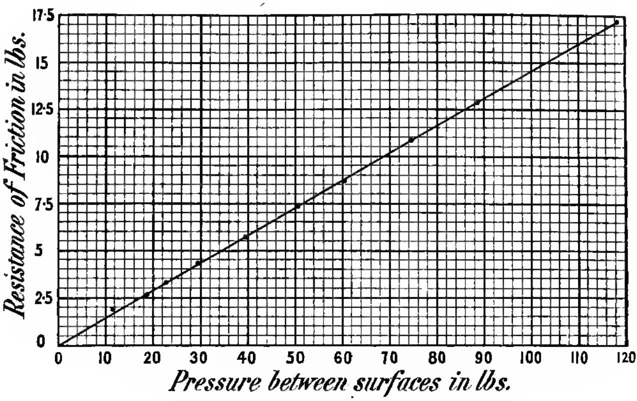


FIG. 21.

and noise to stop it immediately after it has started. This can be done by placing the fingers and thumb of one hand astride of the cord about an inch in front of the slider and pressing firmly against the plank A. This will stop the slider immediately after starting, if the load is not great.

The results are more or less irregular, and it is not an easy matter to carry out the experiment properly. The principal result to be noted is that the friction between surfaces at rest is considerably greater than between the same surfaces in motion.

[1] See Appendix for method of obtaining the equation to a straight line.

These experiments on friction indicate that the force or pressure producing friction acts perpendicular to the surfaces in contact, and that the resistance of friction acts along or parallel to those surfaces, and in a direction opposite to that in which motion takes place.

A further examination of the nature of friction may be carried out in the following way.

A heavy slider, A (Fig. 22), or a slider having a weight K attached to it, is supported by a spring-balance, S. The pressure between the surface of the slider A and the plank P may be produced by a weight E applied at the end D of a bent lever, DBC. At C a roller turns freely on a pin fixed in the lever, and presses A against the plank P. If the arms DB and BC are of equal length, the pressure of the roller on the slider will be equal to the weight E. Let the slider be moved slowly upwards over the plank by pulling steadily on the spring-balance. This is not easily accomplished by hand, but if a cord be attached to a spring-balance, and the other end passed round a drum or axle, which is driven through a worm and worm-wheel or other mechanism for reducing speed, it can be easily done, or, as in the figure, a fly-wheel can be attached to the spindle, the rim of which can be easily moved by hand.

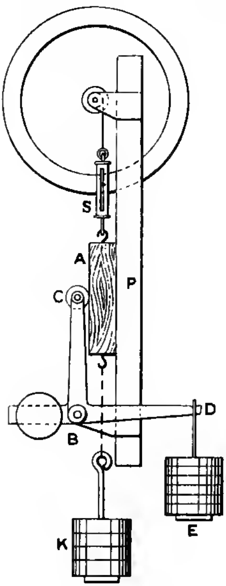


FIG. 22.—Friction apparatus.

The force indicated by the balance is greater than the weight of the slider plus the weight K, the excess of force being that required to overcome friction.

Let W = weight of slider + K;

$$\text{then spring-balance indication} = W + \text{friction}.$$

Now let the slider move downwards. Friction always acts

against motion, and hence its direction is now upwards, thus relieving the spring-balance of some of the weight on its hook.

Then spring-balance indication = W − friction.

Repeat the observations with different weights at D. Then plot the spring-balance indications vertically as in Fig. 23, the corresponding values of E being plotted horizontally. Thus MN represents the spring - balance indication when the weight E is given by ON and the motion is upward. NR represents W, and consequently RM represents the resistance of friction to the same scale. During the downward motion NQ was the balance indication and RQ the friction. RQ = RM, and consequently QM must be twice the resistance of friction. Also if a line RL be drawn bisecting the space between LM and LQ, the ordinate to this line, such as NR, must represent the balance indication when there is no friction. This will be found useful in the next article, and later in the consideration of the friction of some of the machines.

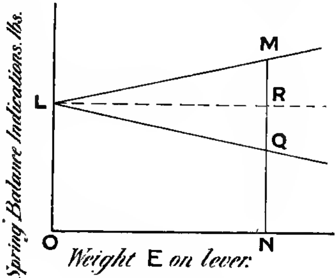


FIG. 23.

It gives a means of measuring both the friction of a contrivance and of the result without friction. There are very few experiments or operations which can be carried out without friction coming into play somewhere or other, and then, if it is appreciable, it must be accounted for and measured.

The Determination of Young's Modulus with a greater degree of accuracy than in the experiment previously described, can with advantage be dealt with here. In Figs. 12 and 14 the difficulty experienced in measuring extensions within the elastic limit was on account of their smallness. To render these more easily measured, they must be greater in magnitude, or more delicate apparatus must be employed for the purpose of measurement. The former method is the one

adopted here. Instead of a wire from 4 to 5 feet long, one from 40 to 100 feet in length is used (Fig. 24). It is made fast at one end to some fixed and rigid material, such as an iron beam in the ceiling of the laboratory, in a manner similar to that shown in Fig. 11, or to a bolt cemented in the wall. The wire is taken horizontally as far as possible and then over a pulley, P, to a slider, S, fitted with a scale and vernier similar to that in Fig. 11. The wire, if over 40 feet in length, should be supported on a couple of pegs driven into the wall at two intermediate positions. This prevents sagging.

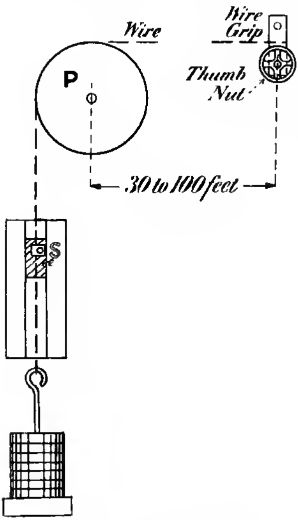


FIG. 24.—Apparatus for determining Young's modulus.

A weight of a few pounds is permanently fastened to the end of the wire, so that it is always ready for use. The load applied is never sufficient to stretch the wire beyond the elastic limit.

The stretching weights are applied with extreme care, so that there shall be no chance of a jerk or shock, and they must be removed with equal care. The following is a record of an experiment.

EXPERIMENT ON A LONG WIRE.

Date, March 12, 1902. Observer, W. Wade.

Object to determine Young's Modulus.
Material, cast steel.
Length of wire = 98 ft.
Diameter = 0·525 mm. = 0·027 in.
Sectional area = 0·000345 sq. in.

Load.	Scale-readings.		Load.	Scale-readings.	
	Loading.	Unloading.		Loading.	Unloading.
lbs.	inches.	inches.	lbs.	inches.	inches.
0	2·94	3·02	10	4·10	4·25
1	3·06	3·15	11	4·22	4·37
2	3·17	3·27	12	4·33	4·48
3	3·28	3·39	13	4·44	4·60
4	3·40	3·51	14	4·56	4·72
5	3·52	3·64	15	4·68	4·83
6	3·63	3·77	16	4·80	4·94
7	3·74	3·88	17	4·92	5·05
8	3·86	4·00	18	5·03	—
9	3·98	4·13			

Now plot the observations as in Fig. 25, the loads being measured upwards and the scale readings horizontally. There will be two lines because there are two sets of observations, one for loading and the other for unloading.

We have seen in Fig. 23 that the vertical intercept between these two lines represents twice the friction of the apparatus, hence draw the dotted line CA, bisecting all the intercepts. Then the abscissa of any point on this line represents the scale-reading which would have been obtained with the corresponding load if there had been no friction.

Evidently this is what we require, hence treat the dotted line CA (Fig. 25) in the same manner that the line BC was treated in Fig. 13. Its slope

$$= \frac{AB}{BC} = \frac{13\cdot5 \text{ lbs. of load}}{1\cdot6 \text{ in. of extension}}$$

Substituting this in the expression for E, we get—

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\frac{\text{load}}{\text{area}}}{\frac{\text{extension}}{\text{length}}} = \frac{\text{load} \times \text{length}}{\text{extension} \times \text{area}}$$

$$= \frac{13\cdot5 \times 98 \times 12}{1\cdot6 \times 0\cdot000345} = 28{,}800{,}000 \text{ lbs. per square inch}$$

The Friction of a Simple Pulley turning on its spindle can be found by suspending weights on the ends of a cord

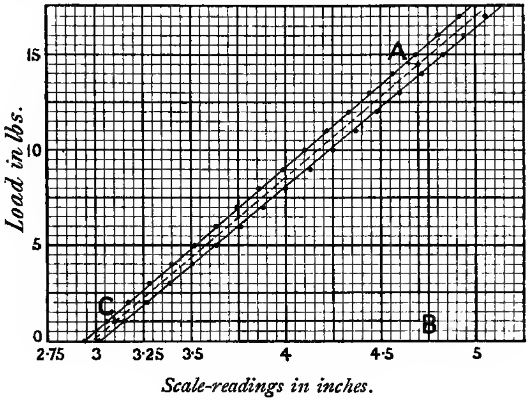


FIG. 25.

passed round the pulley, as shown in Fig. 26. The excess of weight on one end over that on the other represents the amount of friction as measured at the circumference of the pulley.

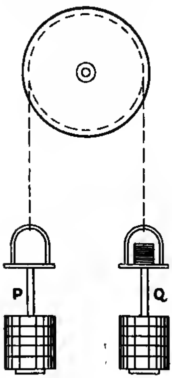


FIG. 26.

A certain weight, say 5 lbs., is placed on each end of the cord, and then further small weights are added to those on one end, until that end moves *slowly* downwards after having been given a start in the same manner as described in the experiment on friction some pages previously. Let P be the total weight on one end of the cord and Q the total weight on the other: then $Q - P$ is the amount of friction measured at the surface of the pulley, and $P + Q +$ weight of pulley is the pressure on the spindle which produces friction.

Use a number of different weights and tabulate the results as below. The actual resistance of friction tending to prevent rotation occurs at the surface of the *spindle*, and not at the

surface of the pulley, and equals that measured at the surface of the pulley multiplied by $\frac{D}{d}$

$$\text{Where P} = \text{diameter of pulley}$$
$$\text{and } d = \quad " \quad " \text{ spindle.}$$

EXPERIMENT ON THE FRICTION OF A SIMPLE PULLEY.

Date, May 1, 1902. Observer, C. S. Scott.

Object of Experiment.—To determine the relation between the friction of a pulley on its spindle and the total load on the pulley producing friction.

Diameter of pulley = 7·5 in. = D

Diameter of spindle = 0·6 in. = d

Ratio $\frac{D}{d} = 12·5$

Weight of each load-pillar = 1 lb.

Observations.

P.	Q.	P + Q.	Q − P.	$\frac{P}{Q}$.
1	1·1	2·1	0·1	0·91
2	2·15	4·15	0·15	0·93
3	3·2	6·2	0·2	0·94
4	4·25	8·25	0·25	0·94
5	5·3	10·3	0·3	0·942
6	6·35	12·35	0·35	0·944
7	7·4	14·4	0·4	0·945
8	8·45	16·45	0·45	0·948
9	9·5	18·5	0·5	0·95
10	10·55	20·55	0·55	0·95
12	12·65	24·65	0·65	0·95
14	14·75	28·75	0·75	0·95

Plot the values of P + Q horizontally, and those of Q − P vertically, and we get the straight line AB (Fig. 27) whose equation is—

$$Q - P = 0·05 + 0·0244\,(Q + P)$$

or—

$$\left.\begin{array}{l}\text{Resistance of friction measured}\\ \quad\text{at periphery of pulley}\end{array}\right\} = 0{\cdot}05 + 0{\cdot}244\,(Q + P)$$

and—

$$\left.\begin{array}{l}\text{Resistance of friction at surface}\\ \quad\text{of spindle}\end{array}\right\} \begin{array}{l}= \text{resistance measured at}\\ \quad\text{periphery} \times \dfrac{D}{d}\end{array}$$

$$= 12{\cdot}5\,[0{\cdot}05 + 0{\cdot}0244\,(Q + P)]$$
$$= 0{\cdot}625 + 0{\cdot}305\,(Q + P)$$

The number 0·305 is the coefficient of friction; but the total load on the spindle = (P + Q + weight of pulley); and the

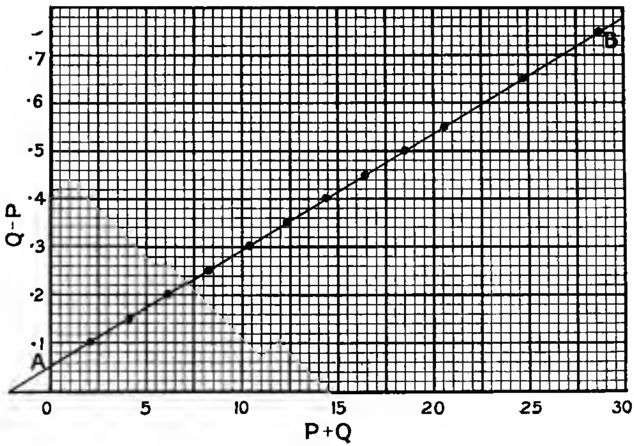


FIG. 27.

resistance of friction at surface of spindle = coefficient of friction × total load = 0·305 (P + Q + weight of pulley). This same resistance of friction also = 0·625 + 0·305 (Q + P). Equating these equals we get—

$$0{\cdot}305 + \text{weight of pulley} = 0{\cdot}625$$

$$\text{and weight of pulley} = \frac{0{\cdot}625}{0{\cdot}305} = 2{\cdot}05 \text{ lbs.}$$

If the line AB, Fig. 27, is produced backwards to meet the base line, it will do so at a point 2·05 to the left of zero. This indicates that if the weight of the pulley could have been ascertained by weighing and added on to P + Q in column three of the table, then the line AB would have been shifted to the right, parallel to itself, a distance 2·05 lbs. on the horizontal scale, and the line AB would then have gone through the origin like the line in Fig. 21.

Example.—A horse pulls a cart which weighs 15 cwt. and is loaded with 1 ton. If the co-efficient of friction at the surface of the axle is 0·2, and the diameters of the axle and cart-wheel are 2 ins. and 4 ft. respectively, what force does the horse exert on the level in hauling the cart; there being no other resistance than the friction of the axle?

Total weight producing friction = 1·75 tons = 1·75 × 2240 lbs.
Resistance of friction at surface of axle = 1·75 × 2240 × 0·2 lbs.
Resistance at the surface of the wheel—

$$= \frac{d}{D} \times 1{\cdot}75 \times 224 \times 2$$

$$= \frac{2}{48} \times 1{\cdot}75 \times 224 \times 2$$

$$= 32{\cdot}6 \text{ lbs.}$$

In the actual case, there is a considerable amount of resistance at the surface of the road in addition to the above. This resistance is of such a nature that we are not able at this stage to investigate its nature and amount. If there had been no wheels to the cart and the coefficient of friction between the cart frame and road were still 0·2; what would be the necessary force to haul the cart and load on the level?

Total load producing friction = 1·75 × 2240 lbs.

Resistance of friction = co-efficient × load.

$$= 0{\cdot}2 \times 1{\cdot}75 \times 2240$$

$$= 787 \text{ lbs.}$$

We are now able to see the use of wheels in reducing the force necessary to haul loads over a rough surface.

Summary of Chapter I.

1. The *extension* of a helical spring is proportional to the load producing extension.

2. *Stress* is the force or load per unit of area. *Strain* is the extension per unit of length.

$$\textit{Young's Modulus of Elasticity} = \frac{\text{stress}}{\text{strain}}$$

3. The Resistance of *Friction* = a coefficient × total perpendicular pressure between the surfaces in contact.

Examples on Chapter I.

1. An iron rod, of 1 in. diameter and 12 ft. in length, stretches $\frac{3}{32}$ in. under a load of 6 tons suspended at its extremity. Determine the stress, and modulus of elasticity of the bar.

Ans. Stress = 7·6 tons per square inch. Young's modulus 11,800 tons per square inch.

2. What do we mean by stress, strain, and modulus of elasticity?

A wire 10 ft. long and $\frac{1}{8}$ sq. in. in sectional area, is hung vertically, and a load of 450 lbs. is attached to its lower extremity, when the wire stretches 0·015 in. in length. What are the stress and strain respectively? And also the modulus of elasticity? *Ans.* 28,800,000 lbs. per square inch.

3. Define the following :—Stress, strain, Young's modulus.

A wire 20 ft. long stretches $\frac{1}{4}$ in. under a load of 9 lbs. Young's modulus 30,000,000 lbs. per square inch. Find the diameter of the wire.

Ans. 0·01914 in.

4. How would you find out for yourself the behaviour of steel wire loaded in tension till it breaks? What occurs in the material? Use the words stress and strain in their exact senses.

5. What do you understand by stress and strain respectively? If an iron rod, 50 ft. long, is lengthened by $\frac{1}{2}$ in. under the influence of a stress, what is the strain? *Ans.* $\frac{1}{1200}$.

6. A steel tie-bar is used to draw together two walls of a building which have bulged. It is 2 in. in diameter; it is placed in position, and the nuts tightened with the bar at a temperature of 120° F.: what pull will it exert to draw the walls together when it cools down to 60° F.? Assume the modulus of elasticity = 12,000 tons per square inch, and that the coefficient of linear expansion per degree F. for steel is 0·000007. *Ans.* 15·84 tons.

7. The connecting-rod of an engine is to have a working stress of 4000 lbs. per square inch, both in tension and compression, and it is required to

transmit a load of 14,950 lbs., its length is 75 in.; what diameter must it be? And how much will it change its length during working? E = 30,000,000 lbs. per square inch.

Ans. Diameter 2·182 in.; change of length 0·02 in.

8. The length of a rod of iron is 15 ft. and Young's modulus of the iron is 24,000,000 lbs. per square inch; what is the diameter of the rod if a load of 5·5 tons produces an extension of 0·121 in.

Ans. Diameter = 0·99 in.

9. A mild-steel tie-rod in a roof is 12 ft. 6 in. long, it is $1\frac{3}{4}$ in. in diameter, and carries a load of 13·6 tons. What is the stress per square inch in lbs., and how much will the rod elongate under this load?

10. Describe carefully how you would carry out a tensile test of a piece of mild steel, so as to be able to draw a stress-strain curve (*a*) for nominal stresses per square inch of original area, (*b*) for the actual stresses on the actual cross-section at each instant.

11. The connecting-rod of a steam-engine has the following dimensions: Length 50 in.; diameter $2\frac{1}{2}$ in. The diameter of the cylinder is 10 in., and the steam pressure 90 lbs. per square inch. How much will the rod alter its length during one revolution? Assume modulus of elasticity 12,000 tons per square inch, and neglect effect of obliquity of the rod.

Ans. 0·0054 in.

12. The diameter of the cylinder of a steam-engine is 9 in., and the steam pressure 60 lbs. per square inch. What must be the diameter of the piston-rod if the tensile stress in the body of the rod is not to exceed 3000 lbs. per square inch? *Ans.* Diameter 1·26 in.

13. Explain carefully, with the aid of a diagram, the behaviour of a piece of good wrought-iron when tested in tension to destruction by gradually increasing the load. Show how you could obtain the value of the modulus of elasticity of the material from the experiment.

14. Define stress and strain. A rod of metal is $\frac{1}{4}$-in. square in section. The modulus of elasticity of the material is 20,000,000 lbs. per square inch, and when the rod is loaded with 800 lbs., the extension is 0·072 in. The elastic limit is not reached. Calculate the original length of the rod.

15. During a test of a piece of $\frac{3}{4}$-in. hard round steel bar in compression it is found that a length of 20 in. is shortened by 0·007 in. under a total load of 2 tons. What is the squeeze modulus of elasticity in lbs. per square inch for this quality of steel?

NOTE.—Squeeze modulus is simply Young's modulus.

16. How would you experimentally determine the nature of the friction between clean, smooth surfaces, say of oak, and what sort of law would you expect to find?

17. A locomotive weighs 54 tons, of which 40 tons are upon the coupled driving wheels. Supposing the co-efficient of friction between wheel and rail to be 0·27. What is the maximum load of the train which it can haul? Assuming the required tractive force to be 20 lbs. per ton on the level.

18. The load upon a spindle is 15 tons, its diameter being 8 inches. What force applied tangentially to the circumference of a wheel 6 ft. diameter fixed on the spindle would be required to overcome the resistance of friction of the bearing; the coefficient of friction being 0·04?

19. The following observations were taken in a friction experiment. Plot them on squared paper and deduce, as accurately as you can, an equation which will represent the relation between friction and the pressure between surfaces in contact.

Friction in lbs.	0·85	1·4	2·25	2·7	4·1	5·6	6·1	7·2	10	10·8
Pressure between surfaces in lbs.	4	7	11	14	21	28	30	36	50	54

20. A wire, 55 in. long and 0·023 in. diameter, is strained with the results given below—

Load (lbs.)	1	3	6	8	9	10	11	12	14
Scale-reading in centimetres	17·17	17·23	17·31	17·37	17·45	19·85	22·9	26·94	44

Calculate Young's modulus and the stress at the last observation.

CHAPTER II.

REPRESENTATION OF FORCES BY LINES.

A force (push or pull) can be completely located and described when its amount or *magnitude* is known, when its position in space or *line of action* is known, and when its *direction* along that line is known.

These three attributes of a force can be completely represented by a straight line; for the magnitude of the force can be represented (to some previously selected scale) by the length of the line; the position or line of action of the force is represented by the position of the line; and the direction of the force by the arrow-head placed on the line. Thus the

Scale—10 lbs. to 1 inch.
FIG. 28.

length of the line AB (2·7 in.) represents a force of 27 lbs. to a scale of 10 lbs. to 1 inch in the horizontal position acting from A to B.

If forces can be represented by lines, then forces must obey the laws of lines, and consequently can be treated in every way just as lines can.

By way of introduction, we will take the following simple problem:—A man walks 2 miles due north, then $2\frac{3}{4}$ miles due east, then 4 miles south-east, and finally 2 miles south-west. How far is he then away from home, or, in other words, what is the net result of his walk, measured from the starting-point?

Select some scale, such as 2 miles to the inch. Also select a starting-point, S (Fig. 29). From S draw ST *in the direction*

in which the man walked, and of such a length that it represents 2 miles to a scale of 2 miles to 1 inch. Similarly from T draw TU to represent $2\frac{3}{4}$ miles in the direction due east. Then from U draw UV to represent 4 miles south-east, and finally VF to represent 2 miles south-west. The man is now a distance represented by SF = 4·75 miles away from home.

In other words, the final *result* (as regards distance from

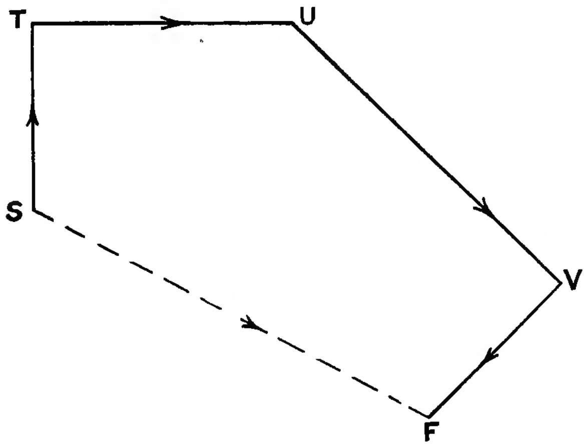


FIG. 29.

the starting-point) is just the same, whether he walks directly from S to F along SF or from S to F along STUVF. The line SF is called the *resultant*, because it represents the *net result* of the man's walk.

It must be noted that whichever path is taken by which to arrive at F (the finishing-point), **the sense of direction as indicated by the arrow-heads** is from S (the starting-point) towards F. **Thus the resultant has the same sense of direction as the components, namely, from the starting-point towards the finishing-point.**

It must also be noticed that the component lines ST, TU, etc., were drawn parallel to the directions in which the man walked.

When we come to represent forces by lines, we shall proceed in exactly the same manner as above, where distances were represented by lines; because we may treat forces in exactly the same manner as straight lines, on account of forces being represented by lines.

Example.—Forces A = 17 lbs., B = 13 lbs., C = 22 lbs., and D = 9 lbs. act together upon a body in the directions shown in Fig. 30. Find their resultant force. (Note that the lines in Fig. 30 only indicate the directions of the forces, and do not represent them in magnitude.)

Proceed in exactly the same manner as in the last problem.

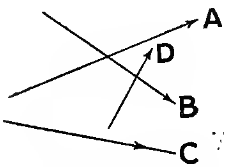


FIG. 30.—Position diagram.

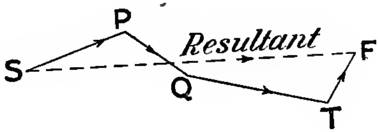


FIG. 31.—Force diagram.

Choose your scale—say 20 lbs. to 1 in. Select any starting-point, S, Fig. 31, and draw from S a line parallel to either of the forces; say SP parallel to A.[1] Mark off SP a length to represent A = 17

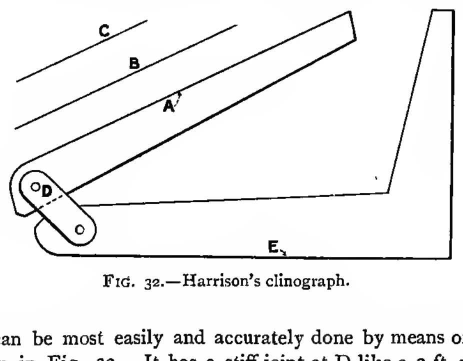


FIG. 32.—Harrison's clinograph.

[1] This can be most easily and accurately done by means of the clinograph shown in Fig. 32. It has a stiff joint at D like a 2-ft. rule, which retains it in any position after being set. It is used with the edge E in contact with the edge of a T-square on a drawing-board. By sliding the

lbs. to the scale of 20 lbs. to 1 inch, and put on it the arrow-head similar to A. Then from P draw the line PQ parallel to and in the same direction as B, and of a length representing B = 13 lbs. to the given scale, at the same time adding its arrow similar to B. From Q draw QT, representing C, and then TF, representing D, to the same scale as before. The resultant is represented by the line SF joining the *starting*- to the *finishing*-point, and its direction is indicated by the arrow-head—**from starting to finishing point.**

We can now write down the general method of finding the resultant of any number of forces in the same plane.

(1) Choose a scale of forces, and write it on your paper near to where the force diagram is to be drawn. (Some multiple of 10 lbs. or tons to the inch will be found most convenient.)

(2) If the position diagram is not given, draw it upon your paper.

(3) Choose a starting-point, S, and from it draw a line parallel to (and in the direction of the arrow on) any one of the forces, and mark off from S the length of the line to represent the given force to the above scale. Put the arrow on the line just drawn.

From the end of this line, remote from S, draw a line parallel to (and in the direction of) one of the remaining forces, and mark off its length to represent the given force to the scale given above. Repeat this construction for all the forces given.

(4) The resultant is represented by the line drawn from the starting-point S to the finishing-point, and its direction is always *from* the starting-point.

(5) It is useful to write near each line the number denoting the magnitude of the force it represents. It may also be convenient to distinguish the resultant by a dotted line or some other artifice.

A number of examples should now be worked by the student before passing on to the next few paragraphs. These will be found at the end of this chapter.

clinograph along the edge of the T-square a number of parallel lines, such as C and B, can be drawn parallel to the edge A.

Example.—The resultant of the forces (Fig. 33) A = 35 lbs., B = 21 lbs., C = 29 lbs., D = 18 lbs., X lbs. and Y lbs. is R = 24 lbs. Find the magnitudes and sense of direction of X and Y.

Method.—As directed above, select a starting-point S (Fig. 34), and draw the lines *a*, *b*, *c*, and *d* to represent the forces A, B, C, and D in Fig. 33. Also draw *r* to represent the resultant R. This, as in the previous figures, must be drawn *from S*.

There are two more lines required to complete the force diagram, and these lines must be parallel to X and Y, and fit in between the

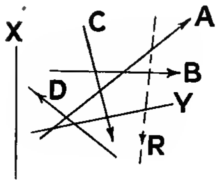


FIG. 33.—Position diagram.

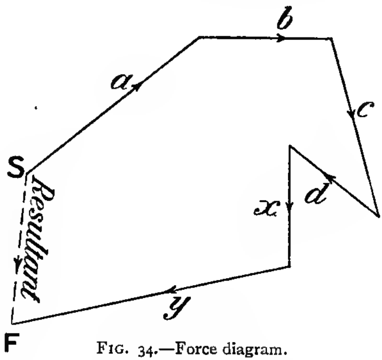


FIG. 34.—Force diagram.

ends of *d* and *r*. Hence through the end of *d* draw a line parallel to X, and through the end of *r* draw a line parallel to Y. These lines will intersect, if produced far enough, and their lengths *x* and *y* will represent the forces X and Y. The arrows on *x* and *y* must follow round the figure in the same direction as those on *a*, *b*, *c*, and *d*.

The student should draw these worked-out examples to a large scale on paper, as directed by the method given above. This will familiarize him with the method quicker than anything else.

Example.—Four forces A = 16 lbs., B = 13 lbs., C = 15·7 lbs., and D = 14·7 lbs. together produce a resultant R = 23 lbs., as shown in Fig. 35. Find the inclination of C and D to the horizon.

Method.—As in the previous example, draw from a starting-point S (Fig. 36), the lines SQ, and QP representing in magnitude

and direction the forces A and B. Then put in the resultant SF. The other two lines must begin at P and terminate at F. Their lengths represent 15·7 lbs. and 14·7 lbs. respectively. Hence with P as centre and radius representing 15·7 lbs. describe an arc. Similarly with F as centre and radius representing 14·7 lbs. describe an arc cutting the other arc in T and T¹. Join PT and FT, and put in the arrows from P towards F. Then PT and TF

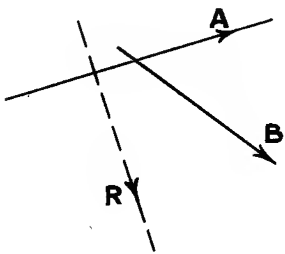


FIG. 35.—Position diagram.

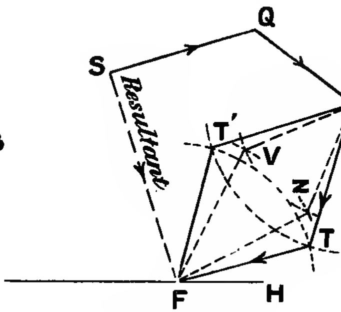


FIG. 36.—Force diagram.

represent in magnitude and direction the forces C and D. Their respective inclinations to the horizon (FH) are 73° and 15°.

It should be noted that these are not the only possible lines that can be drawn under the given conditions. PT¹ and T¹F are also possible positions.

We can also draw two other sets of lines (shown dotted) with arcs described from opposite points, with intersections at V and Z. Thus there are four possible positions for a pair of component forces of 14·7 and 15·7 lbs. to satisfy the given conditions, and these pairs of forces are inclined to the horizon at 75° and 18°, and 27° and 67°.

NOTE.—The scale to which these figures should be drawn is very much larger than that of the illustrations given here.

RESOLUTION OF FORCES.

In the previous section we have shown how the resultant of two or more forces can be found. We are now going to approach the opposite problem, namely, given the resultant to

find two component forces, which, if they acted along given lines, would produce that resultant.

The method is the same as that used in finding the resultant.

Example.—Find the pair of components of the force R = 29 lbs. which act parallel to the lines P and Q respectively (Fig. 37).

Select a starting-point, S, and a suitable scale, and draw SF (Fig. 38) to represent R. Now, one component must begin at S and the other terminate at F; hence draw through S a line parallel to Q, and through F a line parallel to P. These intersect

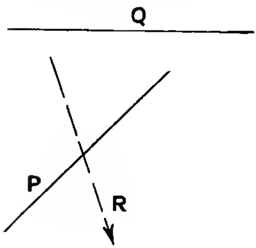


FIG. 37.—Position diagram.

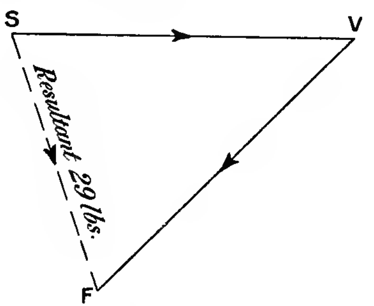


FIG. 38.—Force diagram.

in V. Put the arrows on SV and VF pointing from S towards F; then SV is the component of R parallel to the line Q, and VF is the component parallel to P. Measure off SV and VF, and it will be found that they represent 36·8 lbs. and 38·5 lbs. respectively.

Rectangular Components.—When the directions of the components are at right angles to one another, they are called rectangular components. This is by far the most general case. If the directions of these components are vertical and horizontal, we generally speak of them as vertical and horizontal components respectively.

Example.—Resolve the force AB = 16·3 lbs. (Fig. 39) vertically and horizontally.

Proceeding as in the last example, except that the component

directions are horizontal and vertical; through one end (say A) draw a vertical line, and through the other end B draw a horizontal line cutting the vertical line in C. Then AC represents the vertical component, and CB the horizontal component. Note that the arrows on the components lead in the direction from the starting-point A towards the finishing-point B.

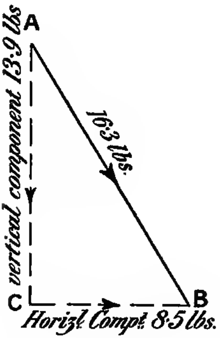


FIG. 39.—Rectangular components.

Example.—The block shown in Fig. 40 is being pulled by a force of 56 lbs. in the direction shown.

If we resolve this force vertically and horizontally we get CA = 22 lbs. and BC = 52 lbs. That is, the force of 56 lbs. is equivalent to a vertical force of 22 lbs. tending to lift the block, and a horizontal force of 52 lbs. tending to drag the block in a horizontal direction.

The student should now redraw the example (Fig. 34), and then resolve each force vertically and horizontally, including

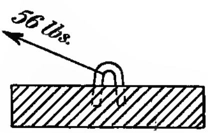


FIG. 40.

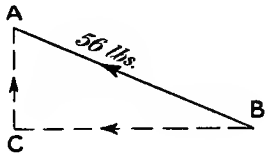


FIG. 41.

the resultant. Prefix a + sign before all those components in the upward direction and a – before those pointing downwards. Similarly, put a + before all those pointing from left to right, and a – before those in the opposite direction.

Find the algebraical sum of the vertical and of the horizontal components of the forces. These should respectively equal the vertical and horizontal components of the resultant. This being so, we may now formulate the statement, *The algebraical sum of the vertical components of a set of forces equals the vertical component of the resultant; and similarly with the horizontal components.*

Summary of Chapter II.

1. A force can be represented by a line.

2. A resultant force is the single force which will produce exactly the same result as the component force acting together.

3. The algebraical sum of the components of a set of forces in any one direction equals the component of the resultant in that direction.

Examples on Chapter II.

COMPOSITION AND RESOLUTION OF FORCES.

1. Two men pull with forces of 30 lbs. and 90 lbs., in directions south-east and north respectively, on a body. Describe briefly and accurately how you would find the single force which would do the same as the above two together. How do you specify a force completely? If the pull of 30 lbs. were reversed, what would be the resultant?

2. Draw two lines AB and AC containing an angle of 120°, and suppose a force of 7 units to act from A to B and a force of 10 units from A to C; find by construction the resultant of the forces, and the number of degrees in the angle its direction makes with AB. *Ans.* 8·75 lbs.

3. Draw an equilateral triangle ABC; a force of 10 units acts from A to B, and one of 15 units from A to C; find their resultant by construction. Also find what their resultant would be if the force of 15 units acted from C to A. *Ans.* 22 and 13.

4. Two forces act at a point along two given lines respectively; state how their resultant can be found by construction.

Draw an angle AOB of 120°, and draw OC within the angle AOB so that AOC may be 45°; if a force of 100 units acts from O to C. Find its components along OA and OB. *Ans.* 85 and 113.

5. Draw AD, AE, lines containing an angle of 60°; draw AB bisecting the angle EAD; also draw AC at right angles to AB in such a way that AD falls within the right angle BAC.

Let a force of 100 units act from A to D. Find the components of a force (*a*) along AB and AC (*b*) along AE and AC.

6. Define the resultant of two forces. Three forces act along the same line; two are forces of 5 and 7 units acting from right to left, the third is a force of 15 units acting from left to right; what is the magnitude of their resultant, and in what direction does it act? *Ans.* 3 units left to right.

7. State how to find the resultant of any number of forces acting along

a line. State what is meant by the "algebraical sum" of two numbers, and give an example. Draw a straight line AB, and let three forces of 15, 10, and 33 units respectively act along it; the former two act from A to B, the last acts from B to A. Find their resultant.

8. Define the rectangular components of a force. Draw two straight lines OA, OB, containing a right angle at O; within the right angle draw OP, such that AOP is an angle of 35°; a force of 18 units acts from O to P. Find, by construction, its rectangular components along OA and OB.

9. Mention the points that go to the specification of a force, and show that a force can be represented by a straight line. Draw two lines Ox, Oy at right angles to each other; two forces act at O; one of 7 units from x to O, and one of 10 units from O to y; draw, to any scale, the straight line (OR) that represents their resultant; and find from the diagram the number of units of force in the resultant, and the number of degrees in the angle xOR.

CHAPTER III.

EQUILIBRIUM—POLYGON OF FORCES.

Equilibrium.—A body is said to be in equilibrium when the resultant of *all* the forces acting *upon* it is zero. In the last paragraph we learnt that the sum of the components of a set of forces in any direction was equal to the component of the resultant in that direction; and as the resultant is zero when the body is in equilibrium, the sum of the components in a given direction is also then zero. This is well illustrated in the next figure (42).

The forces which are in equilibrium are represented by

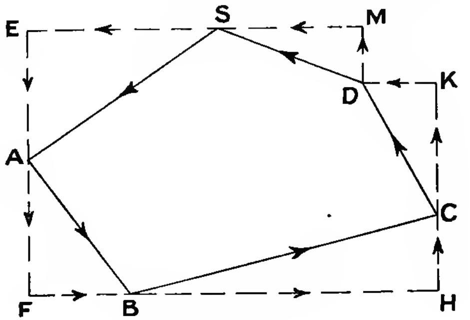


FIG. 42.

SA, AB, BC, CD, and DS, their directions being shown by their arrows. Resolve each force horizontally and vertically, and we get the components SE and EA, FB and AF, etc. Adding up the horizontal components, calling those pointing to the right positive, we get—

$$- \mathrm{SE} + \mathrm{FB} + \mathrm{BH} - \mathrm{KD} - \mathrm{MS} = 0$$

and vertically—

$$- \mathrm{EA} - \mathrm{AF} + \mathrm{HC} + \mathrm{CK} + \mathrm{DM} = 0.$$

This is an illustration of the *First Law of Equilibrium*, which may be thus stated :—

Resolve all the forces acting on a body in equilibrium in any two directions, then the sum of the components in either direction is zero.

Example.—A body is in equilibrium when acted upon by the forces shown in Fig. 43. Find the magnitude of the forces x and y.

As in previous problems, draw the lines *sa*, *ab*, and *bc* to represent the 36 lbs., 23 lbs., and 13 lbs. There are two other forces to be drawn in, the last of these finishing at s, because there

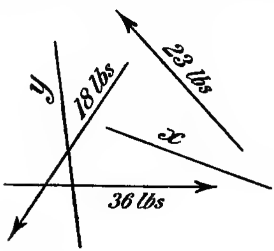


FIG. 43.—Position diagram.

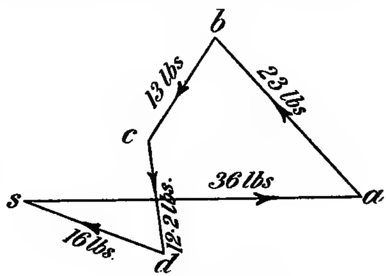


FIG. 44.—Force diagram.

is no resultant, the body being in equilibrium. Hence through C draw a line parallel to y, and through s draw a line parallel to x. As the arrows on all the component forces followed one another round the figure from the starting-point towards the finishing-point, the arrows on *cd* and *ds* will point in the directions shown, cd = 12·2 lbs. and ds = 16 lbs. being the component forces y and x.

We thus see that **when a body is in equilibrium, the arrows follow each other round the force diagram.**

Example.—Five forces of 13 lbs., 29 lbs., 16 lbs., 23 lbs., and 31 lbs. keep a body in equilibrium.

If the directions of the first three forces are given in Fig. 45, determine the inclination to the horizon of the other two.

Draw the lines *sa*, *ab*, and *bc* to scale, as in Fig. 46, to represent the forces given in Fig. 45. The two remaining forces must be drawn from *c* and terminate at *s*. Hence with *c* as centre and one of the forces as radius (say 23) describe an arc. And with *s* as centre and the other force (31 lbs.) as radius describe another arc inter-

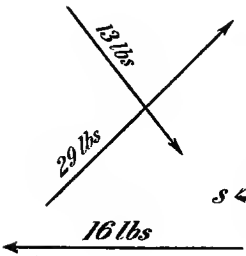


FIG. 45.—Position diagram.

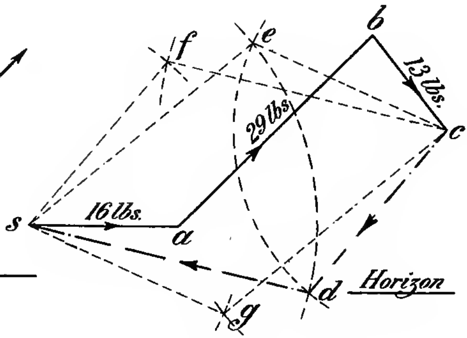


FIG. 46.—Force diagram.

secting the former arc in *d*. Then *cd* and *ds* represent one position of the two forces, 23 lbs. and 31 lbs. Measure off the inclinations of these lines to the horizon with a protractor, and they will be found to be 51° and 167° in the positive direction.

There are other possible positions of the lines *cd*, *ds*. These are *ce*, *es*—*cf*, *fs*, and *cg*, *gs*. Hence there are four possible positions for each of the forces 23 lbs. and 31 lbs.

The Pressure between Surfaces in Contact must act perpendicular to the surfaces at the points of contact. This will be readily seen from the following :—

Let the body (Fig. 47) rest on a smooth horizontal plane, and if the pressure of the plane is not perpendicular to the surface AB, let it act in the direction of the line *sa*. Resolve *sa* parallel to and perpendicular to the plane, giving the components *ba* and *sb*. The component *ba* would make the

body move, because there is no resistance offered to its motion, the plane being smooth. But the body is *at rest*, and therefore there can be no force acting on it which would make it move along the plane; that is, the component along the plane must be zero. If this is so, the force *sa* must be *wholly* perpendicular to the plane.

If three forces maintain a body in equilibrium, the lines of action of those forces must pass through the same point.

Let the forces A, B, and C, in Fig. 48, keep the body in equilibrium. We know that if this is so, the resultant of the

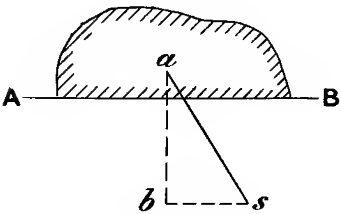


FIG. 47.

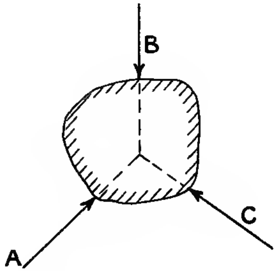


FIG. 48.

three forces is zero. Find the resultant of B and C. Call it R. Then the resultant of R and A must be zero. The only condition under which *two* forces can have zero resultant is when they are equal, and acting in opposite directions along the same line. Hence R and A must be equal in magnitude, and act along the line of action of A. Replace R by its components B and C. Then, as the line of action of the resultant passes through the point of intersection of the component forces, and as R is coincident with A in line of action, the lines of action of the three forces, A, B, and C, must meet in a point.

FORCES ACTING ON A SIMPLE STRUCTURE.

A simple structure is one which is made up of straight parts hinged together, the hinges are assumed to be without friction,

and the parts without weight. The forces are applied to the structure at the hinges. These assumptions are made for the purpose of making easy the solution of a problem.

Not only is a structure in equilibrium as a whole, *i.e.* as a single body, but each individual part of which it is made up must also be in equilibrium. As an illustration, consider the wall-crane in Fig. 49. Not only is the crane itself, with its suspended weight, in equilibrium, but the weight W alone is in equilibrium under the action of the downward pull of the earth and the upward pull of the chain or rope. The pin at B is in equilibrium under the action of the pull of the chain downwards, and the action of each of the rods AB and CB on it. The rod AB is in equilibrium under the action of the pin on it at B, and the other pin on it at A. The rod CB is in equilibrium under the action of the pin at C, and the pin at B.

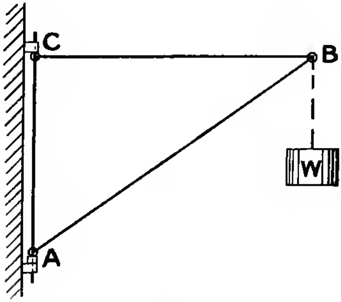


FIG. 49.

To enable us to solve problems relating to simple structures, we must first learn along what line the internal force or stress in the material acts.

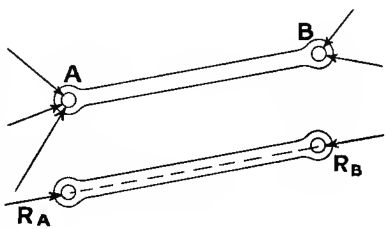


FIG. 50.

Let AB (Fig. 50) be a piece of material, with hinges at A and B. *Let it also be in equilibrium.* We can find the resultant of the forces acting at A, and the resultant of those at B. Call these R_A and R_B respectively. Now, the body is *in equilibrium*, and, therefore, the resultant of R_A and R_B must be zero. For this to be possible, R_A must equal R_B, and both of

them must act along the same line. The only line along which this can happen is that joining the hinges, namely AB. Hence the stress or internal force in a member of a simple structure must act along the member from hinge to hinge.

Example.—To illustrate the above, let 425 lbs. be supported by the rope at B (Fig. 49). It is required to find the force *in* the jib AB and the tie-rod CB.

In solving this problem, **we consider the forces keeping the pin B in equilibrium.** These are three (Fig. 51), namely—(1) the pull of the rope downwards equal to 425 lbs., (2) the pressure of the jib AB on the pin, and (3) the pull of the tie-rod CB on the pin. We know from the preceding paragraph that (2) and (3) must act along the lines AB and BC respectively (Fig. 49).

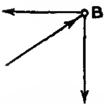


FIG. 51.—Position diagram.

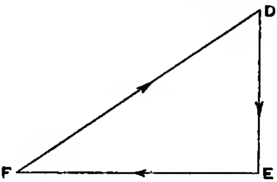


FIG. 52.—Force diagram.

We also know that because the pin B is in equilibrium, the resultant of the forces (1), (2), and (3) must be zero. Hence, draw the line DE (Fig. 52) in magnitude and direction to represent the 425 lbs. to a scale of 500 lbs. to 1 inch. Through either end, E or D, draw a line EF parallel to the tie-rod, and through the remaining end draw a line parallel to the jib. Continue the arrows round the figure, and measure off the lines FE = 640 lbs. and FD = 760 lbs. to the same scale that DE was drawn. Then EF being parallel to the tie-rod CB, represents the force along it, while FD being parallel to the jib, represents the force along that member. We have been considering the equilibrium of the pin B, and the forces mentioned above are the forces acting on the pin. Hence, if the arrow on FD be reproduced on the jib near B, it indicates that the jib *pushes* the pin B, and is consequently in *compression*. Similarly the arrow on the tie-rod indicates that it *pulls* the pin B, and is consequently in *tension*.

Example.—In general the weight is not tied or hooked on to the pin B (Fig. 49), but the cord or chain supporting the weight passes over a pulley at B, and is led to some winding-gear near the crane-post, as in Fig. 53. Find the forces in the tie-rod and jib.

We now have four forces keeping the pin B (Fig. 54) in

equilibrium, of which two are completely known. These are the tensions in the two parts of the rope round the pulley at B. In the right end of the rope the tension is W; and as the pulley is assumed to be without friction, there must be an equal and

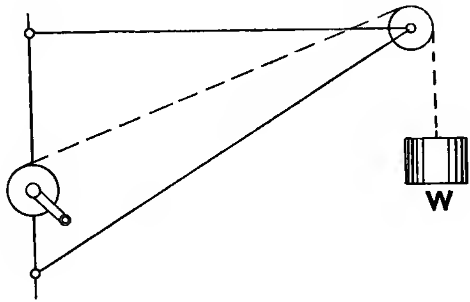


FIG. 53.

opposite pull at the left end of the rope to balance W, because the rope is in equilibrium.

Hence, draw a line *sp* (Fig. 55) representing the pull on the right end of the rope, and then from *p* another line *pq*, representing

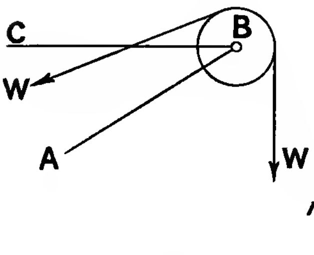


FIG. 54.—Position diagram.

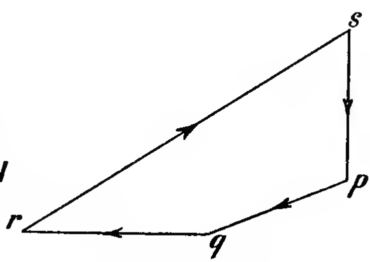


FIG. 55.—Force diagram.

the pull on the left end of the rope. Now draw through *q* a line parallel to say the tie-rod, and, lastly, through *s* draw a line parallel to the jib. Then *qr* and *rs* represent the forces with which the tie-rod and jib act on the pin B.

Example.—A bent lever, ACB (Fig. 56), can turn round a pin

at C. A force of 18 lbs. acts at A in the direction of the arrow. Another force acts at B in the line of action BE.

Find the direction and magnitude of the force at B, and the direction and magnitude of the pressure of the hinge C on the lever, so that it is maintained in equilibrium.

The three forces keeping the lever in equilibrium are the 18 lbs. at A, and the forces at B and C. It was shown on page 52 that the lines of action of these forces must meet in a point; hence, produce the lines of action given, until they meet in D (Fig. 56). The third force is applied at C, and must also pass through D. Hence CD is the line of action of the hinge pressure at C.

Draw a line *sp* (Fig. 57) to represent the force acting at A, and

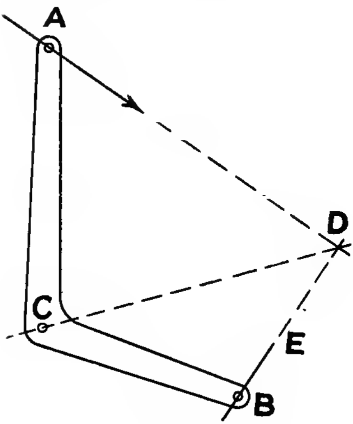


FIG. 56.—Position diagram.

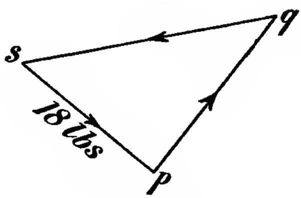


FIG. 57.—Force diagram.

through, say, the end *p*, draw a line parallel to the force at B. Finally, through the point *s* draw a line parallel to the line CD. Then *pq* represents the force 21 lbs. at B, and *qs* the force 27·4 lbs. at C.

Example.—A body (weighing 22 lbs.) resting on a smooth incline (Fig. 58) is kept in position by a cord F. What forces are maintaining the body in equilibrium?

The weight of the body (22 lbs.) acts vertically downwards, the pull of the cord on the body is along the line F, in the upward direction. The pressure of the incline on the body must act along a line perpendicular to the incline (see p. 52). Draw a triangle

(Fig. 59) with sides parallel to the forces 22, R and F, and with the side representing 22 lbs. marked off to some scale. Then the other sides will represent F and R to the same scale.

It should be noted that whatever be the direction of F, the

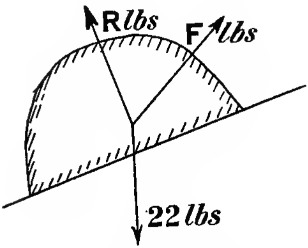


FIG. 58.—Position diagram.

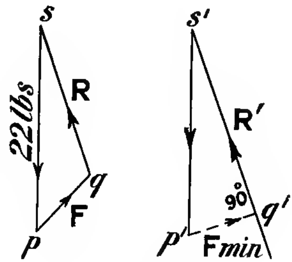


FIG. 59.—Force diagram.

weight 22 lbs. and the pressure R of the incline always act in the same respective directions. If we redraw these lines in Fig. 59, we see that the *shortest* line we can draw from p^1 to s^1q^1 is p^1q^1 perpendicular to s^1q^1; and hence this is the direction in which F must act so that its value must be the least possible.

Example.—A weight of 15 lbs. is suspended, as in Fig. 60, by

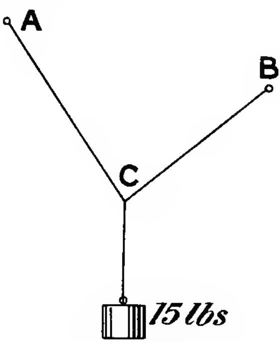


FIG. 60.—Position diagram.

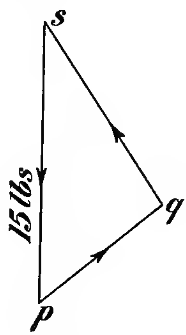


FIG. 61.—Force diagram.

three strings knotted together at C. Find the tensions in the strings CB and CA. If the string CB were cut with a scissors, what would be the tension in the string AC at the instant CB was severed?

The knot C is kept in equilibrium by three forces, namely, the pull of 15 lbs. downwards, the pull in the cord CB, and the pull in the cord CA. Draw *sp* (Fig. 61) to represent the 15 lbs. in magnitude and direction. Through the point *s* draw a line parallel to CA, and through *p* draw a line parallel to CB. Then *pq* represents the pull of the cord CB on the knot C, and *qs*

Fig. 62.

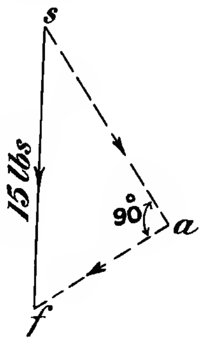


Fig. 63.

represents the pull of the cord CA on the knot C ; *pq* scales off 8·5 lbs., and *qs* 11·5 lbs.

The instant after cutting CB we shall have the weight supported as shown in Fig. 62, and just on the point of moving downwards. We no longer have equilibrium. The component of the 15 lbs. along the string AC is trying to stretch it, while the component perpendicular to it is trying to turn it round A. Hence, draw *sf* (Fig. 63) to represent the 15 lbs., and resolve it parallel and perpendicular to AC. The component *sa* parallel to the cord is trying to stretch it, that is to produce tension in it, and the tension produced is represented by *sa* = 12·5 lbs.

Experimental Illustration of three forces keeping a body in equilibrium.

Three fine cords (whipcord is very suitable) are tied to the ring (Fig. 64). The two upper cords are attached to the hooks of the spring-balances L and R, which should be of the sportsman type, and weigh only a few ounces. A weight of say 14 lbs. is put on the lowest cord.

The positions of the three cords are now marked on the sheet of paper behind them, by marking two points *immediately behind each cord*, such as *a* and *b*. The indications of the spring-balances are then recorded upon the same paper, and the paper removed to a drawing-board. The pairs of

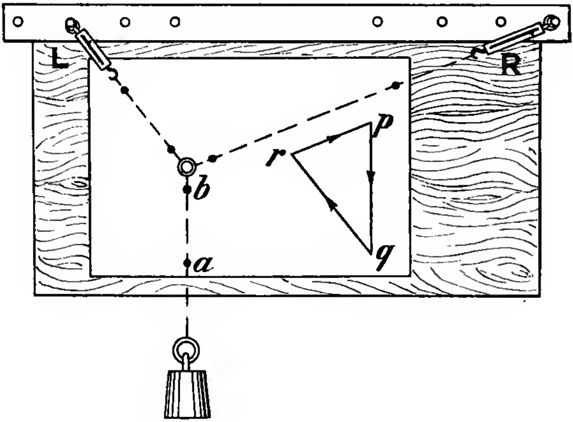


FIG. 64.

points (marked behind the cords) are now joined, indicating the directions of the cords; and the triangle of the forces *p*, *q*, *r*, drawn with *pq* parallel to the pull of the earth on the weight W, and *qr* and *pr* parallel to the pulls of the spring-balances. Measure *pr* and *qr*, and enter the result in the following table :—

	By drawing.	Indicated by spring-balance.
pr . .	6·8	6·75
qr . .	9·5	9·5

The result by drawing the triangle of forces agrees with that

obtained from the balances, well within the limits of error of the balances.

If the experiment has been *carefully* carried out, the lines of action of the three cords will intersect at a point (see p. 52). The scale of forces should be large, not more than 5 lbs. to 1 in. The accuracy of the drawing *pqr* may then be considerable. The experiment should be repeated with a different weight and with the spring-balances attached to other hooks in the frame of the apparatus to show that the same holds good with different angles and different weights.

Experimental Illustration of the Polygon of Forces.

The apparatus much resembles that shown in Fig. 64, but

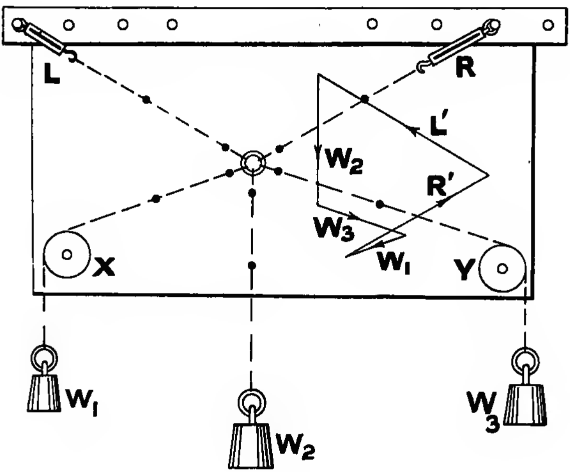


FIG. 65.

there is added a couple of pulleys X and Y (Fig. 65), having very little friction.

A couple of cords tied to the ring pass over these pulleys and support weights W_1 and W_3.

Proceed as in the last experiment and write near each cord the pull exerted on it. Now draw the polygon of forces *as if*

the forces exerted by the spring-balances were known in direction, but not in magnitude. Measure these forces and tabulate them as in the previous experiment. The polygon of forces is shown drawn on the right of the paper.

Experimental Illustration of the Triangle of Forces by means of the inclined plane.

A stout board, EF (Fig. 66), is hinged at E, and supported at its upper end F by a pair of thumb-screws. A heavy roller,

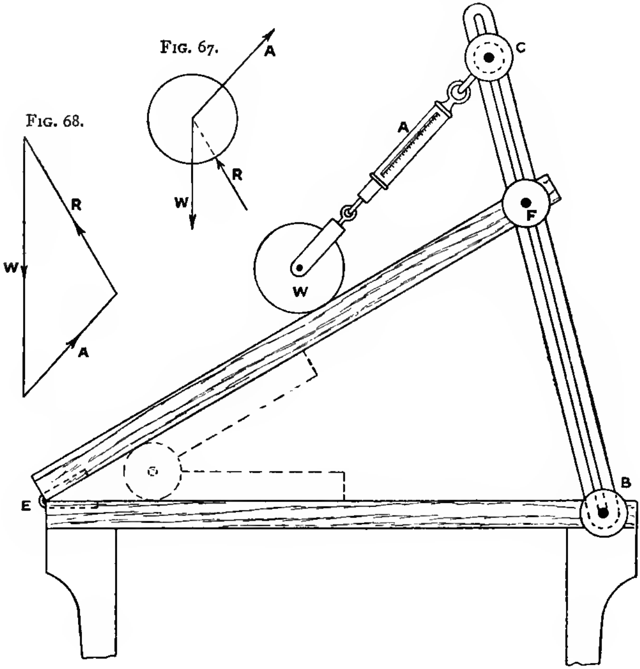


FIG. 67.

FIG. 68.

FIG. 66.

W, rests on the incline, and is prevented from rolling down the plane by a spring-balance, A.

The weight of the roller is unknown. It is proposed to obtain it by the triangle of forces. Measure the apparatus

with a tape so as to be able to draw it to scale on paper. The angle FEB can best be obtained by fitting into the angle a carpenter's rule, as shown dotted, or a clinograph may be used for the purpose. The slope of the plane may be obtained by actual measurement. Observe the spring-balance A. The forces keeping the roller W at rest are, its weight downwards, the spring-balance pull, and the pressure of the plane on the roller perpendicular to the plane (p. 52). These are shown in Fig. 67.

Draw the triangle *scd* (Fig. 68), beginning with *sc*, representing the spring-balance pull. Now detach the balance at C and weigh the roller, and compare this with *cd*, Fig. 68.

The experiment should be repeated with the plane inclined at different angles.

The pressure of the plane on the roller should be checked by attaching another spring-balance to the roller, and pulling the balance by hand in a direction as nearly perpendicular to the plane as possible, until the roller is just off the plane. The indication of the balance should approximately coincide with the force *sd* (Fig. 68).

Tabulate the results as in previous experiments.

Experimental Illustration of the polygon of forces by means of a crane.

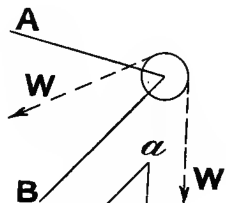
The apparatus is shown in Fig. 69. A load of 28 or 56 lbs. is supported as shown. The indications of the spring-balances A and B are noted and the lengths of the parts are measured with an ordinary canvas tape, so that the skeleton diagram (Fig. 69) may be drawn. The weight W is then removed, and it will be noticed that the tie-rod and jib are not the same length as before.

Adjust the tie-rod to the same length as it was when the weight W was being supported, and then read A and B. These latter readings indicate the forces in those members due to their weight, and must be subtracted from the previous values of A and B, to obtain the forces in those members due to W. Draw the polygon of forces (Fig. 71) *for the forces acting on the pulley* shown in Fig. 70, the forces A and B being assumed unknown in magnitude, then *cd* and *da* represent the forces in

the tie-rod and jib respectively. Tabulate these with those derived from experiment.

Example.—A body weighing 35 lbs. rests on a *rough* inclined plane (Fig. 72), the coefficient of friction between the body and

FIG. 70.



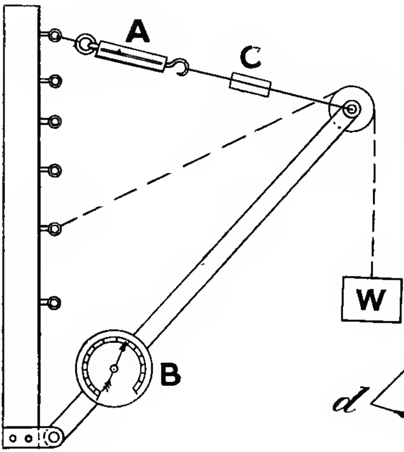


FIG. 69.

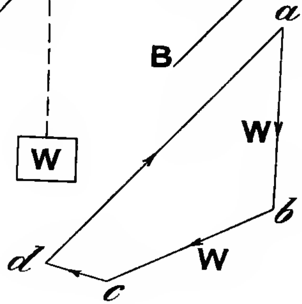


FIG. 71.

plane being 0·3. A force F is applied to the body, and is just on the point of making the body move up the plane. What is the magnitude of F, the resistance of friction, and the pressure of the plane on the body.

Sketch in (Fig. 72) all the forces acting on the body and keeping it in equilibrium. They are W = 35 lbs. vertically downwards, F upwards, the pressure P of the plane on the body upwards, and the resistance of friction f, which must act down the plane; because *friction always opposes motion.*

We know from p. 25 that the friction (f) is produced by the pressure (P) between the surfaces of the body and plane; also that—

Resistance of friction = coefficient of friction × perpendicular pressure

$$\text{or } f = 0{\cdot}3 \text{ P}$$

Now we can find the *direction* of the resultant of f and P thus :—Draw *st* (Fig. 73) representing P, and *tv* representing f ($= 0{\cdot}3$ P); then *sv* represents the resultant of P and f, and can be made to replace those forces if necessary.

It will be noticed that Fig. 73 can be drawn quite easily without knowing the magnitude of P. Both forces, P and f, are multiples

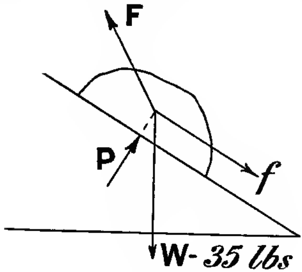


FIG. 72.—Position diagram.

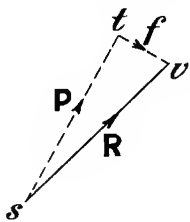


FIG. 73.—Force diagram.

of P. Consequently, if we set off $st = 1$ and $tv = 0{\cdot}3$, we get the same direction of the resultant *sv* as before.

Replacing P and f by their resultant R, we now have acting on the body the three forces F, W, and R (Fig. 74).

Hence, we can draw the triangle of forces (Fig. 75) with three sides parallel to W, F, and R.

Note that the directions of W and R are the same, whatever

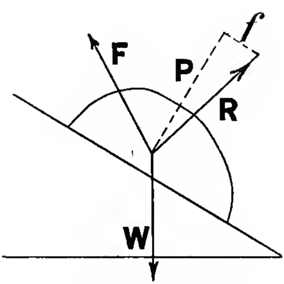


FIG. 74.—Position diagram.

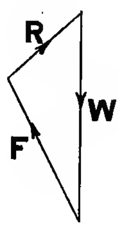


FIG. 75.—Force diagram.

the direction of F. Hence, the least value of F required to move the body up the plane is represented by a perpendicular from the lower end of W (Fig. 75) on to R.

If the body had been on the point of sliding *down* instead of up the plane, the friction f would act *up* the plane (against motion); and the position of R would be that shown in Fig. 76. The

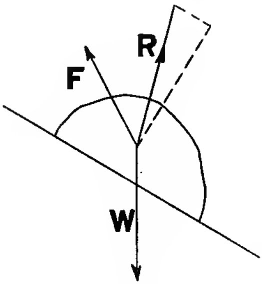


Fig. 76.

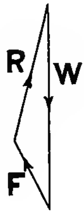


Fig. 77.

triangle of forces is shown in Fig. 77, and it will be seen that F is still acting *up* the plane, and the body will just move down the plane in opposition to the force F.

The Limiting Angle of Friction.—Place a body on a plank which is hinged at one end as in Fig. 78. Now lift up the right end of the plank and keep it at any inclination by

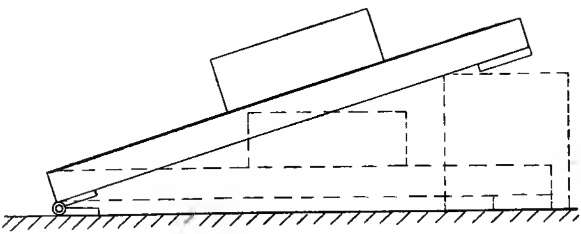


Fig. 78.

inserting a block under the plane. Continue to increase the inclination of the plank until the body resting upon it just begins to slide down the plank.

The forces acting on the body just previous to motion taking place were its weight, W (Fig. 79), the resistance of

friction f, and the pressure, R, of the plane on the body. These can be represented by the sides of a triangle, as in Fig. 80.

Now, W is perpendicular to BC (Fig. 79), and R is perpendicular to BA; hence the angle between W and R (Fig. 80) equals the angle between AB and BC (Fig. 79). Also the angle at C is 90°, so is the angle between f and R (Fig. 80), and consequently the triangle ABC (Fig. 79), and f, W, R

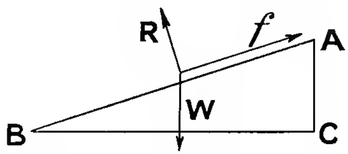


FIG. 79.—Position diagram.

FIG. 80.—Force diagram.

(Fig. 80), are equiangular and therefore similar; that is, their sides are proportional (see Introduction), or—

$$\frac{f}{R} = \frac{AC}{CB} = \tan B$$

But we have seen that—

$$f = \mu \,.\, R$$

and therefore $\dfrac{f}{R} = \mu$

Hence $\mu = \tan B$

$= \text{the tangent of the limiting angle of friction.}$

The Limiting Angle of Friction can be better explained in another way. Let the body (Fig. 81) be acted upon by an oblique force, P, which makes an angle, O, with the normal to the surface AB, along which the body can slide. Resolve P parallel and perpendicular to the surface AB. The vertical component v presses the surfaces together and produces

friction $= \mu v$. The horizontal component h tries to move the body over the surface AB. When O is large, h will be greater than the resistance of friction, and the body will move along the surface AB. Now let the force P be inclined more and more to the horizon; that is, let O grow smaller. There will be a value of O when the force P will no longer be able to move the body. When this happens, the horizontal component h of the force P is just equal to the resistance of friction μv. This value of O is the limiting angle of friction. If O be made still smaller, h will be less than μv, and no motion will be possible; hence, if the force be applied in a direction *within* the limiting angle of friction, no motion will ensue, however large the force may be.

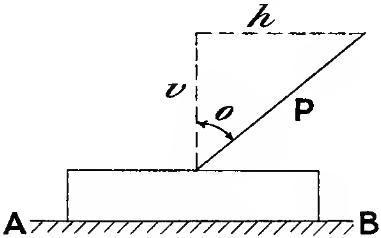


FIG. 81.

This fact is made use of in the cotter joint (Fig. 82), where

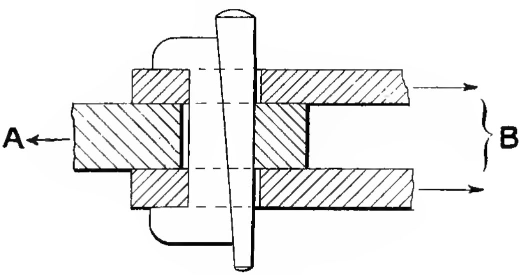


FIG. 82.—Cotter joint (in section).

two pieces have to be coupled together, and at subsequent times adjustment for wear may be required.

The angle of the cotter is less than the limiting angle of

friction, and therefore no amount of force applied in the direction A and B will compel the cotter to come out of its position.

Summary of Chapter III.

1. The resultant of all the forces keeping a body in equilibrium is zero.

2. The sum of the components in any direction, of the forces keeping a body in equilibrium, is zero.

3. The mutual pressure between two surfaces in contact is perpendicular to the surfaces at the points of contact.

4. If three forces keep a body in equilibrium, their lines of action must pass through the same point.

5. The stress in a member of a hinged structure acts along the line joining the centres of the hinges of the member.

Examples on Chapter III.

1. State clearly the theorem called the polygon of forces. When is a body in equilibrium? What is the resultant of a number of forces?

2. Four forces are in equilibrium. Two of them are 14 and 31 lbs. respectively acting at an angle of 35°. The other two are 17 and 22 lbs. respectively. What are their directions?

3. State the theorem called the triangle of forces. AB is an inclined plane; C is a given point above the plane. P is a particle of given weight at rest on the plane, and it is supported by a thread tied to C. State what forces act on P, and show how to draw a triangle for them.

4. Draw ABC an equilateral triangle with the base AB horizontal, and C downwards. Let a weight at C be tied by threads AC, BC, to fixed points at A and B; if the thread BC is cut, show that the tension of AC is suddenly increased by one half.

5. ABCD is a rectangular body, which can turn freely in a vertical plane round a hinge at A; AB is 10 ft. long and BC is 6 ft.; the body weighs 300 lbs.; it rests with AB against a fixed point P, vertically below A, and at a distance of 8 ft. from A; find the reaction of P against the body. *Ans.* 112·5 lbs.

6. A weight resting on a smooth inclined plane is held in position by a horizontal force. Draw a diagram exhibiting the directions and magnitudes of the forces acting; and calculate the force necessary to hold a

hundredweight on a plane tilted 30° from the level (*a*) if acting horizontally, (*b*) if acting at the best angle.

7. A picture, weighing 56 lbs., is slung over a nail in the ordinary way by a cord attached to two eyes in the top horizontal bar of its frame. If the height of the nail above this bar is half the distance between the eyes, what is the tension in the cord? Under what circumstances would the tension be equal to or greater than the whole weight of the picture?

8. Draw a horizontal line CD, take a point A vertically over C, and another point B, between C and D; join AB. Suppose that AB represents a uniform rod that can turn freely round a hinge at A, and rests with the end B against a smooth horizontal plane, CD; name the forces which keep the rod at rest, and show in a diagram how they act. If the weight of the rod is 10, find the numerical values of the other forces.

9. A rod AB is placed on a smooth inclined plane, and the end A is tied by a string AC to a fixed point C on the plane; show how to find the pressure on the plane, and the tension of the string. If the length of the base of the plane is 12 ft., and the height of the incline is 5 ft., and if the rod weighs 10 lbs., find the numerical values of the pressure and tension. *Ans.* 9·25 lbs. and 4 lbs.

10. If a body is pressed against a smooth surface, in what direction does the mutual action take place? Draw a circle, suppose that its plane is vertical, and that A is its lowest point; draw a chord AB equal in length to the radius. If we suppose AB to represent a rod placed inside a circle, and that there is no friction, name the forces that act on the rod and show them in a diagram. Explain why the forces cannot keep the rod at rest.

11. A load, W, of 2000 lbs. is hung from a pin, P, at which pieces AP and BP meet like the tie-rod and jib of a crane. The angles WPB and WPA are 30° and 60° respectively. Show by a sketch how to find the forces in AP and BP. Distinguish as to a piece being a strut or a tie. *Ans.* 1900 lbs., 3400 lbs.

12. Draw a triangle, ABC, whose sides BC, CA, and AB are respectively 12, 9, and 7 units long, and let BC be horizontal and A downwards. Let B and C represent two fixed points to which the threads AB and AC are fastened; a weight of 10 lbs. hangs by a thread from the point A, at which the three threads are tied together; state what are the forces which support the weight, and find them by construction. *Ans.* 6·9 and 8·2 lbs.

13. Draw a horizontal line, CD, take a point, A, vertically over C, and another point, B, between C and D, join AB. Suppose that AB represents a uniform rod that can turn freely round a hinge at B, and rests with the end A against a smooth vertical plane, CA; name the forces which keep the rod at rest, and show in a diagram how they act. If the weight of the rod is 25 lbs., find the numerical values of the other forces.

14. A rod or lever is capable of turning freely round a fixed point or fulcrum, and is acted on by a force at each end; putting the weight of the lever out of question, state the relation which must exist between the

forces when the rod stays at rest. Draw an equilateral triangle ABC. BC represents a weightless lever acted on at B by a force of 7 units from A to B, and at C by a force of 9 units from A to C; if the lever is at rest, find by construction or otherwise, the position of the fulcrum; find also the magnitude and direction of the pressure on the fulcrum.

15. A weight of 20 lbs., suspended by a string from a peg P, is pulled aside by another string knotted to the first at a point K, and pulled horizontally. Find the force necessary to pull it until PK is 60° from the vertical; and find, at the same time, the force on the peg. *Ans.* 34·6 and 40.

16. What are the conditions of equilibrium of a particle acted on by any number of forces in one plane?

A body weighing 10 lbs. rests upon a plane inclined to the horizontal at an angle of 45°, the angle of friction being 30° under the action of the least force which will prevent it sliding down the plane; find this force in magnitude and direction. *Ans.* 2·7 lbs. at 15° to the horizon.

17. When two smooth bodies are pressed together, in what direction does the mutual action take place? If the bodies are rough, what other force may be called into play?

A particle of given weight is placed on an inclined plane and stays at rest; what is the magnitude of the friction called into play? Under what circumstances would the particle stay at rest if the inclination of the plane were increased?

18. A body on a smooth plane is subject to the action of forces.

What are the conditions of equilibrium?

Find in magnitude and direction the least force which will keep a body weighing 100 lbs. at rest on a smooth plane, inclined at an angle of 35° to the horizontal. *Ans.* 58 lbs. along the plane.

19. In what direction does a smooth plane exert a reaction on a body which is pressed against it?

One end of a string is fastened to a fixed point P, and the other end to the end B, of a rod AB; the lengths are such that AB comes to rest in an inclined position, with the end A on a smooth horizontal plane below P; explain why the position must be such that the string shall be vertical.

20. State the laws of friction.

The centre of gravity of a circular disc is at a distance from the centre equal to half the radius. The disc rests with its plane vertical on the rough horizontal table, and is supported by two smooth pegs at the extremities of a horizontal diameter. Show that if the coefficient of friction be greater than half, every position is one of equilibrium.

21. A uniform rod rests in limiting equilibrium against a smooth fixed point C, with one end on a rough horizontal plane; the height of C above the plane, the inclination of the rod to horizontal, and the length of the rod are 6 in., 45°, and 11 in. respectively. Find the pressure of the fixed point C and the coefficient of friction. What is the least possible pressure on the point C, and what is then the position of the rod?

22. Draw a vertical line, AB, 3 in. long. A being above B. Produce

AB to C, making BC 2 in. long. E is a point to the right of AB, such that ABE is 56° and BAE 90°. Similarly D is a point on the left of AB, such that BAD = ABD = 48°. The structure represents a partially balanced crane, with a footstep bearing at C and a horizontal support at B. 5 tons is suspended from E, and 2 tons from D. Find the supporting forces and the stresses in all the members. *Ans.* 9·66 tons at B and 11·95 at C.

23. State and prove the relation between the weight, W, of a body resting on a smooth inclined plane, the reaction, R, from the plane, and the force, P, necessary to just balance the weight: (1) when the force, P, acts parallel to the plane, (2) when it acts parallel to the base, (3) when it acts at an angle, θ, to the plane.

24. If 150 lbs. per ton is a sufficient tractive force to draw a loaded waggon along a horizontal road, what tractive force per ton will be required to draw the load up an incline 1 in 10? *Ans.* 374 lbs. per ton.

25. The triangle ABC represents the skeleton diagram of a simple crane. The post AB is vertical and 10 ft. long; A being above B. The angle ABC is 45°, and BAC 110°. The rope, which suspends 2 tons, is passed over a pulley at C, and then parallel with BC, until it reaches the winding-drum at B. Find the stress in the jib and tie-rod, and write down their magnitudes. *Ans.* Stress in jib 6·48 tons, in tie-rod 3·36 tons.

26. The weight of a chain hanging from two points of support is 500 lbs. Its inclinations to the horizontal at the points of support are 30° and 50° respectively; what are the tensions at the points of support?

Ans. 439 lbs. and 325 lbs.

27. State the proposition known as the triangle of forces. A picture hangs by a cord from a nail. The ends of the cord are attached to rings in the frame of the picture, 2 ft. apart, and the nail is 1 ft. above the line joining the rings, which is horizontal. Make a drawing to scale of the triangle whose sides represent the forces acting at the nail. Use your diagram to find the tension of the string when the picture weighs 10 lbs.

Ans. $7\frac{1}{8}$ lbs.

28. Find the stresses on the jib and tie-bar respectively of a crane, whose jib measures 20 ft. in length, when the tie-bar and post are 16 ft. in length respectively, and a weight of 15 cwt. is suspended from the end of the jib. The line of direction of the chain, after leaving the barrel or drum, is parallel to the tie-bar.

29. A jib foundry crane consists of a vertical post, AB, 16 ft. long, fitted with pins working in sockets at both A and B. From the upper end, A, of the post extends a horizontal member AD, 28 ft. in length, and from the foot B is a strut BC, which meets AD at a point 16 ft. from A. A load of 20 tons being suspended from D, find the shearing stress on the pin at A, and the stress along the strut BC.

30. An incline is such that the ratio of height to base is 1 to 10. A body is placed on the plane, the coefficient of friction being 0·2. What force applied at an angle of 30° to the incline (upwards) would make the body move? Also what is the least force, and what is its direction?

31. Enunciate the proposition known as the triangle of forces. One end of a string is attached to the fixed point A, and after passing over a smooth peg B, in the same horizontal plane, sustains a weight of P lbs.; a weight of 50 lbs. is now knotted to the string at C, midway between A and B. Find P so that in the position of equilibrium AC may make an angle of 60° with AB. *Ans.* P = 25 lbs.

32. Two pieces of a hinged structure meet at a pin, and a load is applied to the pin. Explain by the aid of a sketch how you would completely determine the total stress in each piece when the load is known.

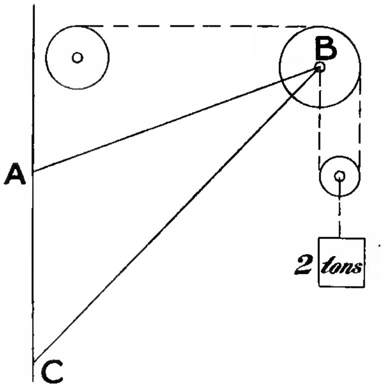


FIG. 83.

33. A crane is represented in Fig. 83, in which AC = 4 ft., angle CAB = 110°, and the angle ACB = 45°. Find the forces in AB and AC. Find also their sectional area if 5 tons per square inch and 1 ton per square inch are the respective stresses permitted.

34. A ladder 10 ft. long and weighing 45 lbs. passes over a smooth wall, and projects 2 ft. beyond the top of the wall. The ladder makes an angle of 60° with the ground, and its lower end is fixed. Find the pressure on the wall when a man weighing 150 lbs. stands on the ladder at a distance of 5 ft. from the fixed end.

35. ABC and D are the angles of a square taken in order. Forces of 8 lbs., 9 lbs., 13 lbs., and 10 lbs. act respectively from B to A, B to C, C to D, and A to D. Describe clearly *how* you would proceed to find their resultant.

36. Referring to the figure in the previous example, two forces of 7 and 13 lbs. act in the directions A to B and A to D, while a third force is, with the two above, in equilibrium. Describe how you would find the third force in magnitude and direction.

37. If a kite in equilibrium in the air makes an angle of 45° with the horizontal plane, and if the pressure of the air on the kite is equal to three times the weight of the kite, indicate, by a carefully drawn figure, the direction of the string at its point of attachment to the kite, and the magnitude of the tension of the string in terms of the weight of the kite. *Ans.* Tension = 2·4 W at 28° to horizon.

38. A string ABCD (Fig. 84) is fastened to supports at A and D. Two weights 10 lbs. and x lbs. are tied to the string at B and C. Find x and the tensions in AB, BC, and CD. BC is horizontal.

39. W (Fig. 85) resting on the plane = 30 lbs. $\frac{AB}{BC} = 0\cdot5$. Find the force acting along the plane that will keep it at rest. Also the force parallel to CB. Also the force along MN when AMN = 30°.

If the coefficient of friction were 0·2, what would each of the above forces be?

40. The rod in Fig. 86 is at rest. The planes are smooth. Where is the centre of gravity of the body, and why?

41. If a body is in equilibrium under the action of a set of forces, what

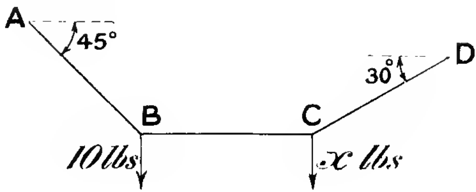


Fig. 84.

must be the conditions fulfilled by the forces. Draw AB horizontally, A being to the left of B. Set off AC at an angle of 35° with AB; also AD at 125°, and AE at 260°. Forces of 35, 125, 260, and 360 lbs. act from A to B, from A to D, and E to A respectively. Are these forces in

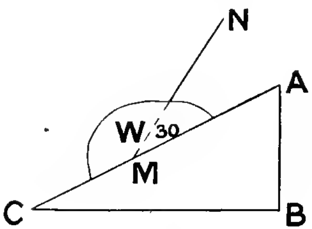


Fig. 85.

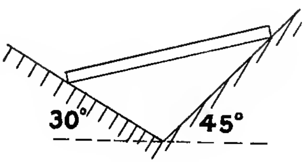


Fig. 86.

equilibrium, and why? How could they be rendered in equilibrium by the application of a single force, and what is its value and direction?

42. If three forces act on a body and maintain it in equilibrium, state what you know about those forces, and use the information in solving the following problem. A smooth plane is inclined at 35° to the horizon, and a body is supported upon it by a string which is inclined to the plane at 25° in an upward direction. The body weighs 10 lbs. Find the tension

in the string. If the inclination of the string had been 25° to the horizon, what would have been its tension?

43. Discover which of the members of the accompanying frame (Fig. 87) are in tension or thrust, and find the value of these quantities.

44. ABC is an equilateral triangle with BC horizontal. A force of 23 lbs. acts at A in a direction south-east, and another (unknown) at B in a direction north-east. In what direction must a third force act at C to maintain equilibrium? What is its value?

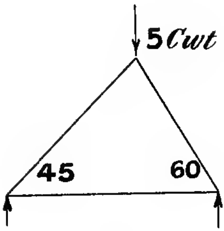


FIG. 87.

45. A and B are two points a given distance apart, A below B, the line AB is of given length and inclined at a given angle to the horizon; a thread of given length has its ends fastened to A and B; a given weight is hung on the thread by a smooth hook; find the position in which it comes to rest, and the tension of the thread.

46. If a heavy box be wholly suspended by a strap attached to the handles at its end, show that the tension in the strap is diminished if its length is increased.

47. A barrel weighing 5 cwt. is lowered into a cellar down a smooth slide inclined at an angle of 45° with the vertical. It is lowered by means of two ropes passing under the barrel, one end of each rope being fixed, while two men pay out the other ends of the ropes. What pull in lbs. must each man exert in order that the barrel may be supported at any point? *Ans.* 99 lbs. nearly.

CHAPTER IV.

THE LAWS OF EQUILIBRIUM.

WE have found in the last chapter that when a body is in equilibrium there can be no resultant force acting on it.

This statement does not indicate the most convenient method of solving problems in which results have to be *calculated* instead of drawn.

It will, in this investigation, be best to proceed at once to the following experiment.

A rod, AB, is suspended by a couple of spring-balances, as

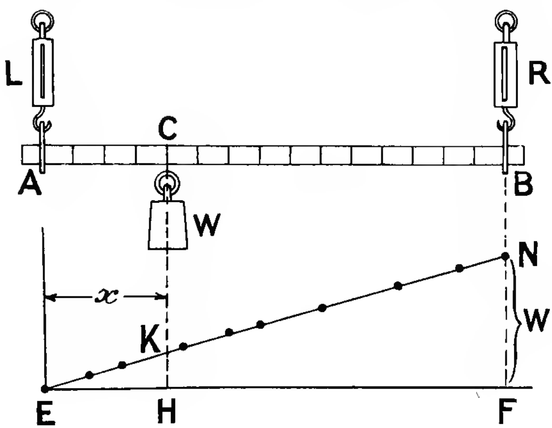


FIG. 88.

shown in Fig. 88. It is convenient to have the rod divided in inches. Note the spring-balance indications without any weight on, and enter them in the table below. Now suspend

a light weight of, say 14 lbs., by a piece of fine cord, as shown in Fig. 88, and note the balances and the distance AC. Enter them in the table. Shift the weight to some other position, and note the result. Do this for about a dozen positions. Subtract the values of R and L with no-load W, from those values found, and the results will be the values of R that would be *obtained* if the rod had no weight.

WEIGHT SUSPENDED = 14 LBS.

x.	Actual left spring-balance reading.	Left balance reading due to W only = L.	Actual right balance reading.	Right balance reading due to W only = R.
inches.	lbs.		lbs.	
—	1·9	0	1·6	0
27	3·3	1·4	14·3	12·7
23	5·1	3·2	12·5	10·9
18	7·5	5·6	9·9	8·3
14	9·4	7·5	8·1	6·5
12	10·4	8·3	7·2	5·6
9	11·6	9·7	5·9	4·3
8	12·2	10·3	5·3	3·7
5	13·5	11·6	3·9	2·3
3	14·5	12·6	3·0	1·4
0	15·9	14·0	1·7	0·1
30	1·9	0	15·6	14·0

Now draw a base line, EF, parallel to the rod and of the same length. Plot upwards from EF *immediately under the weight*, the value of R for that position of the weight. Thus, when AC = 8, R = 3·7. Plot EH = 8, and HK = 3·7. Plotting all the numbers in the fifth column of the adjoining table, we get the straight line EN. We can find its equation by the method given in the Appendix. It is—

$$\text{Ordinate} = \text{intercept} + \text{slope} \times \text{abscissa}$$

Now, an ordinate represents R, and an abscissa represents x; therefore, as the intercept is zero, the line passing through the origin—

$$R \text{ (without weight of rod)} = \text{slope} \times x = \frac{FN}{EF} \times x$$

Now, FN is the right supporting force (without rod) when the weight W is at F. In this position the whole of the weight is supported by the balance, and hence FN = W. We then have—

$$R \times EF = W \times x$$

The product of a force into its perpendicular distance from a point is called the **moment of the force round the point**, and is the numerical value of the tendency of the force to turn a body round that point. The above equation indicates that the moment of R round E in anti-clockwise direction is exactly equal to and balanced by the moment of W in clockwise direction round the same point.

We may, if we like, write the above equation as—

$$R \times EF - W \times x = 0$$

This simply expresses the fact that if a body (the rod) is in

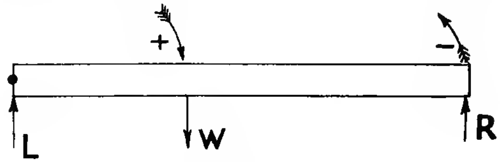


Fig. 89.

equilibrium, and clockwise moments are called positive, while anti-clockwise moments are called negative, the sum of the moments of the forces acting on the rod (round any point) is zero.

This is the **second law of equilibrium**, and it holds good, whether the forces are parallel or not.

It will be noticed that the moment of L round E is zero, it

being situated at no distance from E, and therefore its product into that distance will be zero.

It must also be clearly understood that we are considering the equilibrium of the *rod*, and therefore the positive or negative sign of a moment will depend on whether *the force is trying to turn the rod round the point in question* in the positive or negative direction.

Returning for a moment to the table on p. 76, we see that within the limits of error in reading the spring-balances the sum of the forces in each case is zero.

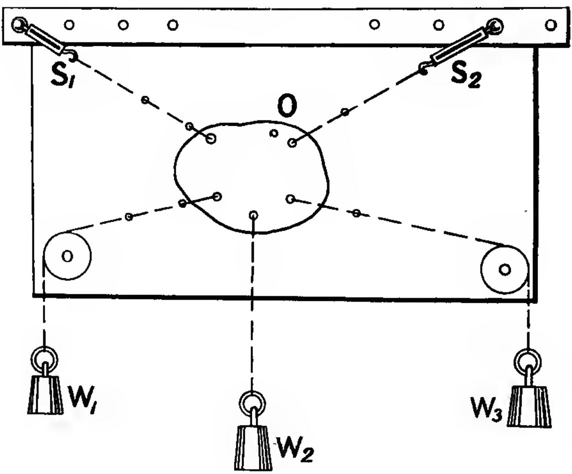


FIG. 90.

This is simply another illustration of the first law of equilibrium given on p. 50.

The second law may be further illustrated in the following manner :—

Take a piece of sheet celluloid, or strong cardboard, or wood, and attach it to the cords of the apparatus in Fig. 65, as shown in Fig. 90. Its weight must be small in comparison with W_1, W_2, and W_3, or it will introduce some error which our

knowledge, so far as we have gone, does not permit us to take into account.

The sheet is in equilibrium under the action of five forces, W_1, W_2, W_3, and S_1 and S_2. If the second law of equilibrium is true, the sum of the moments of those forces round *any* point, such as o, must be zero.

As in Fig. 65, mark off two points under each cord on a large sheet of paper pinned on the board behind the cords. These are shown by small circles.

Then remove the paper and draw in the lines of action of the forces. Take any point, O, on the paper, and from it draw perpendiculars O*a*, O*b*, O*c*, etc., on to the lines of action of the forces. These are shown dotted in the next figure (91). Add together the moments of these forces (calling those in one direction positive, and those in the opposite direction negative), and it will be found that their sum is approximately zero.

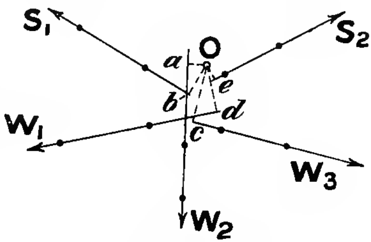


FIG. 91.—Position diagram.

Thus, $W_2 \times Oa - S_1 \times Ob - W_1 \times Od + W_3 \times Oc + S_2 \times Oe = O$.

We may now write down the laws of equilibrium, always remembering to use proper signs for both forces and moments.

1st Law.—If a body is in equilibrium, the sum of the components (in any two directions) of the forces acting on it is zero.

2nd Law.—If a body is in equilibrium, the sum of the moments of the forces acting on it, round any point in space, is zero.

Example.—A beam, whose weight may be left out of the calculation for the present, until we have dealt with the properties of the centre of gravity of a body, supports three loads of 2, 3, and 5 tons, as shown in Fig. 92. Find the supporting forces, L and R, at the ends of the beam.

The beam is in equilibrium under the action of five forces,

2, 3, 5, L, and R tons, therefore we can apply the second law of equilibrium. Take moments round a point A in the line of action of L. The moment of the 2 tons is $2 \times 3 = 6$ tons-feet, and this *tries to turn the beam round A* in clockwise direction, which we will call positive. Similarly, the moment of the 3 tons round A is $3 \times 4 = 12$ tons-feet in the positive direction, and the moment of

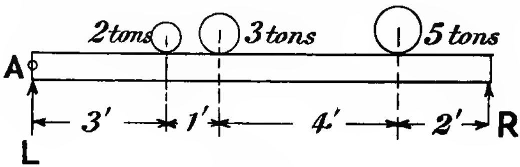


Fig. 92.

the 5 tons round A is $5 \times 8 = 40$ tons-feet, also positive. The moment of R round A is $R \times 10$, and is negative, because it tries to turn the beam round A in anti-clockwise direction. The moment of L round A is $L \times O$, which is zero.

The second law of equilibrium tells us that if these moments are added together the result must be zero, that is—

$$6 + 12 + 40 - 10\,R = 0$$
$$\text{or } R = 5{\cdot}8 \text{ tons}$$

The other supporting force, L, may be found in a similar manner by taking moments round some point in the line of action of R. But it is quicker to apply the first law of equilibrium, by which we know that if we add up all the forces (they being all vertical) their sum is zero. That is—

$$L - 2 - 3 - 5 + R = 0$$

Substituting 5·8 tons for R, we have

$$L = 10 - 5{\cdot}8 = 4{\cdot}2 \text{ tons}$$

Another application of the first law of equilibrium is found in determining the thickness of a cylindrical shell to withstand a given internal pressure.

Let t equal the thickness of the shell, l its length, and d its internal diameter. One half of the shell balances the other

half, and the pressure of one half of the steam balances the equal and opposite pressure of the other half; the halves being divided by a plane, ABCD, containing the axis of the cylinder.

The short arrows shown in the figure represent the tension in the material or the stress produced by internal pressure.

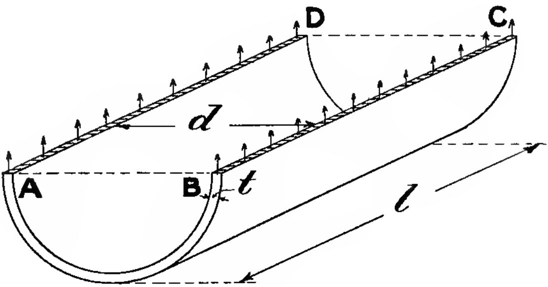


FIG. 93.

This is balanced by the downward pressure of the steam (in the half removed) on the rectangular section ABCD, the pressure being communicated to the semicircular shell by the steam contained between ABCD and the lower half of the shell.

Let p = pressure of steam per square inch.

Total pressure of steam on ABCD $= p \times d \times l$ lbs.

total resistance of material $= t \times l \times 2 \times f$ lbs.

where f = stress or resistance in lbs. per square inch.

Equating the above (by the first law of equilibrium) we get—

$$p.d.l. = 2.t.l.f.$$

$$\text{or } t = \frac{pd}{2f}$$

Example.—Solve the problem on page 56 by calculation.

From C drop perpendiculars a and b on to the lines of action of

P and E. Then, by the second law of equilibrium, the sum of the moments of the forces P and E round C must be zero, that is—

$$P \times a - E \times b = 0$$

Measure a and b, and insert in this equation, and E can be found.

Next, to find the third force acting through C. Resolve P and

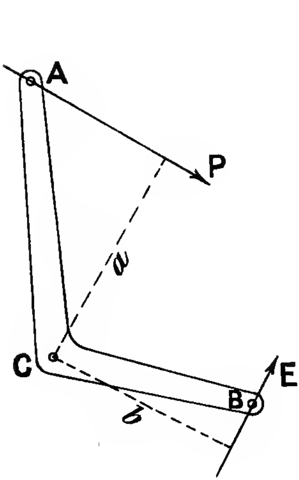


FIG. 94.

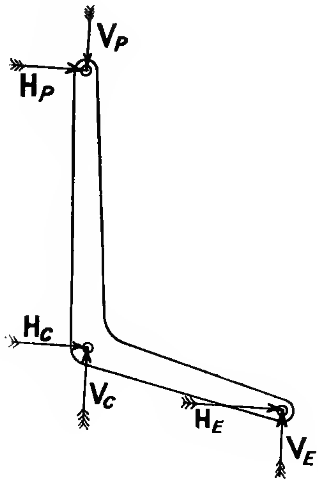


FIG. 95.

E vertically and horizontally, and let V_P and V_E (Fig. 95) be the respective vertical components, while H_P and H_E are the corresponding horizontal components.

Let the third force at C be also resolved horizontally and vertically, the components being H_C and V_C.

Then by the first law of equilibrium the sum of the components of the forces acting on lever in any direction is zero, or—

$$V_E + V_C - V_P = 0$$

This gives—

$$V_C = V_P - V_E$$

Similarly—

$$H_P + H_C + H_E = O$$
$$\text{And } H_C = -(H_P + H_E)$$

The force R at C, of which H_C and V_C are the components, is given by

$$R^2 = H^2{}_C + V^2{}_E$$

because H_C and V_C are at right angles to one another. (See Introduction.)

In this manner the problem may be solved by calculation, though the author would strongly recommend a student to draw it out to scale.

Centre of Gravity or Centre of Mass.—We are now in a position to deal with the centre of gravity of a body or bodies.

Let the bodies shown in Fig. 96 be assumed to be rigidly

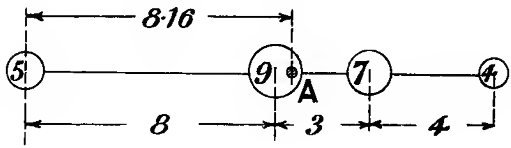


FIG. 96.

connected together by a weightless wire. Where can a single support be applied so that they will remain balanced as shown?

Let the supporting force be F, and situated at a distance x from the centre of the weight 5.

By the first law of equilibrium—

$$F - 5 - 9 - 7 - 4 = 0$$
$$\text{or } F = 25$$

By the second law the sum of the moments of all the forces round any point, such as the centre of the weight 5, must be zero, or—

$$(4 \times 15) + (7 \times 11) - Fx + (9 \times 8) + (5 \times 0) = 0$$

Putting in 25 for F, we get—

$$\frac{60 + 77 + 72}{25} = x$$
$$= 8{\cdot}16$$

The single support F must be placed 8·16 to the right of the weight 5, or 0·16 to the right of the weight 9. The point A in the centre line of the weights, through which the support acts, is called the *centre of gravity* of the bodies, and is really the point through which the resultant of the weights acts. This resultant is equal to the supporting force F, but acts, of course, in the opposite direction, that is, in the same direction as the weights themselves.

The point A is also sometimes called the centre of the parallel forces, but we shall return to this in a future problem.

The centre of gravity is thus seen to be the point about which the body or bodies will balance, or it is the point through which the resultant of all the weights must pass.

Example.—The figure (97) is cut out of a sheet of iron of uniform thickness. Find its centre of gravity.

Divide the outline into a number of parallel strips of equal width, as described in finding the area of an irregular figure. (See Appendix.)

Let w be the weight of a strip, with a suffix denoting which strip is referred to, such as w_3 for the third strip. Also let x_3 denote the horizontal distance of the centre of the third strip from the line AB; while the distance of the centre of gravity of the whole figure is X from AB.

Using the method of the last article, we know that if we take the moment of each strip round a point in AB, and then the moment of the single supporting force through the centre of gravity round the same point, the sum of these moments must be zero, from the second law of equilibrium.

Let W be the single supporting force, then—

$$w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + w_5x_5 + w_6x_6 - \mathrm{WX} = 0$$

$$\text{or } \frac{w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + w_5x_5 + w_6x_6}{\mathrm{W}} = \mathrm{X}$$

But from the first law of equilibrium we have—

$$w_1 + w_2 + w_3 + w_4 + w_5 + w_6 - W = O$$

therefore W = the sum of the weights of the strips.

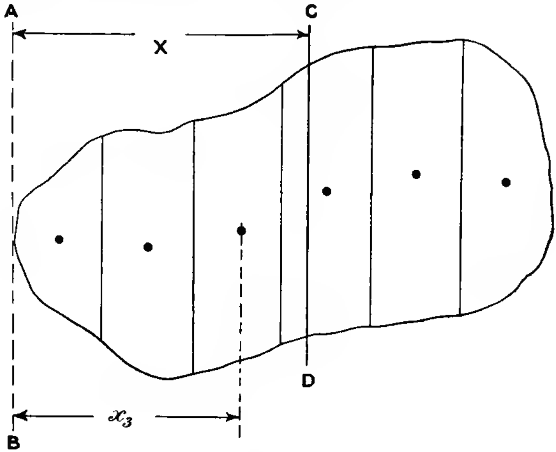


Fig. 97.

Now, the weight of a strip = area of strip × thickness × weight of 1 cubic inch.

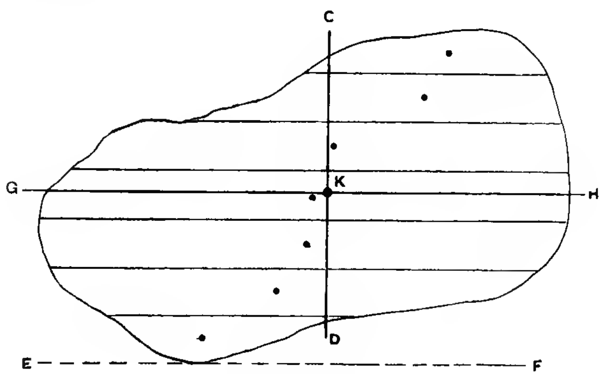


Fig. 98.

Or w = width of strip × mean height of strip × t × i

$= b \times h \times t \times i$ (say).

The thickness is constant, and so is the width of a strip and the weight i of a cubic inch, hence we may take these quantities outside the bracket and write the above equation for X thus –

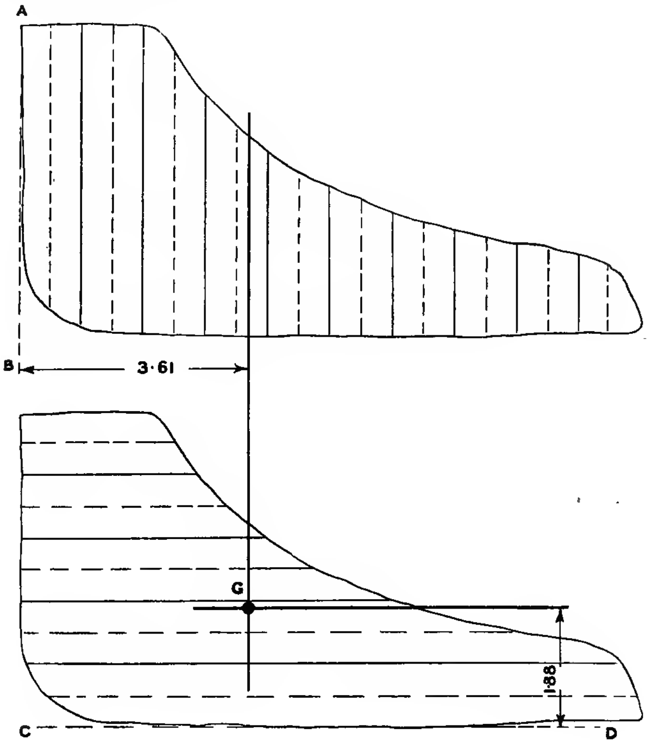


Fig. 99.

$$X = \frac{bti(h_1x_1 + h_2x_2 + h_3x_3 + h_4x_4 + h_5x_5 + h_6x_6)}{bti(h_1 + h_2 + h_3 + h_4 + h_5 + h_6)}.$$

Hence the rule—

Multiply the middle height of each strip by its distance from

AB. Add these products together, and divide the result by the sum of the middle heights. This result is the distance of the centre of gravity from AB.

We have now found the line CD (parallel to AB) which contains the centre of gravity; but we have not yet found whereabouts in CD the centre of gravity is.

Now divide the figure into strips parallel to the line EF (Fig. 98), and repeat the above calculation. Another line GH will then be found which passes through the centre of gravity. As both lines, CD and GH, contain the centre of gravity, it must lie at their point of intersection K.

In Fig. 97 the diagram was divided into six strips. This is generally too few, ten being the least number which should generally be used.

Example.—Find the centre of gravity of the area given in Fig. 99.

The student should always measure off the middle heights in decimals, and not express them as vulgar fractions.

Measure off the middle heights, and their distances from some fixed line, and tabulate them as below:—

Middle heights in inches, upper fig. 99.	Product of the middle heights into their distances from AB.	Middle heights in inches, lower fig. 99.	Product of the middle heights into their distances from CD.
4·43	2·22	2·5	11·3
4·88	7·32	3·3	11·6
4·46	11·15	4·8	12·0
3·31	11·59	7·6	11·4
2·67	12·01	8·8	4·4
2·2	12·10		
1·8	11·70		
1·53	11·48		
1·37	11·70		
0·97	9·23		
27·62	100·50	27·0	50·7

$$\text{Distance of centre of gravity G from AB} = X = \frac{100·5}{27·62} = 3·61 \text{ in.}$$

$$\text{Distance of centre of gravity G from CD} = Y = \frac{50·7}{27} = 1·88 \text{ in.}$$

These numbers should be checked experimentally by cutting out the figure in zinc or tin, and hanging up by a piece of thread (Fig. 100), and drawing a vertical through the point of suspension

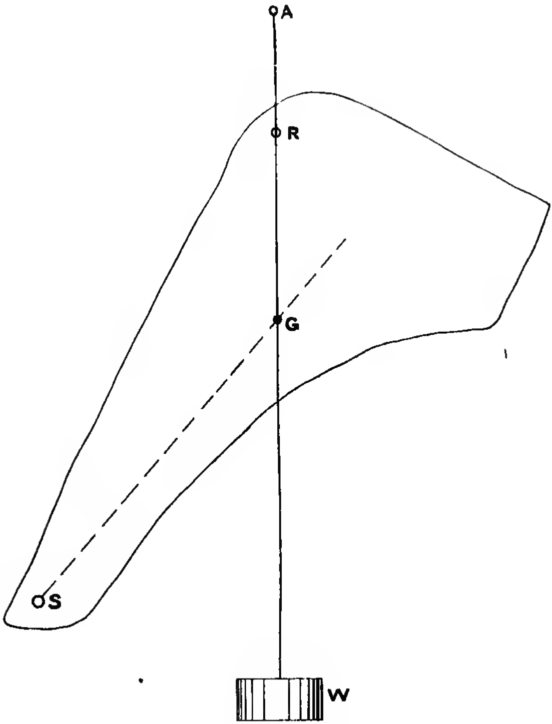


FIG. 100.

by means of a plumb-line. Repeat the process with some other suspension-point such as S. The intersection of the verticals is the centre of gravity.

Example.—A beam weighs 5 cwt., and is 10 ft. long. It is supported at one extremity, and 6 ft. from that extremity. It is loaded at two points, 1 ft. and $6\frac{1}{2}$ ft. from the overhanging end, with 9 and 13 cwt. respectively. Find the supporting forces.

The centre of gravity through which the weight of the beam

must act will be at its middle point. Putting in the forces, as in Fig. 101, and taking moments round A, we know from the second law of equilibrium that the sum of these moments is zero. That is—

$$(R \times 6) - (13 \times 2{\cdot}5) - (5 \times 1) + (9 \times 3) = 0$$
$$\text{or } R = 1{\cdot}75 \text{ cwt.}$$

By the first law of equilibrium we know that the sum of all the forces must be zero, or—

$$9 - L + 5 + 13 - R = 0$$
$$\text{But } R = 1{\cdot}75$$
$$\text{hence } L = 24{\cdot}25 \text{ cwt.}$$

Example.—A ladder, AB, rests against a smooth wall at A, and

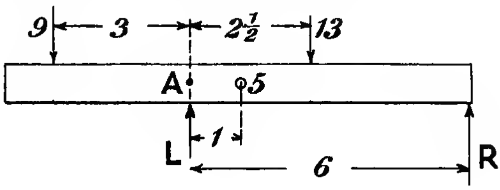


Fig. 101.

on the ground at B. The centre of gravity of a man and the ladder is at G (Fig. 102), the two together weighing 210 lbs. Find the pressure of the wall on the ladder, and that of the floor on the ladder, and get the coefficient of friction between the floor and ladder.

The weight of man and ladder will act vertically downwards through G (Fig. 102). The pressure of the wall will act perpendicular to the surface of the wall at A. These two lines of action will intersect at K, hence the third force (the pressure of the floor on the ladder) must act through K. It also acts through B. Hence join BK. This is its line of action.

Draw the triangle of forces, *o*, *b*, *c* (Fig. 103), for the three forces passing through K, then *co* is the pressure of the wall, and *bc* the pressure of the floor on the ladder.

Reproduce *bc* (Fig. 103) in Fig. 104. This is the resultant of

the vertical pressure of the floor on the ladder, and the resistance of friction in a horizontal direction. Hence, resolve *bc* vertically and horizontally (Fig. 104), and we get *bm*, the pressure perpendicular to the surfaces in contact, which produces the friction *mc*.

$$\text{As friction} = \mu \times \text{perpendicular pressure}$$

$$mc = \mu \times bm$$

$$\text{and } \frac{mc}{bm} = \mu$$

It will be seen, by comparing Figs. 103 and 104, that *mc* is

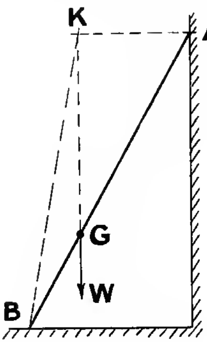


FIG. 102.

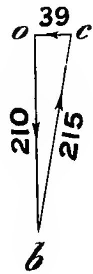


FIG. 103.

FIG. 104.

equal and opposite to *co*. This is only what we should expect from the first law of equilibrium.

It should be noticed in connection with this problem that as the man ascends the ladder, the centre of gravity of the two also moves up the ladder. Let it have moved to M (Fig. 105), then K has moved to L, and BL is less inclined to the horizon than BK.

As *bm* (Fig. 106) equals the weight of man and ladder, it remains constant, and consequently *mc* must have increased to *md*, so that *bm* and *md* may have a resultant in the direction *bd*. This shows that the friction *mc* must have increased to *md*. If the component *md* becomes greater than the greatest possible friction, the foot of the ladder will slide outwards.

This shows that the higher the man ascends the ladder the more likely its lower end is to slide outwards. To prevent this, the centre of gravity must be brought lower down the

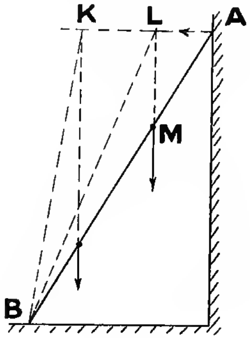


FIG. 105.

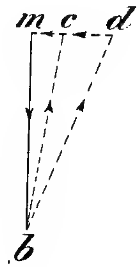


FIG. 106.

ladder, and this can be done by a person standing on the lowest round of the ladder, as is often done.

Example.—Find the centre of gravity of the given Figure 107.

Let W represent the weight of the complete rectangle ABCD, supposing no piece to be cut out of it, and let w be the weight of the piece EFGH cut out; then the weight of the piece ABHGFECDA must be $W - w$.

Let M be the centre of gravity of the pieces weighing $W - w$, and K that of the piece EFGH. The centre of gravity of the complete rectangle is at L. It is required to find the distance of M from Q.

Taking moments round Q, we must have the moment of the piece GE + the moment of the remainder

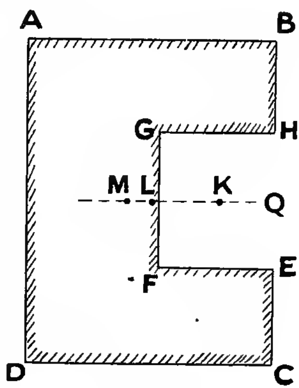


FIG. 107.

ABHGFECD = moment of complete rectangle ABCD.

$$\text{Or } w \cdot KQ + (W - w)MQ = W \cdot LQ$$

i.e.
$$MQ = \frac{W \cdot LQ - w \cdot KQ}{W - w}$$

Parallel Forces.—In a previous chapter we learnt that the resultant of a number of forces was the equal and opposite of the equilibrant or force required to produce equilibrium. We have learnt in the present chapter that the equilibrant of a number of parallel forces added to the sum of the forces (with proper signs) equals zero.

Hence the resultant of a number of parallel forces is their algebraical sum.

The *position* of the equilibrant is found by using the second law of equilibrium, as in the problems on beams, given a few pages previously. As the equilibrant is equal and opposite to the resultant, the position of the latter is the same as that of the former.

Parallel forces are said to be *like* when they act in the *same direction*, but they are said to be *unlike* when they act in *opposite directions*.

Centre of Parallel Forces.—If the directions and points of application of a number of parallel forces are given, find the line of action of the resultant as described above. Turn all the lines of action of the forces through any angle, and find the new position of the resultant. The point in which the two resultants intersect is called the centre of parallel forces. Notice that a centre of gravity is a centre of parallel forces.

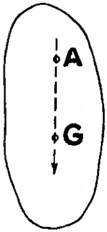


FIG. 108.

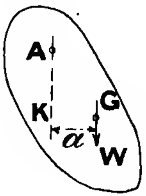


FIG. 109.

Stable and Unstable Equilibrium.—From the work already done we know that if a body be suspended, as in Fig. 109, from the point A, and its centre of gravity is at G, and further, if it is displaced from its position of rest, as shown in the right illustration, the force of gravity acts vertically downwards through the centre of gravity G, and produces a moment W × *a*, tending to rotate

the body back towards its position of rest. The same will happen if it is displaced to the left instead of the right. There is no moment resisting the above moment, hence the force of gravity will turn the body towards its position of rest, and in that position there will be no moment acting on the body (from second law of equilibrium). The above moment was $W \times a$; hence, when at rest (as the moment is zero), a must be zero; that is, the centre of gravity, G, must lie vertically *under* the point of suspension A.[1] This is a case of *stable equilibrium*, which we may define as follows:—A body is in stable equilibrium when it tends to return to its original position after being slightly displaced.

If the body tends to move further from its original position, after being slightly displaced, it is said to be in *unstable equilibrium.*

If it does not tend to move further from or return to its original position, it is said to be in *neutral equilibrium.*

A sphere on a horizontal table is a case in point.

The Balance shown in Fig. 261 is used for weighing, and we require to know the conditions which govern the construction of an accurate balance. Its principle depends upon the fact that if two bodies are suspended from the arms of a lever or beam, and balance one another, the moment of one body round the turning-point equals the moment of the other body round the same point. If the length of each arm is the same, then the weight of each body must be the same.

We have here considered the weights of the bodies alone, neglecting the weight of the beam. In every balance the beam has weight which must be considered.

Let the beam of a balance be represented in Fig. 110 by the line AB, and let the centre of gravity of the beam and vertical pointer be in the bearing edge of the knife-edge.

If the beam balances, and the areas AD and DB are equal, the weights, W_1 and W_2, must be equal. But we have at once this defect, that although the weights are equal, we are not able

[1] This is another way of stating that the centre of gravity of a body in equilibrium is at its lowest possible position.

to detect that they are equal from using the balance because the beam will come to rest in any position such as that shown dotted; the conditions holding equally well for the dotted as for the original position.

To make the balance useful we must compel the beam to come to rest in a horizontal position, and it must be *stable* in that position, *i.e.* it must return to its position of rest if displaced.

We have seen in the previous article that, for this to happen, the centre of gravity of the beam must lie in the same vertical line as the knife-edge, and *underneath* it, when at rest.

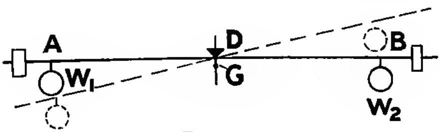


FIG. 110.

Assuming the beam comes to rest in a horizontal position, the centre of gravity of the beam must be at some point, G, Fig. 110. The further this point is from the knife-edge, the more rapidly will the beam return to its position of rest, but at the same time the balance will be less sensitive. Hence the position of G is the result of a compromise between sensitiveness and rapidity of weighing.

The bodies are supported in scale-pans resting upon knife-edges fixed at A and B on the beam. The bearing edges of

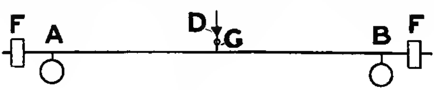


FIG. 111.

these knife-edges should be in the same horizontal line as the knife-edge at D, for if not, let the central knife-edge be above the others, as in Fig. 111. When the beam is displaced, the scale-pans are no longer at equal distances from the vertical through the central knife-edge, as shown in Fig. 112. The

left scale-pan is nearer by an amount y than when the beam is horizontal, while the other scale-pan is at almost the same distance as before. Also the centre of gravity, G, is displaced to the right of its true position of rest, producing a moment equal to the weight of the beam multiplied by x. In this

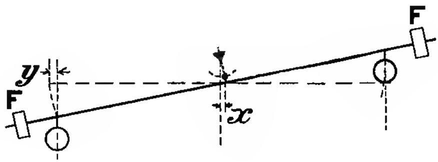


FIG. 112.

condition the balance is too stable, and not sensitive enough except for rough work. Should the outer knife-edge be above the central one, the beam would be unstable because its centre of gravity would be above the centre of support, and would then be altogether useless. The little weights F screw on to the ends of the beam, and permit of the balance of the beam being adjusted.

The Unadjusted Balance.—The balance has not been adjusted properly unless the arms are of equal length, the scale-pans of equal weight, and the centre of gravity of the beam is exactly underneath, and almost coincident with the knife-edge suspension. At the same time an unadjusted balance may be used to get the true weight if it is not permanently deranged.

When the arms are of unequal length, let a be that of one, and b that of the other arm. Also let W be the true weight of a body, while P is the weight in one scale-pan required to balance it, and Q the required weight when it is put in the other pan. Taking moments about the point of suspension in each case, we have—

$$P \times a = W \times b$$
$$Q \times b = W \times a$$

Multiplying, we get—

$$PQ\,a\,b = W^2\,a\,b$$

$$\text{or } \sqrt{PQ} = W$$

Again—

$$\frac{P}{W} = \frac{b}{a}$$

$$\text{and } \frac{W}{Q} = \frac{b}{a}$$

$$\text{or } \left(\frac{b}{a}\right)^2 = \frac{P}{Q}$$

$$\text{and } \frac{b}{a} = \sqrt{\frac{P}{Q}}$$

Hence if the arms are of unequal length, the above method of *double weighing* enables the true weight to be found, it being the square root of the product of the weights of the body in both scale-pans. The ratio of the unequal arms is given by the square root of the dividend of the two weights.

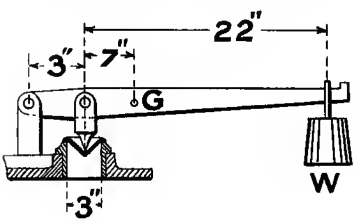


Fig. 113.

Another method is to balance the weight of the body by shot, sand, or other material. Replace the body by proper weights until a balance is obtained. The weights equal the weight of the body.

Example.—The safety valve in Fig. 113 is 3 in. in diameter, and is just on the point of blowing off steam. The lever weighs 4 lbs., and its centre of gravity is situated at G.

If the weight on the end of the lever is 40 lbs., what is the pressure in the boiler? The weight of the valve is 2 lbs.

Let p = the absolute steam pressure in pounds per square inch.

Then $\frac{\pi}{4} \times 3^2 \times p$ = total pressure of steam on valve upwards.

Let a = atmospheric pressure in pounds per square inch.

Then $\frac{\pi}{4} \times 3^2 \times a$ = total pressure of atmosphere on valve downwards.
Weight of valve = 2 lbs. downwards.

Total upward force on lever $= \frac{\pi}{4} \times 3^2(p - a) - 2$

Taking moments of forces round pin at left end, we have—

$$\left[\frac{9\pi}{4}(p - a) - 2\right]3 - (4 \times 10) - (40 \times 25) = 0$$

$$\text{or } \frac{27}{4}\pi p - \frac{27\pi a}{4} - \frac{6}{4} - 40 - 1000 = 0$$

Putting in a = 15 lbs. per square inch, we get—

p = 64·4 lbs. per square inch absolute

or 64·4 − 15 = 49·4 lbs. per square inch above the atmosphere.

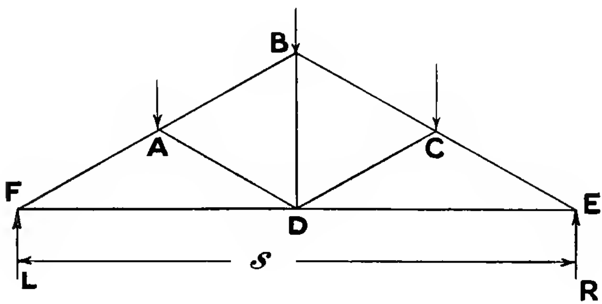


FIG. 114.—Structure or position diagram.

This latter is the pressure indicated by the ordinary pressure-gauge.

Example.—A framed structure (Fig. 114) is loaded as shown at the joints A, B, and C. All the joints[1] are assumed to be frictionless hinges. It is required to determine the force in each member of the structure.

[1] A joint is the coupling together of any two members.

This problem is most quickly solved by a mechanical method, based on the laws of equilibrium, but at this stage we shall simply apply the information we have already obtained.

Applying the second law of equilibrium by taking moments round F, and calling the span of the structure s, we have—

$$Rs - 2000 \times \frac{3s}{4} - 3000 \times \frac{2s}{4} - 1000 \times \frac{s}{4} = 0$$

Cancelling s, we have R = 3250 lbs.

By the first law—

$$R + L - 1000 - 3000 - 2000 = 0$$

$$\text{and } L = 6000 - R$$
$$= 6000 - 3250$$
$$= 2750 \text{ lbs.}$$

The *pin* of the hinge at F is kept in equilibrium by three forces, namely L, one along DF, and one along AF, therefore these forces

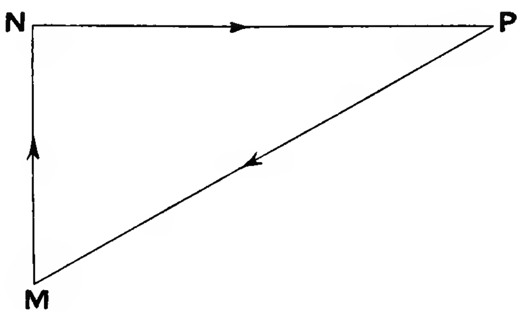


Fig. 115.

can be represented by a triangle. Draw MN (Fig. 115) to represent L = 2750 lbs. to some convenient scale, say 1000 lbs. to 1 inch, and through one end (say N) draw a line parallel to the force in FD. (We have seen, on page 53, that the force in FD must act along FD.) Then through the other end M draw a line parallel to the force in AF, intersecting NP in P.

Put the arrows on the sides NP and PM. These are the directions in which the forces in FD and FA act upon the *pin*

of the hinge to keep the *pin* in equilibrium. Put in the arrows on the members near F. The member FD pulls the pin F, therefore it must be in tension. Similarly the member AF pushes the pin F, and consequently it must be in compression.

A member in compression at one end must be so throughout its length, that is, it must push whatever it is acting on at its other end. The member FA is acting on the pin at A, hence it must push it. At A we have four forces acting, two of which are known. Draw TU (Fig. 116) parallel to and representing 1000 lbs., then

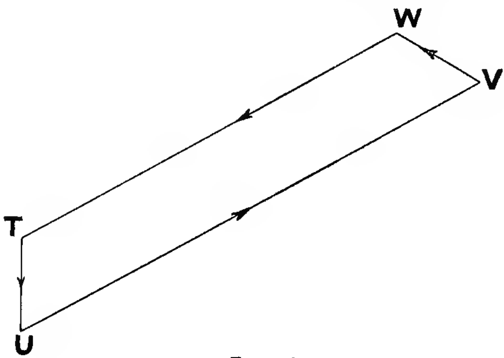


FIG. 116.

UV parallel to and representing the force in FA at A. Through V draw a line parallel to DA, and lastly through T draw a line parallel to AB, and meeting VW in W. Put the arrows on the lines as in Chapter III., then VW represents the force in DA, and it pushes A, therefore it is in compression. Similarly WT represents the force in BA, which is for the same reason in compression. Put the arrows in Fig. 114.

Proceeding in this way round the structure point by point, the forces can all be found, that in BD being tensile.

Forces in an Engine Mechanism.—An engine mechanism (Fig. 117) consists essentially of four parts, the cylinder and framework D, which also acts as a support or bearing for C; a piston, A, sliding in the cylinder to and fro along the centre line AC; a crank, BC, turning round C, and a rod AB connecting A to B.

The path of B is shown by the dotted circle.

As in the roof frame in the previous example, we must take

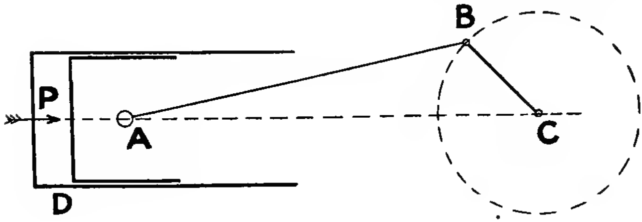


FIG. 117.

each body separately. Consider first of all the gudgeon-pin A. It is acted on by three forces (Fig. 118), namely the pressure, P, of the working fluid, the pressure of one of the guides vertically, and the pressure of the connecting-rod in the direction along the rod. Drawing the triangle of forces (Fig. 119) for the pin A, we get the forces PQ and R, the last being upwards, and the direction of Q indicating that the connecting-rod is in compression; that is, it must push the pin B from left to right.

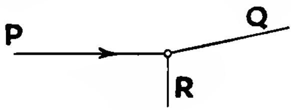


FIG. 118.

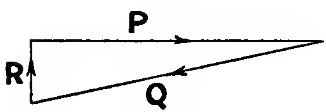


FIG. 119.

Resolve this push, Q, into two components (Fig. 120) parallel and perpendicular to the crank. The component N cannot assist in turning the crank, as it acts through the centre of the shaft, but the component E is entirely devoted to the turning of the crank, its moment round the centre of the shaft being E × length of crank.

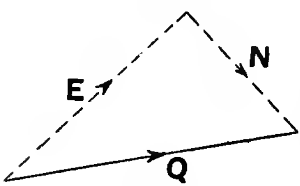


FIG. 120.

This latter is generally called the *turning moment* on the crank-shaft, and the component E is called the *crank effort.*

The turning moment can be more easily found in another way.

Produce the crank CB (Fig. 121) to cut the vertical through A in F. Put in the figure all the forces, keeping the connecting-rod in equilibrium. They are the crank effort E,

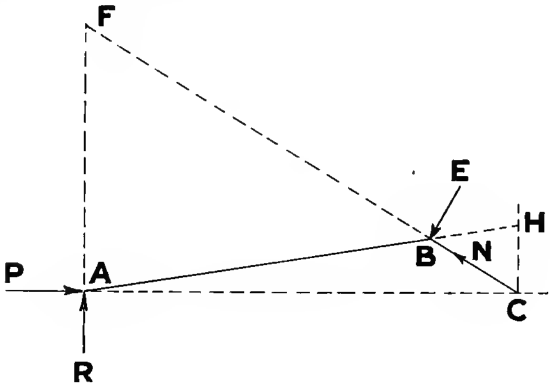


FIG. 121.

the radial pressure N along the crank, and the pressures P and R at A. The second law of equilibrium tells us that if we take the moments of these forces round any point in space, their sum must be zero. Take moments round F—

$$\text{Then } (E \times BF) + (N \times 0) + (R \times 0) - P \times AF = 0$$

$$\text{or } E = P \times \frac{AF}{BF}$$

And the turning moment on the crank-shaft—

$$= E \times BC = P \times \frac{AF}{BF} \times BC$$

Now HC and FA are parallel, and FB and BC are in the same line, as also are AB and BH; hence the angle FBA

= the opposite angle HBC, also the angle FAB = the alternate angle CHB and AFB = BCH (Euclid I. 29). Therefore the triangle ABF is similar to the triangle CBH (see Introduction), and consequently the sides of one triangle are proportional to the corresponding sides of the other.

The angle at A is opposite the side FB, and in the other triangle the angle H is opposite the side BC; then FB and BC are corresponding sides in the two triangles.

Similarly, AF and CH are corresponding sides, therefore—

$$\frac{FA}{FB} = \frac{CH}{CB}$$

$$\text{or } \frac{FA}{FB} \times CB = CH$$

Substituting this in the above equation, we get—

$$\text{Turning moment} = E \times BC = P \times CH$$

$$\text{and crank effort } E = P \times \frac{CH}{BC}$$

The variation in the turning moment can be best shown by a diagram.

Draw a horizontal line TV (Fig. 122) equal in length to

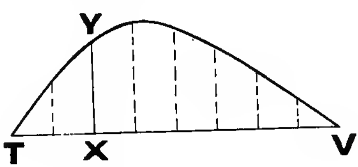


FIG. 122.—Crank effort diagram.

half the circumference of the dotted circle in Fig. 117, that is equal to the half the path of the crank-pin. Divide this into a number of equal parts, not less than ten. Mark off corresponding points round the half circumference, giving the positions of the crank-pin (Fig. 123). Produce the connecting-rod to cut

the vertical through the centre of the crank-shaft in H. Then $P \times CH$ is the turning moment, and $P \times \frac{CH}{BC}$ the crank effort. Set up this quantity, XY, to a convenient scale at X, and repeat the process for every point in the half circumference. We

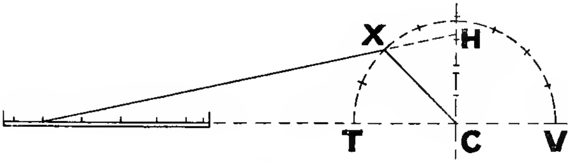


FIG. 123.

then get the outline TYV (Fig. 122), the ordinate to which gives the crank effort. The student should read the account in Chapter VIII. of an experiment to verify the above.

Summary of Chapter IV.

***1st Law of Equilibrium.*—The sum of the components of forces keeping a body in equilibrium is zero in any direction.**

***2nd Law of Equilibrium.*—The sum of the moments of the forces keeping a body in equilibrium is zero round any point.**

The centre of gravity of a body is the point about which it will balance, or it is the centre of parallel forces of gravity upon its particles.

Examples on Chapter IV.

1. A dead-weight safety-valve is 4 in. in diameter; the weight of the valve and spindle is 20 lbs. What dead-weight would require to be added so that steam should blow off when the pressure reaches 80 lbs. per square inch? *Ans.* 980 lbs.

2. Draw a square, ABCD, and a diagonal BD; forces of 10, 12, and 20 units act from B to C, D to C, and B to D, respectively; if the side of the square is 3 units long, find the moment with respect to A of the resultant

of the three forces, and state in what direction the forces tend to turn the square round A.

Define the moment of a force with respect to a point. *Ans.* 36·5.

3. Define the moment of force with respect to a point.

State the relation that holds good between the moments of two forces, and the moment of their resultant; all the moments being taken with respect to a point in the plane of the forces.

A, B, C, D are the angular points of a square taken in order, a force of 20 units acts from A to B, and a force of 15 units from A to D; by means of the rule you have enunciated, find the moment of the resultant about the middle point of DC, a side of the square being 6 units long.

4. Draw a square, whose corners in order are A, B, C, D and whose side is 12 units long; a force of 7 units acts from A to B, and a force of 10 units acts from A to D.

(*a*) If the moments of the forces are taken about C, will they have the same signs or opposite signs, and why?

(*b*) Find the algebraical sum of the moments about C.

(*c*) Find the point in CD about which the moments are equal but of opposite signs. *Ans.* − 36 and 8·4 from D.

5. Draw a square ABCD; a force of 8 units acts from A to D, and two parallel forces of 12 units act from A to B and C to D; find the resultant. Also find what the resultant would be if the first force acted from D to A. *Ans.* 8 and 8.

6. A uniform straight bar, 14 in. long, weighs 4 lbs.; it is used as a lever, and an 8 lb. weight is suspended at one end. Find the position of the fulcrum when there is equilibrium. *Ans.* $2\frac{1}{3}$ ins. from 8 lbs.

7. The handle of a claw-hammer is 15 in. long, and the claw is 3 in. long. What resistance of a nail would be overcome by the application of a pressure of 50 lbs. at the end of the handle?

You are required to show, by a diagram, the manner in which you arrive at your result. *Ans.* 250 lbs.

8. Define "moment of a force." How is it measured? A bar of metal of uniform section weighs 5 lbs., and a weight of 10 lbs. hangs from one end. It is found that the bar balances on a knife-edge at 9 in. from the end at which the weight hangs; what is the length of the bar? *Ans.* 54 in.

9. Weights of 1, 2, 3, and 5 lbs. are hung from points 1 ft. apart on a string 5 ft. long, the 1 lb. weight being 1 ft. from one end. The ends of the string are attached to two points 3 ft. apart on the same horizontal line, and the form assumed by the string is drawn on a piece of paper held behind it. Show how to find from measurements on the figure thus formed, and a graphical construction, the tension in the various parts of the string.

10. A body consists of two portions; the centre of gravity and weight of each portion is known; show how to find the centre of gravity of the whole body.

A square lamina is divided into four equal squares by lines parallel to the sides, a circle is inscribed in one of these squares and the portion of the

lamina within the circle is removed; find the centre of gravity of the remainder.

11. A force (P) acts at a point A, and a parallel force (Q) at a point B; specify the force (R) that will balance them (*a*) when they are alike, (*b*) when they are unlike parallel forces (*i.e.* when they act (*a*) in the same direction, (*b*) in opposite directions).

If AC = 3 ft., and P = 10 units, and Q = 12 units, specify R when P and Q act in opposite directions.

12. Find the resultant of two parallel forces for the case when they act in opposite directions.

There are 3 bricks each 10 in. long of the same shape and material. The lowest one lies on the ground, and the others are placed upon it so that each projects x in. over the one immediately beneath it. If the bricks are just on the point of falling over, find the value of x.

13. Define the centre of gravity of a body.

A wheel of known weight has a lump of lead of known weight attached to its rim. Explain how you could determine, experimentally, the position of the centre of gravity of the whole, and how you calculate the position of the centre of gravity of the lead.

14. Find the centre of gravity of three equal weights not in the same straight line.

A table standing on four legs has two flaps, each of the same size and weight as the middle piece of the top of the table. If one flap is hanging down and the other horizontal, find what weight placed on the outside edge of the horizontal flap will just upset the table, the weight of the supports being neglected. *Ans.* Weight of one flap.

15. Define the centre of two parallel forces, and find its position in the case of two unlike parallel forces acting at given points.

A, B, C, D, in order, are the angular points of a square; unlike parallel forces, D and C, act at A and B. Find the parallel forces that must act at C and D if the centre of the four forces is at the middle point of the square, and show that the problem can be solved in only one way.

16. What is meant by the moment of a force about a point?

An irregularly shaped beam balances about the middle point of its axis when masses of 20 and 40 lbs. are suspended one from each end, and about a point one-third of the length from one end when the masses are interchanged. Find the mass of the beam and the distance of its centre of gravity from one end. *Ans.* 120 lbs. and one-third its length.

17. Find the sectional area of the members AB, BC, AC, and DB (Fig. 124), if tension members are allowed a stress of $1\frac{1}{4}$ tons per square inch, and compression members a stress of $\frac{1}{2}$ ton per square inch. If there were a member joining B to E, find the stress in this member.

18. A triangular frame, ABC (Fig. 125), is supported at A by a hinge, and at C by a rope which passes over the pulley D. Find the stress in each member AB, BC, and CA, and in the rope CD.

NOTE.—Begin at the point B.

19. What conditions must be fulfilled by a body in equilibrium? Illustrate your answer by working the following problem :—A, B, and C are three points such that AB = 3 ft., BC = $1\frac{1}{4}$ ft., and AC = $3\frac{1}{2}$ft. ABC is a bent lever turning about B. A force of 13 lbs. is exerted at A in a direction inclined at 45° to AB, and another force acts at C in a direction inclined at 60° to BC. Find the force at C and the third force at B.

Solve the problem also by the aid of the triangle of forces. Describe each step carefully, and give reasons for taking it.

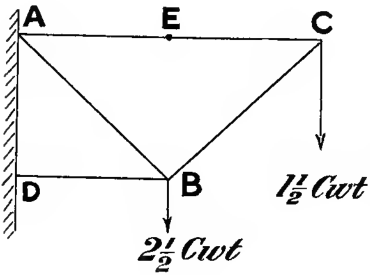


FIG. 124.

20. AB is a horizontal line, and C is a point 2 in. away from B and $1\frac{1}{4}$ in. above its level. D is another point 2 in. from C and the angle BCD = 150°. AB represents a rough floor, BC a plank resting on it at B, and DC a cord. The weight of the plank is 60 lbs. Determine the coefficient of friction between the plank and the floor.

21. A rectangular trap-door, measuring 4 ft. square, and weighing 75 lbs.,

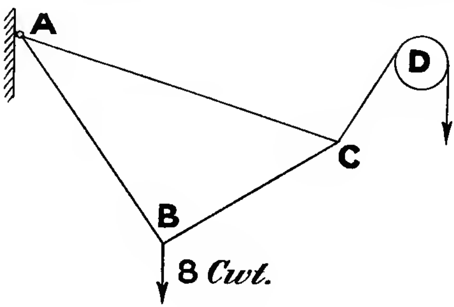


FIG. 125.

is hinged with one edge horizontal, and is supported in the horizontal position by a chain which is connected with the middle point of the outer edge of the trap-door, and with a point vertically over the middle point of the edge in which the hinges are fixed, but 7 ft. above it. Sketch the arrangement, and determine the tension upon the chain and the reaction on the hinges. *Ans.* 41 and 41 lbs.

22. Draw a vertical line AB, 3 in. long, A being above B. Produce

AB to C, making BC 2 in. long. E is a point to the right of AB, such that ABE is 56° and BAE 100°. Similarly, D is a point on the left of AB, such that BAD = ABD = 48°. The structure represents a partially balanced crane with a footstep bearing at C, and a horizontal support at B. A mass of 5 tons is suspended from D, and of 2 tons from E. Find the supporting forces, and the stresses in all the members.

23. State the rule for finding the resultant of two like parallel forces.

What is meant when two parallel forces are said to be "like"?

A uniform rod AB, 6 ft. long, rests on two points in a horizontal line, 5 ft. apart; one of the points is under A. If the weight of the rod is 20 lbs., find the pressure on each point.

24. A uniform rod AB, 5 ft. long, weighs 40 lbs. A mass of 60 lbs. is attached to it at a point 1 ft. from A. A cord $3\frac{1}{2}$ ft. long is fastened to the end A, and another 4 ft. long to B. If the free ends of the cords are made fast to a peg, how will the whole hang, and what will be the tension in the cords?

25. A beam 10 ft. long is fastened at its ends to a couple of ropes. The free end of one is made fast to a fixed point. The free end of the other is pulled by a force of 5 cwt. in a horizontal direction, the weight of the beam being 1 ton. Find the tension in the other cord, its position, and the position of the beam.

26. Define the centre of gravity of a body. If a body is suspended from one point round which it can turn freely, what statement does this circumstance enable us to make as to the position of its centre of gravity? If the body were a piece of tin plate cut in the form of a parallelogram, and hung up by a point in one of its edges? What further statement can we make as to the centre of gravity?

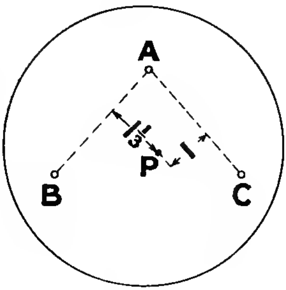


FIG. 126.

27. A round table is supported on 3 legs at A, B, and C. AB = 3 ft., AC = 3 ft., BC = 4 ft. A weight of 100 lbs. is placed at P, and the distance of P from AB and AC is $1\frac{1}{3}$ ft. and 1 ft. Find how much each leg supports.

Ans. A = 22·3 lbs., B = 33·3 lbs., C = 44·4 lbs.

28. The weight of the rod (Fig. 127) is 7 lbs. and the load on the rod is 28 lbs. at 9 in. from the left end. Find the supporting forces R and L.

29. A bent lever, one arm of which is 2 ft. 2 in. long, and the other arm 5 in. long, has a force of 33 lbs. applied at the end of the longest arm; what force must be applied at the end of the shorter arm to keep it in equilibrium?

Do you know anything about the direction of the supporting force?

30. AB (Fig. 128) is a non-uniform body weighing 100 lbs. Describe how you would find its centre of gravity if it is suspended by two cords, CA and BD, from pegs C and D, in the position shown. Also how would you find the tension in the cords and in AB?

31. Four particles, whose masses are m, n, p, q, are placed at the angles of a square; find the position of their centre of gravity, and the condition that the centre of gravity may be at the point of intersection of the diagonals.

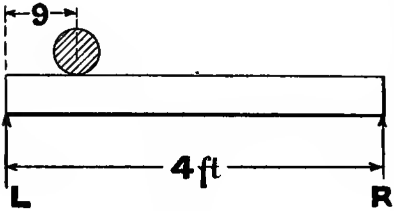


FIG. 127.

32. A body rests on a rough inclined plane. Angle of inclination = $\tan^{-1}\frac{1}{4}$. Coefficient of friction between plane and body 0·3. (1) Find what force applied upwards, parallel to the plane, will just move the body. (2) Downwards, parallel with the plane. (3) Upwards and making an angle of 30° with plane. (4) Upwards, and making an angle of 30° with plane (below the plane). Also find the least force which will make the body move. *Ans.* (1) 0·56W; (2) 0·07W; (3) 0·52W; (4) 0·78W; (5) 0·05W down, 0·52W up.

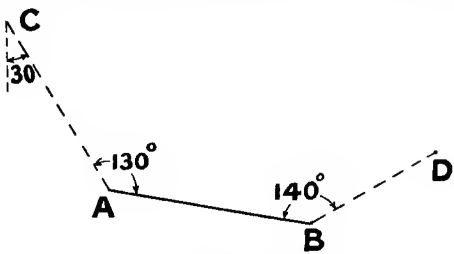


FIG. 128.

33. In what sense may an area be said to possess a centre of gravity?

ABCD is a plane quadrilateral having the sides AB and CD parallel. Determine the position of its centre of gravity.

34. Define the moment of a force with respect to a given point.

Draw an equilateral triangle ABC, and let a side represent a length of 3 ft.; a force of 10 lbs. acts along BC. Find the numerical value of its moment with respect to A.

The force may act either from B to C or from C to B. How would the moment in the one case be distinguished from the moment in the other case? *Ans.* 26 lbs.-feet.

35. What do you mean when you say a body is in equilibrium? Illustrate your answer by finding the position of a second supporting force of 2 tons that must be applied to the following beam to produce equilibrium. The beam weighs $\frac{3}{4}$ ton, and the shaded area represents a uniform load of $\frac{1}{4}$ ton per foot run. The beam is 20 ft. long (Fig. 129).

36. Explain in detail the steps you take, and why you take them, in solving the following problem.

A post, CD, is hinged to the floor at C (Fig. 130). A rope, AB, is

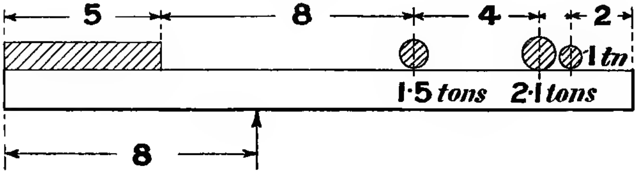


Fig. 129.

fastened to the post at B and to the floor at A. The other rope, ED, is pulled with a force of 5 cwt. Find the tension in the rope AB, and the pressure of the hinge on the post at C.

Ans. Tension in AB, 16 cwt.; pressure at C, 16·1 cwt.

37. Find the resultant of the forces A, B, C, D, E, and F (Fig. 131).

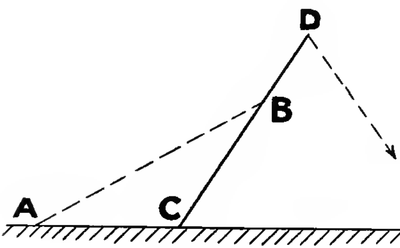


Fig. 130.

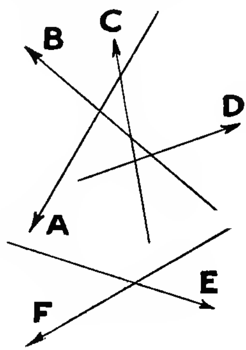


Fig. 131.

Find also the sum of the horizontal and vertical components of these forces, and of the resultant.

38. Two points, A and B, situated 3 ft. apart, but the level of A is 2 ft. higher than that of B. Three strings, AO, BO, and WO, are tied together

at O, and a weight of 20 lbs. is supported by the string OW. Obtain, by drawing, the tensions in AO and BO when AO = 2·2 ft. and BO = 1 ft.

39. Describe how you would experimentally determine the position of the centre of gravity of an irregular solid, such as a bicycle.

(*a*) Does the non-rigidity of the body affect the problem?

(*b*) Does the fact that the wheels can rotate affect it?

40. A uniform bar, 10 ft. long, balances over a rail, with a boy weighing three times as much as the bar hanging on to the extreme end of it. Draw a figure showing its balancing position.

41. A uniform isosceles triangle has its two equal sides each 5 ft. long, and its base 8 ft. long; find its centre of gravity. If its weight be 5 lbs., and a weight of 10 lbs. be hung at the vertex, find the centre of gravity of the whole.

42. Find the load which must be placed at one corner of an equilateral triangular plate to bring the centre of gravity to the middle of a perpendicular bisector (or median line).

43. A uniform beam, 14 ft. long and weighing 120 lbs., is attached to two props, one of which is 3 ft., and the other 5 ft., from its centre; calculate the forces on the props when a weight of 100 lbs. is placed first at one end and then at the other end of the beam.

44. Explain fully the circumstances under which a rectangular block, standing on a plank which is being gradually tilted, shall topple over, being prevented from sliding by a small obstacle. As an example, take the case of a block 8 × 5 × 5 cub. in.

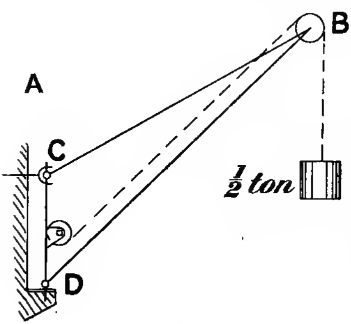


FIG. 132.

45. If in Fig. 84 the string BC were cut with a scissors, what would be the tension in AB immediately after cutting?

46. The mass of an equilateral triangle is 4 lbs. Masses of 1, 1, and 2 lbs. respectively are placed at its angular points. Find the centre of mass of the system.

47. A piece of thin sheet-iron is cut into the form of a T with the following dimensions: width of head 8 in.; breadth of head 1 in.; length of tail 6 in., width $1\frac{1}{4}$ in. Calculate the exact position of the centre of gravity. How could you find the position experimentally?

48. ACB = 60° (Fig. 132), CDB = 45°. Find the stress in each member and the tension on the wall-stay at C.

49. A frame (Fig. 133) is arranged as shown in sketch. Find the stress in each member, and state whether tension or compression. Also find the two supporting forces.

50. A frame, as in sketch (Fig. 134), is loaded as shown. The dotted line is a rope. Find the tensile or compressive stresses in each member.

51. DAC (Fig. 135) is a heavy rod whose centre of gravity is at G, and weighs 50 lbs. MBC = 50°, BCD = 35°, BC = 6 ft., CG = 3 ft., CA =

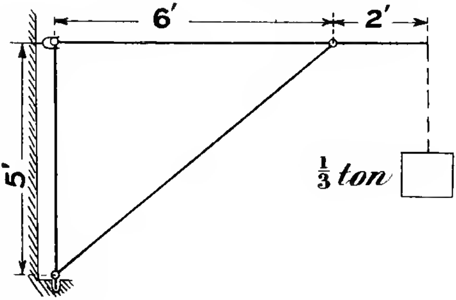


FIG. 133.

4·5 ft. At A the rod rests against a rough cylindrical peg. The coefficient of friction between rod and peg is μ. Find μ. If the friction were removed, how and why would the rod begin to move? *Ans.* $\mu = 0·3$.

52. The lever of a safety-valve is balanced and is 24 in. in length; the distance between the fulcrum and the end of the valve-spindle is 3 in., the

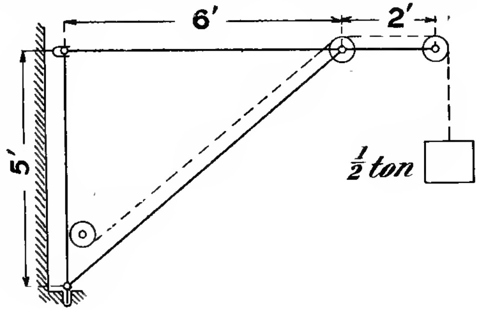


FIG. 134.

diameter of the valve being $2\frac{1}{2}$ in. Find the weight to be put on the end of the lever in order that the steam may escape at a pressure of 50 lbs. per square inch, the weight of the valve being neglected. *Ans.* 30·5 lbs.

53. Define centre of mass.

A uniform wire, 10 in. long, is bent so as to consist of two pieces at

right angles to each other; one of these pieces is 3 in. long, the other 7 in.; find the perpendicular distances of the centre of mass of the system from the two portions of the wire. *Ans.* 2·45 in. and 0·45 in.

54. A bar projects 6 in. beyond the edge of a table, and when 2 oz. is hung on to the projecting end the bar just topples over; when it is pushed out so as to project 8 in. beyond the edge, 1 oz. makes it topple over. Find the weight of the bar, and the distance of its centre of gravity from the end. *Ans.* 2 oz. and 12 in.

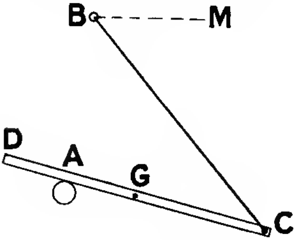


FIG. 135.

55. A stone, weighing 1 ton, is suspended in the air by a chain; a rope fastened to the stone is pulled so that the chain makes 30°, and the rope 60°, with the vertical. Draw a very careful figure, showing the three forces acting on the stone, and a triangle representing them. Find the pull on the rope. *Ans.* (1) ½ ton. (2) 1 ton.

56. A rod without weight rests horizontally on two points, A and B, 10 ft. apart; between A and B take points C, O, D, such that AC = 2 ft., AO = 4 ft., AD = 7 ft.; a weight of 100 lbs. is hung at C and one of 90 lbs. at D. Find the algebraical sum of the moments with respect to O of the forces on one side of O.

57. A wire is stretched horizontally between two points 6 ft. apart, and a 1-lb. weight is then hung by a string from the middle of the wire, which is pulled down by the weight 1 in. below the horizontal. Draw a figure showing the forces which act at the point of attachment of the weight to the wire, and find approximately the pull on each of the end points to which the wire is fastened. *Ans.* 18 lbs.

58. A cubical block of stone, weighing 150 lbs., rests on the ground, which is so rough that the block will not slide. It is to be tilted up so as to rest on the edge. Find the least force which, applied at the opposite edge in a horizontal direction, will begin to tilt the block, and find the least force in any direction at that edge which will begin to tilt it.

Ans. 75 lbs. and 53 lbs.

59. A flat board is placed on a table. How could you find its centre of gravity by pushing it over the edge of the table?

60. ABCD is a square, the sides AB and AD being taken as axes of co-ordinates; masses of 1, 2, 3, 4 units are placed at A, B, C, D respectively. Find the co-ordinates of the point where a mass of 5 units must be placed that the centre of gravity of the whole may be at A. Show in a diagram the position indicated by the co-ordinates.

61. Show by the aid of a figure, or of figures, how the degree of stability of a coach or cart depends on the height of the centre of gravity.

The wheels of a coach are 5 ft. apart, and the centre of gravity is 10 ft.

from the line of contact of the wheels and ground on either side. To what height may the wheels on one side run up a bank before the coach is upset?

62.[1] A tripod whose vertex is A, and whose legs are AB, AC, AD, of lengths 8, 9, and 9·5 ft. respectively, sustains a load of 2 tons. The ends B, C, D form a triangle whose sides are BC = 7 ft., CD = 6 ft., BD = 8 ft. Find by graphical construction the compressive stress in each leg.

Ans. AB, 1·3 tons; AC, 0·44 tons; AD, 0·45 tons.

63. A sheer-legs is formed of two sheer-poles, BC, DC, each 25 ft. in length, and secured to a base-plate in the ground at B and D. The wire guy or tension rope, AC, is attached to the ground at a point A, which is 60 ft. distant from BD. The vertical from the top, C, of the poles meets the ground at a distance of 10 ft. from the centre of the line BD, which is 15 ft. long. Find the tension in tons on the guy when a weight of 20 tons is suspended. *Ans.* 11·3 tons.

64. A uniform plank is placed across 2 trestles, the middle of the plank being midway between the trestles. Find how the pressure on a trestle changes as a man, whose weight is 5 times that of the plank, walks along the plank from one trestle to the other.

65. If G is the centre of a mass of a uniform triangular lamina ABC, show that forces represented in magnitude by GA, GB, GC will be in equilibrium.

66. In a wharf-crane, the post, tie-rod, and jib measure 15, 20, and 30 ft. respectively, what would be the nature and amount of the stresses in each of the three members when a load of 7 tons is suspended over the pulley at the jib-head, (1) when the lifting chain passes from the pulley to the drum or barrel parallel with the jib, (2) when the drum is placed so that the chain passes from the jib-head parallel with the tie-rod?

67. A weightless rod, 3 ft. long, is supported horizontally, one end being hinged to a vertical wall and the other attached by a string to a point 4 ft. above the hinge; a weight of 180 lbs. is hung from the end supported by the string. Calculate the tension in the string, and the pressure along the rod.

68. Explain why, in a common scale-pan or letter-balance it does not matter whereabouts on the pan the weights are placed; although they may be sometimes near, and sometimes further off, the fulcrum. (See Mechanism.)

69. A uniform beam 12 ft. long, and weighing 56 lbs., rests on and is fastened to two props 5 ft. apart, one of which is 3 ft. from one end of the beam. A load of 35 lbs. is placed (*a*) on the middle of the beam, (*b*) at the end nearest a prop, (*c*) at the end furthest from a prop; calculate the weight each prop has to bear in each case.

70. A rod 10 in. long, weighing 8 lbs., has a mass of 4 lbs. suspended

[1] This example requires a knowledge of solid geometry, and is too difficult for most students.

from one end, and 6 lbs. from the other ; find the position of its centre of gravity if it remains horizontal when supported at the middle point.

71. State how to find the centre of gravity of two particles whose masses are given, and which are placed at a given distance apart. If the masses are 5 and 7 units, and they are placed 3 ft. apart, how far is their centre of gravity from each of them?

72. Describe the essential parts of a common balance, and say how the balance must be constructed if it is to be (*a*) true, (*b*) quick, (*c*) sensitive.

Could you weigh an object properly if the arms of the balance beam were of different lengths? Could you do so if the beam was all right, but one pan was heavier than the other? If you could, say how in each case.

73. In a pair of pincers the jaws meet at $1\frac{1}{2}$ in. from the pin forming the joint. The handles are grasped with a force of 50 lbs. on each handle at a distance of 8 in. from the pin. Find the compressive force on an object held between the jaws, and also the pressure upon the pin.

74. A series of particles, weighing 3, 20, 5, 16, 30, and 36 lbs., have respectively the following co-ordinates (x and y) 3 and 5, 7 and 9, 3 and 0, -4 and 1, -6 and -5, 2 and 2. Place the particles in position, and calculate the position of their centre of gravity.

75. A man sells pounds of sugar with a balance the scale-pans of which differ by $\frac{1}{2}$ oz. Supposing he uses each pan alternately, does he lose or gain anything?

76. What are the qualities of a good balance? If the arms are unequal show how it may be used to weigh correctly. How could you show that there was a difference in the length of arms?

CHAPTER V.

RECIPROCAL FIGURES AND THE FUNICULAR POLYGON.[1]

THE example on page 97 was solved by taking each hinge separately and drawing the polygon of forces for each hinge. The process was somewhat tedious, and it would be more convenient if a shorter method could be used. Such a method has been devised, and it embodies a new method of lettering the structure diagram. In this we shall no longer letter the hinges as in previous work, but instead letter the *space* between two forces; thus, in the adjoining figure the force represented by the arrow on the left is called AB, and the force represented by the arrow on the right is called BC.

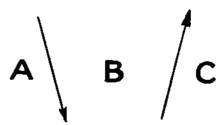


FIG. 136.

Take as an example the roof-truss shown loaded in Fig. 114, page 97. It has been redrawn in Fig. 137, with the new method of lettering.

The load of 1000 lbs. lies between B and C, and is called BC. Similarly, the left supporting force lies between the space A and the space B, and is called AB. *Note that all the letters in the structure diagram are capitals.*

Calculate the supporting forces AB and EA as in previous problems. Now begin to draw the force diagram, but this will be lettered in the old way at the ends of each line *with italic letters;* at the same time the letters in the structure and force

[1] Students reading this book for the first time may omit this chapter or defer the reading of it till a later period. Those who take Building Construction as their principal subject will generally find it advantageous to take this chapter in the order given.

diagrams must correspond. Thus, in Fig. 138, take any point *b*, and draw *bc*, *in the direction of* BC, to represent the force BC to a scale of 2000 lbs. to one inch. **Write the scale on your paper near the diagram before beginning to draw any lines.** Now from *c* draw *cd* in the direction of CD to represent CD, and from *d* draw *de* to represent DE. Then from *e* draw or mark off *ea* to represent the supporting force EA, and the remaining force *ab* represents the remaining supporting force AB. The line *bcde*, *eab* is sometimes called the load line, and beginning at *b*, then drawing to *e* and back again to *b* simply indicates that the structure is in equilibrium, the

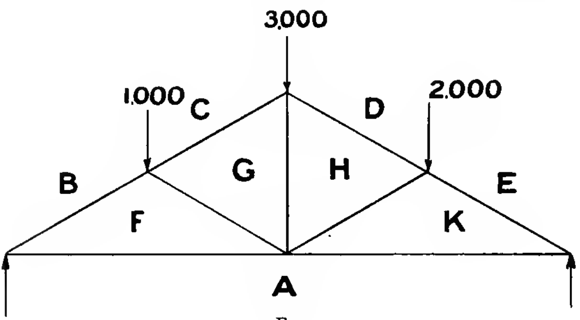


FIG. 137.

two lines being really the sides of a polygon of forces in which the starting and finishing points are the same.

Now letter the spaces F, G, H, K, between the members in Fig. 137. Begin with one end of the structure diagram, say the left end, and through *b* (in Fig. 138) draw a line parallel to the member BF, then *f* must lie somewhere on the line just drawn.

Next, through *a* draw a line parallel to AF. The point *f* must be on this line also, hence it must be at the point where this line cuts the previous line.

Now through *f* draw a line parallel to FG, then *g* must be on this line. Similarly through *c* draw a line parallel to CG, then *g* must be on this line also. Hence it must be at the point of intersection of these two lines.

Proceeding in this way, draw a line through *g* parallel to GH, and another through *d* parallel to DH, and the point *h* is found. Through *h* draw a line parallel to HK, and another through *e* parallel to EK. Draw the remaining line *ak*, and

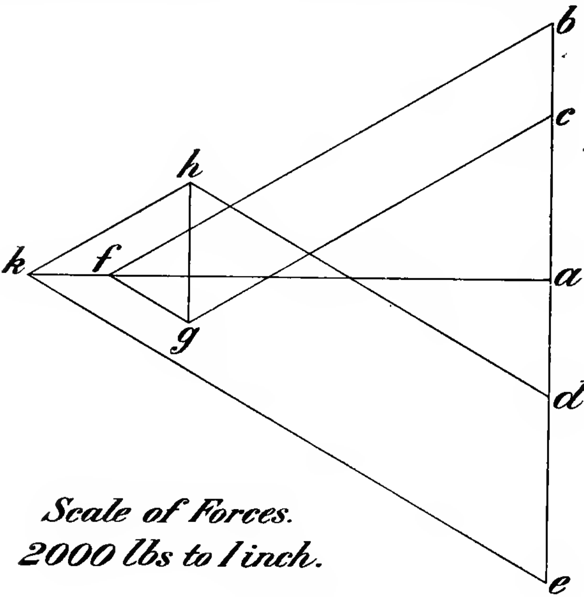


FIG. 138.

if this line is parallel to AK the force-diagram has been accurately drawn; if not, something has been put in wrong.

A table should now be made, giving the forces in the various members, as measured on the force diagram (Fig. 138).

Member.	Force.	Member.	Force.
	lbs.		lbs.
AP	+ 4700	DH	− 4470
BP	− 5470	HK	− 2000
PG	− 1000	AK	+ 5600
CG	− 4500	EK	− 6450
GH	+ 1500		

The only thing that remains to be done is to determine the kind of force in each member; that is, whether the member is in tension or compression.

As previously shown, a polygon of forces can be drawn for each hinge, and the directions indicated by the arrows on the sides of the polygon are those in which the corresponding members act *on the pin of the hinge.* Take, as an example, the hinge at which the central load of 3000 lbs. acts. The known force of 3000 lbs. acts *downwards*, the corresponding line in Fig. 138 being from *c* to *d* (not *d* to *c*). Move round the hinge in the direction from C to D, D to H, H to G, and G to C. Then move round the corresponding polygon in Fig. 138 in the same direction, namely from *c* to *d*, *d* to *h*, *h* to *g*, and *g* to *c*, putting an arrow on each line.

These are the directions in which the members act upon the hinge in question. The member CG *pushes* the hinge, therefore the member is in compression. The member GH pulls the hinge, hence it is in tension. Because a tension stretches or increases the length of a member a tensile force is indicated by a *plus*, and a compressive force by a *minus*, because it causes the member to become shorter.

The Funicular Polygon.—In the case of the roof-frame just discussed, the supporting forces at the ends were *calculated*. We can, if we like, find these by a process of drawing something similar to that just described.

It does not matter whether the forces are supposed to act on a roof, a beam or any other *rigid* structure, as long as their lines of action are not changed or disturbed in any way.

In the demonstration which follows, the roof-frame has given place to a single rigid rod on which two forces, W_1 and W_2, act as shown in Fig. 139.

Put in lines to represent the position and direction of the supporting forces, and letter the spaces between the forces in the same manner as in the last problem.

Draw the force polygon *bc*, *cd* (Fig. 140) representing the loads W_1 and W_2. We now require the position of the point *a* in *bd*. This we obtained in the last problem by calculation. We now want to get it by drawing.

Take *any* point O on your paper (Fig. 140) and join *oc.* Now, anywhere through the space C (Fig. 139) draw a line, such as *km*, parallel to *oc* in Fig. 140. Join *od*, and through the space D and the point *m* draw *mn* parallel to *od*, cutting AD in *n*. Join *ob*, and through the space B and the point *k* draw a line parallel to *Ob*, cutting the left support in *e*.

We now have lines *ek*, *km*, *mn*, in Fig. 139, drawn across all the spaces except A, parallel to corresponding lines in Fig. 140, drawn from O to the letters corresponding to the spaces.

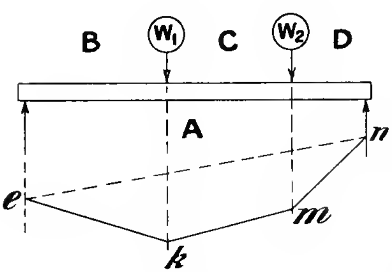


FIG. 139.—Structure diagram.

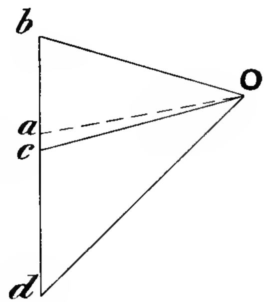


FIG. 140.—Force diagram.

Join *en* (Fig. 139), and as every line in Fig. 139 has a parallel line in Fig. 140, we must now draw through O a line parallel to *en*. This line is O*a*, giving the required point, *a*, in the load line. The left support is AB, in Fig. 139, and, therefore, in Fig. 140 it will be *ab*. Similarly, the right support DA is *da* in magnitude.

It will save confusion to redraw that part of Fig. 139 required for the proof of the correctness of the above method.

Consider any vertical section of the rod distant x from the left end and x_1 from W_1 (Fig. 141). The triangle *krs* has its sides drawn parallel (by construction) to the sides of the

[1] A space is *anywhere* between the lines of action of two forces; for instance, the space B is anywhere between the verticals through *e* and *k*, whether above or below the rod.

triangle *bco* (Fig 142); hence their corresponding sides are proportional. That is—

$$\frac{bc}{co} = \frac{sr}{rk} \text{ or } \frac{bc}{sr} = \frac{co}{rk}$$

Also, in the same manner, if *op* and *kt* are horizontal lines—

$$\frac{op}{kt} = \frac{co}{rk} = \frac{bc}{sr}$$

Now, $kt = x_1$ in Fig. 141, and let *op* be called H. Also $bc = W_1$. Then substituting, we get—

$$\frac{H}{x_1} = \frac{W_1}{sr} \text{ or } W_1\, x_1 = H \,.\, sr$$

But $W_1 x_1$ is the moment of W_1 about a point in the line of

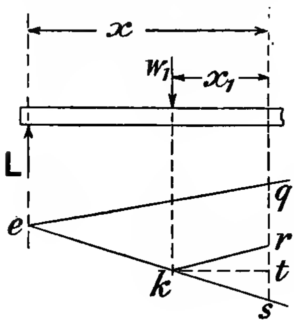


FIG. 141.

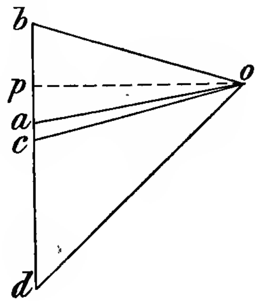


FIG. 142.

section we are considering; and the equation shows that this moment is equal to the constant distance op = H multiplied by the intercept *rs* cut off from the line of section by two lines, *kr* and *ks*, which have been drawn from a point in the line of action of W_1 parallel to the radial lines *oc* and *ob*, which terminate at the pole O at one end and at the extremities of W_1 at their other ends.

Similarly, the moment $L \times x$ of the supporting force L about the section under consideration, is H multiplied by the intercept *qs* cut off on the line of section by the two lines, *eq*

and *es*, drawn from a point in the line of action of L parallel to the two lines joining the pole O and the extremities *a* and *b* of the force L.

The sum of the moments of the forces to the left of the section (round a point in the line of section) is—

$$L \times x - W_1 x$$

Which equals—

$$\begin{aligned} H \times qs - H \times sr & \\ &= H (qs - sr) \\ &= H \times qr \end{aligned}$$

= H multiplied by the intercept of the line of section cut off by the funicular[1] polygon.

If the line of section be coincident with the right support, the second law of equilibrium says that the sum of the moments of the forces on the rod must be zero. The intercept cut off by the funicular polygon at *n* is zero. The same is the case at *e*.

It must be carefully noted that Fig. 139 is a diagram of lengths only, and Fig. 140 is a diagram of forces only, every line in it representing a force real or imaginary.

Remembering this, we see that *ek*, *km*, and *ne* may be a system of links or rods acted on by forces L, W_1, W_2, and R at their joints, and thus maintained in equilibrium. Also that the forces in these links would be represented by the lines *ob*, *oc*, *od*, and *oa*, in Fig. 140, to the same scale as the loads W_1 and W_2 are represented by *bc* and *cd*. Also that *bd* is vertical, and consequently the horizontal *op* = H represents the horizontal component of the forces *bo*, *co*, *ao*, and *do*, which in consequence is constant throughout the linkage. Further, we took *any* pole, O, and, as every pole will give a different funicular polygon, there are an infinite number of funicular polygons, all of which have the same length of intercept at the same place, the corresponding pole being in a line parallel to the load-line, at the same distance, H, from the line of loads.

[1] The figure *ekmne*, Fig. 139, is called the funicular, or link, polygon.

Example.—Given the Warren girder loaded, as in Fig. 143, determine the supporting forces and the force in each member.

Letter the spaces between the forces acting on the structure, remembering that the whole of the upper space between the supporting forces is D.

Draw the load line, *e*, *a*, *b*, *c*, *f*, and take any pole O. Draw the radial lines *oe*, *oa*, etc., and across the corresponding spaces, E, A, etc., in the structure diagram draw parallel lines forming the funicular polygon, *i*, *t*, *u*, *v*, *w*, *x*. Draw the closing line *ix*, and from the pole O draw a parallel line cutting the load line in *d*. Then, *de* is the left supporting force, and *cd* the right supporting force.

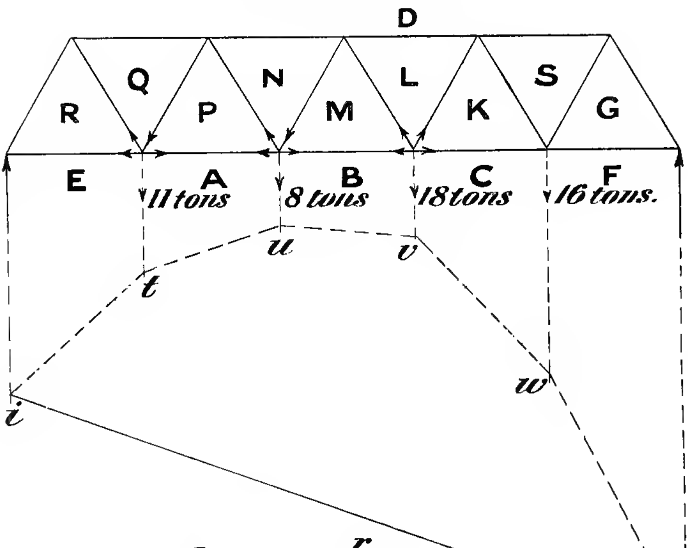
Now letter the spaces in the structure. Beginning at the left end, through *d* draw a line parallel to the member DR, and through *e* draw a line parallel to the member ER. These intersect in *r*. Again, through *r* draw a line parallel to the member RQ, and through *d* draw a line parallel to the member DQ. These intersect in *q*. Carrying out this method, the complete force diagram can be drawn.

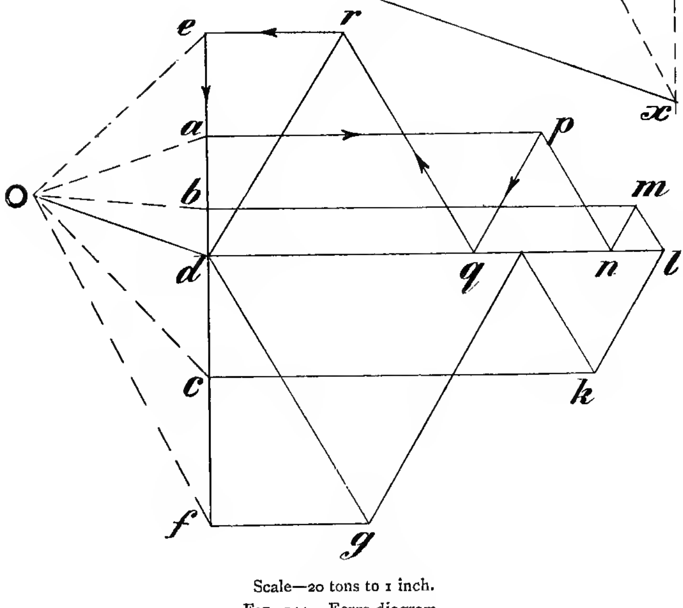
Next determine the kind of force in the different members, following out the method given on p. 118. Take the case of the hinge at which the load of 11 tons is applied. The known force is EA or AE. As it is downward, it must be from *e* to *a* in the force diagram, and consequently we must proceed round the hinge in the direction from E to A, A to P, P to Q, Q to R, and R to E. Passing over the corresponding lines in the force diagram, leaving an arrow on each line indicating the direction of procedure, we get the directions in which the members act *on the pin* of the hinge.

The forces in the structure can then be tabulated as below.

Member.	Force.	Member.	Force.
	tons.		tons.
DR	− 27·5	DQ	− 28·5
RQ	+ 27·5	DN	− 43·5
QP	− 14·5	DL	− 49·1
PN	+ 14·5	DS	− 33·9
NM	− 5·5	ER	+ 15·0
ML	+ 5·5	AP	+ 36·0
LK	+ 15·0	BM	+ 46·0
KS	− 15·0	CR	+ 41·6
SG	+ 33·5	FG	+ 17·0
GD	− 33·5		

FIG. 143.—Structure diagram.





Scale—20 tons to 1 inch.

FIG. 144.—Force diagram.

A Peculiar Case is presented in the structure shown in Fig. 145, and it is given here to show the student that the conditions laid down in the example on page 52 must be followed, or the problem cannot be solved by ordinary means, if at all.

The structure shown in the figure represents a roof-truss or frame, and is invariably constructed of wood. The members AB and BC are formed in one continuous piece. This at once violates the conditions mentioned above, and consequently, the method previously given cannot be used here. The same objection applies to the pieces CE and AE. We are then left

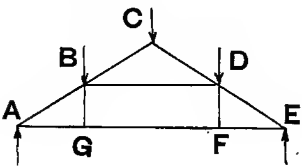


FIG. 145.

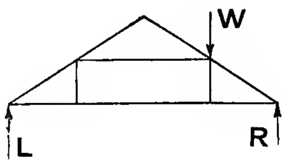


FIG. 146.

with a choice between taking the truss exactly as it is, and being unable to solve the problem, or—what is almost as bad—to make the assumption that there are hinged joints at BDG and F.

It will be interesting to follow out the solution of the problem for the instruction it affords, on the assumption that there are joints at BDF and G.

Assume for the moment that the frame is loaded with a force, W, Fig. 146. The right support, R $= \frac{3}{4}$ W, and the left support, L $= \frac{W}{4}$. This latter is trying to urge the part of the frame to the left of W in the upward direction. And there is nothing to prevent it from turning the rectangle into the dotted position, Fig. 147; in fact, we have here a mechanism made up of four hinged rods. But we have this force L in Fig. 146, and hence the frame will collapse, and is quite unable to sustain any load as suggested in Fig. 146.

It will now be necessary to enquire what is and what is not a structure which is capable of sustaining a load.

Examine any hinged structure we have already dealt with, say the Warren girder in Fig. 143, and let us gradually build it up.

A triangular frame will not give in the manner suggested

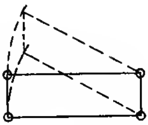


FIG. 147.

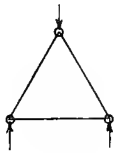


FIG. 148.

in Fig. 146 if two joints are fixed. We have already learnt that it is perfectly stable. Here (Fig. 148) we have three joints and three members. Now add two more members, as in Fig. 149, and we have five members and four joints. Add two

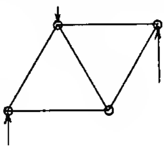


FIG. 149.

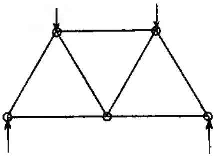


FIG. 150.

more members (Fig. 150), and we have seven members and five joints. It should be noticed that every two members added simply adds one joint.

Proceeding in this way until the whole structure is completed, we can record at every stage the number of members and joints in the structure as in the following table :—

Members.	Joints.	Members.	Joints.
3	3	13	8
5	4	15	9
7	5	17	10
9	6	19	11
11	7		

We want the relation between the number of members and joints.

Plot the number of members vertically and the number of joints horizontally, as in Fig. 151. The points lie on a straight line whose equation is[1]—

$$\text{Number of members} = 2 \times \text{number of joints} - 3$$

Returning to Fig. 145, we see there are 7 joints; hence there should be $2 \times 7 - 3 = 11$ members. We see that there

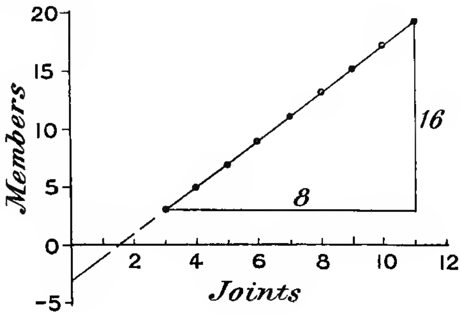


FIG. 151.

are only ten, hence one of the necessary members is missing, and must be supplied before we can proceed with the solution of the problem.

The missing member is either of the diagonals of the rectangle, Fig. 147. Putting in this member, the problem can be finished in the ordinary way, as previously explained.

If a structure has more members than that given by the equation—

$$\text{Members} = 2 \times \text{joints} - 3$$

it has too many, and it is impossible to solve the problem by the graphical means described in the early part of this chapter; while, if it has less than the above number of members, it is unstable, and will probably collapse.

[1] See Appendix for method of getting equation.

EXAMPLES ON CHAPTER V.

1. Calculate and find, by drawing, the supporting forces in Fig. 152. Also find the stress in each member.

2. Find the stress in each member of Fig. 153, and find the direction of the force at the hinge.

3. If the upward support in Fig. 153 had been made to slope upwards from left to right at an angle of 45° to the horizon, find the force at the hinge and the stresses in the members.

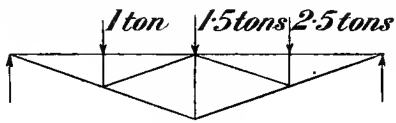


FIG. 152.

4. If weights of 2 cwt. were placed at each of the points in Fig. 153, where the members meet, where would be the centre of gravity of those weights?

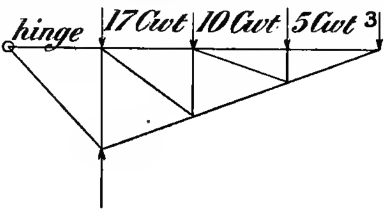


FIG. 153.

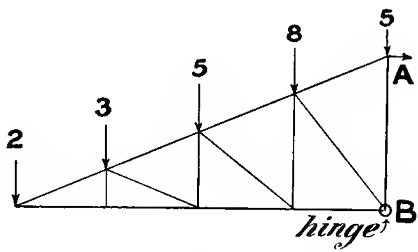


FIG. 154.

5. The numbers in Fig. 154 represent the loads in cwts. Find the stress in each member and the supporting forces at A and B.

6. If the hinge in Fig. 154 had been at A, and the horizontal force at B, what would then have been the supporting forces and the stresses in the members.

7. The numbers in Fig. 155 represent cwts. Find the stresses in the members.

8. In Fig. 143 replace the weights 11, 8, 18, and 16 tons by 13, 0, 17, and 4 tons respectively, and find the stresses in the members.

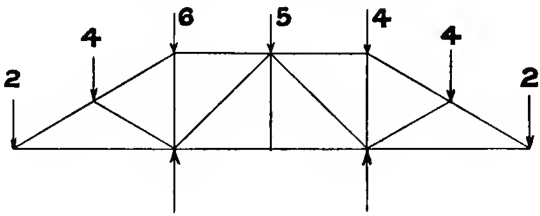


Fig. 155.

9. The lines in Fig. 156 represent links hinged at A, B, C, D, E and F. A load of 5 cwt. is suspended from E. If the diagram gives the actual

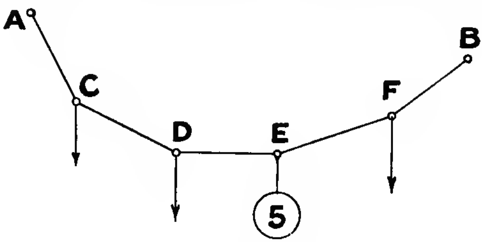


Fig. 156.

position of the links, find the loads at C, D, and F, and the stresses in the links.

10. Find the magnitude and *position* of the resultant of the forces in Fig. 131.

11. Find the stresses in the members of the roof frame, Fig. 157.

12. Find the stresses in the members of the structure in Fig. 158, assuming it to be hinged at A, and the load at C is 2 tons. There is also a horizontal supporting force at B.

13. The dotted lines in Fig. 159 represent links hinged together at A, B, C, D, and E. Find the forces necessary to maintain equilibrium and the given shape of the linkage. Find also the tension in each link.

14. If in the last example there were further links joining B to D, and A to D, find the necessary forces to produce equilibrium and the stresses in the members.

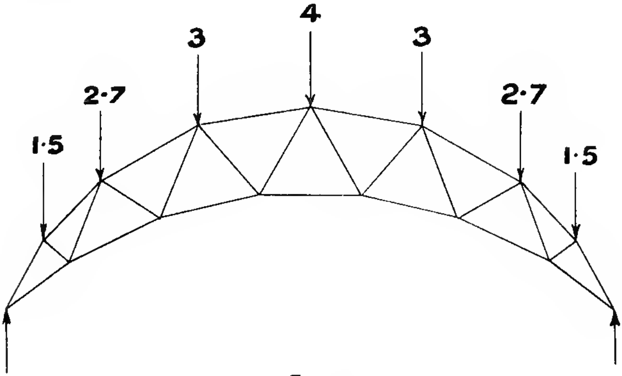


FIG. 157.

15. Assuming a smooth floor, find the tension in the cord, Fig. 160. What would the tension be if the load were placed at the joint?

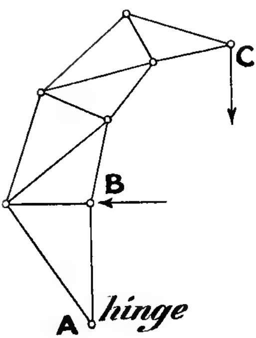


FIG. 158.

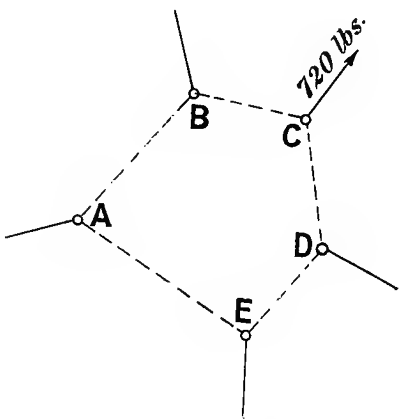


FIG. 159.

16. If in Fig. 160 there were rollers on the back legs of the steps, and the coefficient of friction between the front legs and the floor were 0·2,

find the tension of the cord, and say what forces keep the front legs in equilibrium.

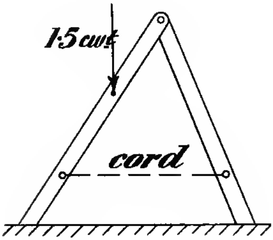


FIG. 160.

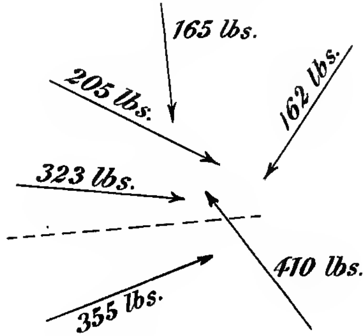


FIG. 161.

17. Determine the resultant of the forces in Fig. 161, and give its components parallel and perpendicular to the dotted line.

CHAPTER VI.

DYNAMICS.

The Motion of a Body.—We may represent the position of a body, or, in other words, its distance, from a given fixed point, in the following manner:—

Let a body be 15 ft. from a given point when we begin to measure time (as we can do with a stop-watch), and let it be

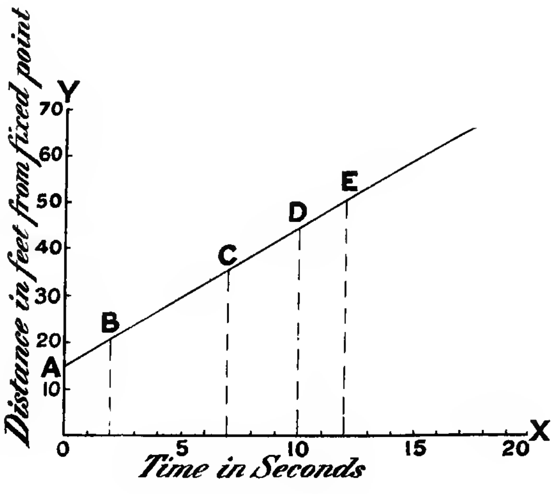


FIG. 162.

21 ft. away from the same point in 2 secs., 36 ft. in 7 secs., 45 ft. in 10 secs., 51 ft. in 12 secs. Take a piece of squared paper, as in Fig. 162, and make a scale of seconds along the

base oX, and a scale of feet along oY. Plot on this squared paper the points representing the above quantities as shown at A, B, C, D, and E. The equation to this line may be found by the method given in the Appendix. It is—

$$\left.\begin{array}{l}\text{Distance in feet from}\\ \quad\text{the fixed point}\end{array}\right\} = 13 + 3 \times \text{time of motion in seconds.}$$

The distance moved in 1 sec. is 3 ft.; for if we put 1 sec. for the time in the above equation, the distance of the body from the fixed point is then 16 ft., that is, 3 ft. further from the fixed point than at the beginning of the second. Again, at the end of 5 secs. the body is 28 ft. away, and at the end of 6 secs. it is 31 ft. away, or the distance moved over in 1 sec. is 3 ft., the same as before. This is called the *velocity* of the body. But the slope of the line AE in Fig. 162 is the number 3 in the above equation; hence we may define the *velocity* of a body as *the slope of the distance curve on a time base.*

In the above example the slope and consequently the velocity was *constant.* Also the distance from the fixed point at the end of the time, *minus* the distance from the fixed point at the beginning of the time = distance moved over during the time; and from the above equation to the line in Fig. 162 it equals slope × time, which again equals velocity × time, because velocity is the slope. Hence *when the velocity is constant*—

The distance moved over in t secs. = velocity × time t.

Speed is another name for velocity, though it does not specify *direction*, as velocity is supposed to do.

Example.—A train moves with a constant speed or velocity for 1 hour and 29 mins., and during that time moves over 78 miles: what is its velocity?

When the velocity is constant, we have—

$$\text{Distance moved over} = \text{velocity} \times \text{time of motion}$$

$$\text{or } 78 = \text{velocity} \times 1\tfrac{29}{60} \text{ hours}$$

$$\text{and } \frac{78}{1\frac{29}{60}} = \text{velocity in miles per hour} = 52{\cdot}5$$

Example.—Convert 30 miles per hour to feet per second, and then to yards per minute.

Referring to our general equation, we have—

$$\text{Distance} = \text{velocity} \times \text{time}$$

$$\text{or } \frac{\text{distance}}{\text{time}} = \text{velocity}$$

Now, miles per hour is a velocity, and therefore equals—

$$\frac{\text{distance in miles}}{\text{time in hours}}$$

To convert this to feet per second, we must first convert miles to feet in the numerator, and hours to seconds in the denominator ; thus—

$$\text{feet per second} = \frac{\text{distance in feet}}{\text{time in seconds}}$$

$$= \frac{\text{distance in miles} \times 5280}{\text{time in hours} \times 60 \times 60}$$

$$= \frac{30 \times 5280}{1 \times 60 \times 60} = 44$$

It is rather convenient to remember that 88 ft. per second is the same speed as 60 miles per hour.

Variable Velocity.—Let the distance moved over in a given time be indicated by an ordinate to the curve OPBC in Fig. 163, where we have time measured along the base and distance vertically upwards, as in Fig. 162. We have already learnt (p. 132) that *the velocity at any instant is the slope of the distance-time curve at that instant.* For example, the velocity at the end of the time represented by ON (Fig. 163) is the slope of the curve at P, which is the same as the slope of the tangent to the curve at P, which equals—

$$\frac{FD}{DE} = \frac{75}{3{\cdot}96} = 18{\cdot}9 \text{ ft. per second}$$

EF being the tangent to the curve at P.

At N_1, vertically below N, plot upwards P_1N_1 equal to this

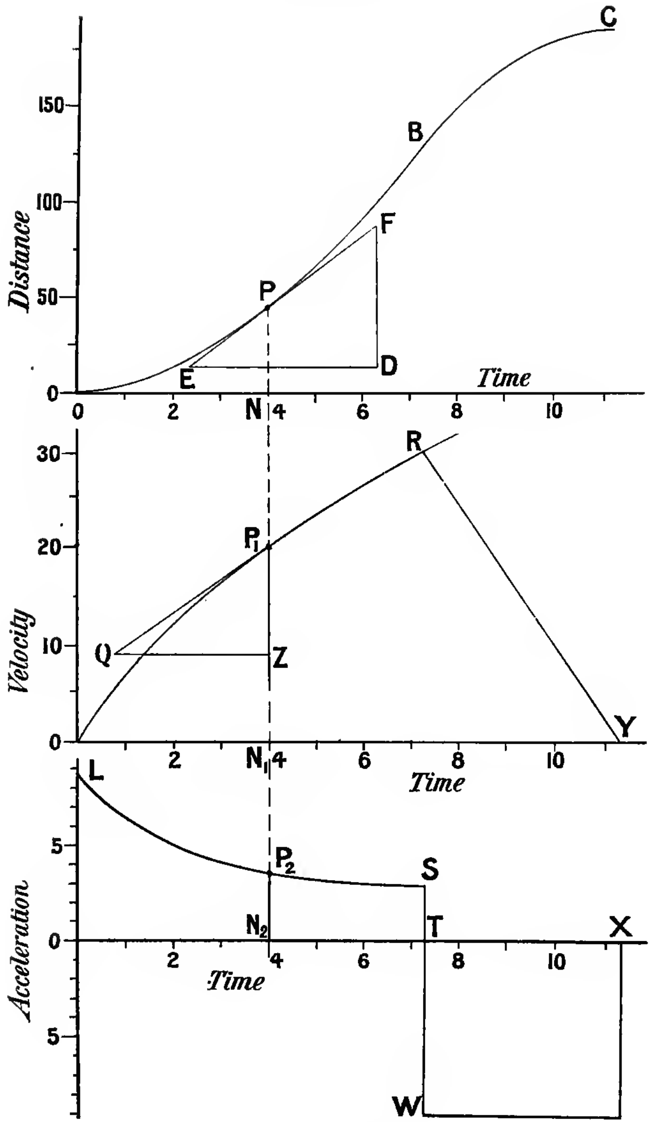

C
B
150
100
F
Distance
P
50
E
D
Time
0
0
2
N
4
6
8
10
R
30
P1
20
Velocity
10
Q
Z
0
Y
2
N1
4
6
8
10
Time
L
5
P2
S
Acceleration
N2
T
X
0
2
4
6
8
10
Time
5
W

FIG. 163

velocity, and do the same for a number of points on the original curve OPC. We then get the velocity-time curve OP_1RY.

We may now examine the change that takes place in the velocity. Referring for a moment to Fig. 163, we saw that the velocity was given by the slope of the curve, which is measured at P by FD (a distance) divided by DE (a time). Dividing a distance by a time, we get "*the distance moved over in one unit of time*," or the gain of distance per unit of time, which is the *change of distance per unit of time* (the distances being measured from a fixed point).

Reasoning from analogy, we see that the **slope** of the velocity-time curve (Fig. 163) must give us the **change of velocity per unit of time.** The name we give to this quantity is **Acceleration.** The slope at different points in the velocity-time curve is plotted vertically in Fig. 163 from the base OX, the positive values upwards and the negative values downwards.

We may also notice that the acceleration at P_1 = slope at $P_1 = \frac{\text{change of velocity}}{\text{time}} = \frac{P_1Z}{QZ} = \frac{11 \cdot 3}{3 \cdot 25} = 3 \cdot 46$ ft. per second per second. Hence acceleration × time = change of velocity during that time.

We may now define *velocity* as the slope of the distance-time curve, and *acceleration* as the slope of the velocity-time curve.

Let us now examine the case in which the acceleration is constant. The acceleration-time curve will be a horizontal straight line, PQ (Fig. 164). We have just learnt that—

$$\text{Acceleration} \times \text{time} = \text{change of velocity.}$$

But the area included between any two verticals, such as OA and RV and the curve AR (Fig. 164)

$$= \text{acceleration OA} \times \text{time OV.}$$

Hence this area must represent the change of velocity

which takes place between O and V; that is, during the interval of time OV.

Now take a piece of squared paper (Fig. 165), and make a

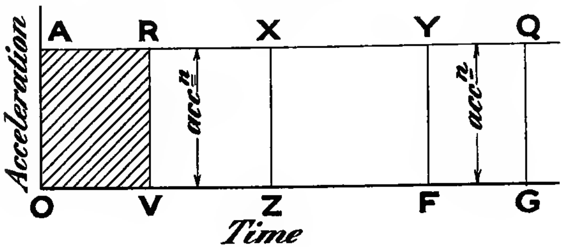


FIG. 164.

scale of time along the base, but plot upwards the change of velocity, that is, the area between OP and the ordinate in question. Thus set up VA to represent the area OARV, and

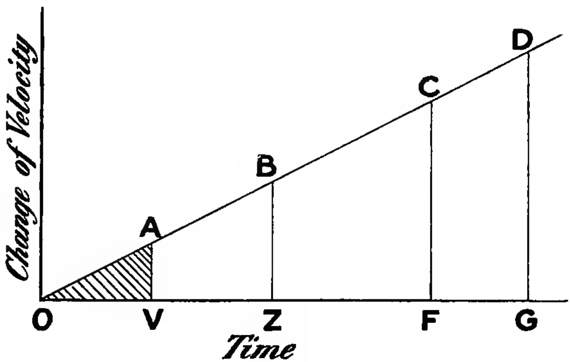


FIG. 165.

ZB to represent the area OAXZ; also FC to represent the area OAYF, and so on. We then get the straight line OABCD.

If the body had an initial velocity before the increase of velocity began, we should represent this initial velocity by OM (Fig. 166), and plot the change of velocity above OF. Thus,

at the end of the time MH, the velocity of the body will be the initial velocity + the change that has taken place during the time MH, which is represented by HZ + ZB. The equation to the line OD is—

$$\left.\begin{array}{l}\text{Velocity at the end of}\\ \quad\text{a given time}\end{array}\right\} = \text{OM} + \text{slope} \times \text{time}$$
$$= \text{initial velocity} + \text{acceleration} \times \text{time}$$

It will be convenient if we call the velocity at the end of a given time simply the final velocity, and we then write—

Final velocity = initial velocity + acceleration × time

We also found on p. 132 that if the velocity were constant—

$$\text{Velocity} \times \text{time} = \text{distance moved over}$$

In Fig. 166 the velocity is not constant, but we could, if

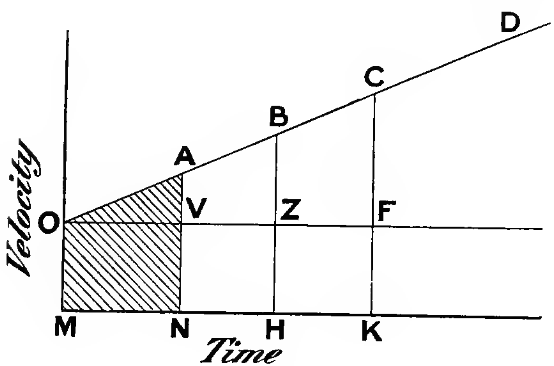


Fig. 166.

necessary, find an *average* velocity which would make the body move over a given distance in the same time as the variable velocity. Then—

$$\text{Average velocity} \times \text{time} = \text{distance moved over in that time}$$

As velocity is measured upwards (Fig. 166), average velocity

$\times$ time = area included between OM and the ordinate at the end of the time. Hence the distance moved over in a given time

= area between ordinates at the beginning and end of the time

In Fig. 166 the area—

$$\begin{aligned} \text{MOBH} &= \text{average height} \times \text{base} \\ &= \frac{\text{MO} + \text{HB}}{2} \times \text{MH} \\ &= \left(\frac{\text{initial velocity} + \text{final velocity}}{2}\right)\text{time} \end{aligned}$$

Substitute for the final velocity from the equation in heavy type on p. 137, and we get—

$$\text{Area} = \left[\frac{\text{init. vel.} + (\text{init. vel.} + \text{acceleration} \times \text{time})}{2}\right]\text{time}$$

$$= \text{initial velocity} \times \text{time} + \tfrac{1}{2}\,\text{acceleration} \times \text{time}^2$$

But area represents distance moved over during the time, hence—

Distance moved over = initial velocity × time + $\frac{1}{2}$ acceleration × time^2

The above results can be briefly summarized thus—

Let s represent the distance moved over by a body in the time t, with a *constant acceleration* of (a), the initial velocity being V and the final velocity v.

Then—

$$v = \mathrm{V} + at \quad . \quad . \quad . \quad . \quad \text{I.}$$

$$s = \mathrm{V}t + \tfrac{1}{2}at^2 \quad . \quad . \quad . \quad \text{II.}$$

$$\text{and } v^2 = \mathrm{V}^2 + 2as \quad . \quad . \quad . \quad \text{III.}$$

The last equation is obtained from equations I. and II., thus—

Square both sides of equation I. Then—

$$\begin{aligned} v^2 &= V^2 + 2\,V\,at + a^2t^2 \\ &= V^2 + 2\,a\,(Vt + \tfrac{1}{2}\,at^2) \\ &= V^2 + 2\,as \end{aligned}$$

Example.—In starting from a station a train increases its speed by 3 ft. per second during each second. How far will it move in 30 seconds, and with what velocity will it be moving at the end of 10 seconds?

Inserting the given values in equation II., we get—

$$\begin{aligned} s &= Vt + \frac{1}{2}at^2 \\ &= (0 \times 30) + \frac{1}{2} \times 3 \times 30^2 \\ &= 0 + 1350 \text{ ft.} \end{aligned}$$

Again, using equation I., we have—

$$\begin{aligned} v &= V + at \\ &= 0 + 3 \times 10 \\ &= 30 \text{ ft. per second} \end{aligned}$$

Example.—A body is moving at a given instant with a velocity of 50 ft. per second. During its motion its velocity is reduced by 8 ft. per second in every second. How far will it move in 3 seconds, and what will then be its velocity? What will its velocity be after moving over 100 ft., and how far did it move during the fourth second of its motion?

Using equation II., we have—

$$s = Vt + \frac{1}{2}at^2$$

And as the acceleration is negative, that is, a retardation, or as the acceleration is in the opposite direction to the initial velocity V, we must write -8 for a.

$$\begin{aligned} s &= (50 \times 3) - \frac{8}{2} \times 9 \\ &= 114 \text{ ft.} \end{aligned}$$

Its velocity at the end of 3 seconds is given by the equation—

$$v = V + at$$
$$\text{or } v = 50 - 8 \times 3$$
$$= 26 \text{ ft. per second}$$

the negative sign being due to the negative direction of the acceleration.

Its velocity after moving 100 ft. is best obtained from the equation—

$$v^2 = V^2 + 2as$$
$$\text{or } v^2 = 50^2 - 2 \times 8 \times 100$$
$$= 900$$
$$\therefore v = 30 \text{ ft. per second}$$

Lastly, the distance moved during the fourth second of motion

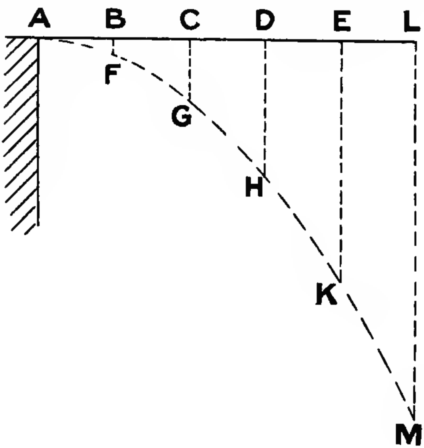


Fig. 167.

will be obtained by finding s when $t = 3$, and then again finding s when $t = 4$. The first has already been found to be 114 ft. The second will be found in the same manner to be 136 ft. The difference of these two values of $s = 22$ ft., which is the distance moved over in the fourth-second of motion.

Example.—A body is projected horizontally over a cliff with a

velocity of 50 ft. per second. Trace the path of the body during 5 seconds.

It is found by experiment that when a body is allowed to fall freely its acceleration is constant and equal to 32·2 ft. per second per second. Hence the vertical component of its motion will be given by the three equations for motion with constant acceleration.

At the end of 1 second the body will have moved 50 ft. horizontally with constant velocity. It will also have moved vertically a distance given by—

$$\begin{aligned} s &= Vt + \tfrac{1}{2}at^2 \\ &= 0 + \tfrac{1}{2} \times 32 \times 1^2 \\ &= 16 \text{ ft.} \end{aligned}$$

Measure AB horizontally (Fig. 167) to represent 50 ft., and then BF vertically downwards to represent 16 ft. The body will have arrived at F at the end of 1 second.

At the end of 2 seconds it will have moved 100 ft. = AC horizontally, while vertically it will have moved—

$$\begin{aligned} s &= Vt + \tfrac{1}{2}at^2 \\ &= 0 + \tfrac{1}{2} \times 32 \times 4 \\ &= 64 \text{ ft.} = \text{CG} \end{aligned}$$

In this way any number of points in its path may be found, and the path traced out.

Example.—Two points, A and B, are in the same vertical line, 200 ft. apart, A being above B. A body is let fall from A, and simultaneously another body is thrown upwards from B with a velocity of 50 ft. per second. Where will they meet?

Let them meet at a distance s from the level of B. This meeting-point will therefore be $(200 - s)$ from A.

Let t = time in seconds before they meet; then the distance travelled by the lower body is given by—

$$\begin{aligned} s &= Vt + \tfrac{1}{2}at^2 \\ &= 50t - \frac{32}{2}t^2 \end{aligned}$$

the negative sign indicating that the acceleration is in the opposite direction to the initial velocity.

Again, the second body traverses, in the same time, the distance—

$$200 - s = 0 + \frac{32}{2}t^2$$

Adding these equations together, we get—

$$200 = 50t$$

$$\text{or } t = 4 \text{ seconds}$$

In 4 seconds the upper body will have fallen through a distance—

$$s = Vt + \tfrac{1}{2}at^2$$

$$= 0 + \frac{32}{2} \times 16$$

$$= 256 \text{ ft. below A}$$

$$56 \text{ ft. below B}$$

Motion down an Incline.—We can conduct an experiment to determine the accuracy or otherwise of the statement

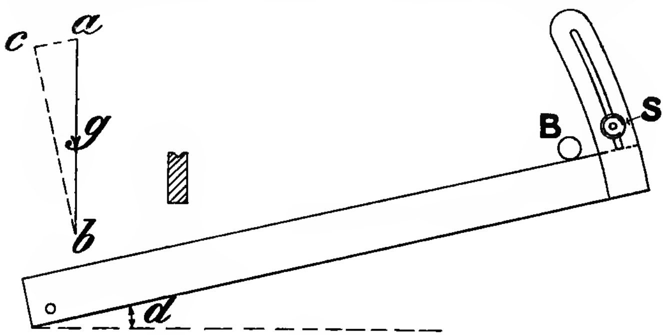


Fig. 168.

in heavy type on page 141, namely, that the acceleration of a falling body is constant.

For that purpose a long grooved plank (Fig. 168) can be used, which can be tilted through a small angle and fixed in that position by a thumbscrew, S.

A ball, B, is allowed to roll down the groove, and the time

of motion can be recorded by a stop-watch, or, preferably, by a water-clock something like that shown in Fig. 171. The distance moved over can be varied from zero up to the length of the plank.

The acceleration vertically downwards is g feet per second in each second.

The component of this parallel to the plane is ac on the left of Fig. 168, which is constant if g is constant. Hence, if we find by experiment that the acceleration down the plane is constant, then g must be constant.[1]

Keep the incline fixed at a constant angle, d. Let the ball roll through different distances along the plane, and record the time for each distance. Tabulate the observations, and plot the distances vertically and the times horizontally, as in Fig. 162. Repeat the process of getting the velocity and acceleration from this curve that was used in Fig. 163. It will be found that the acceleration down the plane is constant, and therefore g is constant.

Angular Motion.—We have just discussed linear motion, or motion along a line, when the acceleration was constant. When an arm or rod is made to turn round an axis, like the hand of a clock or the crank of an engine, the *end* of the hand moves over a *linear* distance, while the hand or crank sweeps out an *angle*, *i.e.* passes over an angular distance.

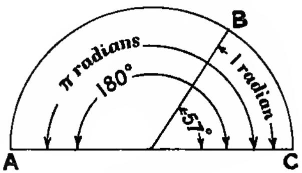


FIG. 169.

The angle passed over in one second is called the angular velocity of the arm, and is measured[2] in radians per second or in turns per minute.

[1] It should be noted that the acceleration of the ball down the plane will not equal the component of g along the plane, because the ball is compelled to roll or rotate as well as move longitudinally, and part of the component of g must be used for rotation, while the part which is measured is that used for translation.

[2] There are π radians in a semicircle or 2π radians in a circle, *i.e.* an angle of π radians = an angle of 180 degrees (see Fig. 169).

The equations on page 138 hold for *angular* as well as for *linear* motion. In that case V represents the initial angular velocity, and v the final angular velocity, in radians per second, while a is the angular acceleration in radians per second per second, and s the angular distance or angle turned through.

Example.—A fly-wheel is turning at a given instant at the rate of 90 times per minute. It is allowed to slow down until its speed is 50 turns per minute, and during the interval it has made 25 complete turns. What was the angular acceleration, and how long was it in slowing down?

From page 138 we have—

$$v^2 = V^2 + 2as$$

Substituting 90 turns per minute when converted to radians per second for V, and $2\pi 25$ for s, we have—

$$\left(\frac{50 \times 2\pi}{60}\right)^2 = \left(\frac{90 \times 2\pi}{60}\right)^2 - 2a \times 2\pi 25$$

from which $a = 0{\cdot}196$ radians per second per second

$$\text{Also } v = V + at$$

$$\text{or } \frac{50}{60} \times 2\pi = \frac{90}{60} \times 2\pi - 0{\cdot}196t$$

that is $t = 21{\cdot}3$ seconds

Force and the Motion it produces.—We have so far only studied the geometrical aspect of motion, and have not yet endeavoured to ascertain the relation between the force producing motion and the amount of motion produced by it. Experiments, conducted with this end in view, are by no means easy to carry out; on account of the difficulty of measuring time with a fair degree of accuracy, with inexpensive apparatus; which is the condition under which the majority of students must work.

At the outset it is absolutely necessary that the nature of the problem to be solved should be clearly understood. We want to find how much velocity will be imparted to a given body by a definite force acting during a given time, or, in

other words, we want to find an equation connecting these quantities.

The velocity imparted to a body in a given time depends upon the acceleration, and therefore the quantities we are chiefly concerned with are—force, quantity of matter acted upon, acceleration produced, time of action, and space moved over during that time.

These need not all be included in the same result, as was evident from a consideration of the equations deduced for motion having constant acceleration.

The term *quantity of matter* used above requires to be clearly defined before we proceed further.

In the first chapter, in a footnote, it was stated that the *weight* of a body varied according to its distance from the earth's centre. If the body could be carried to the centre of the earth its weight would be zero, but the quantity of matter in the body would still be the same as it was at the earth's surface. This indicates that the quantity of matter in a body and its weight are two entirely different things, and must not be confused one with the other.

The Quantity of Matter in a Body is called its Mass.

The unit of mass is the quantity of matter in the piece of platinum called the *standard pound*, and the mass of any body can be found by balancing the body in one pan of a balance against a number of duplicates of the standard pound in the other pan.

It will be seen that this method is independent of the position or locality in which the operation is carried out, as any variation of the pull of the earth on the constituents of one scale-pan must be the same as that on the constituents of the other pan.

We see from the above that when we weigh a body with a balance such as a chemical balance, or those used for weighing tea, sugar, etc., we are really comparing the *mass* of the body with our standard and are not finding its *weight*; for we should get the same result if we removed the balance and body to some other locality where the pull of the earth (weight) was different.

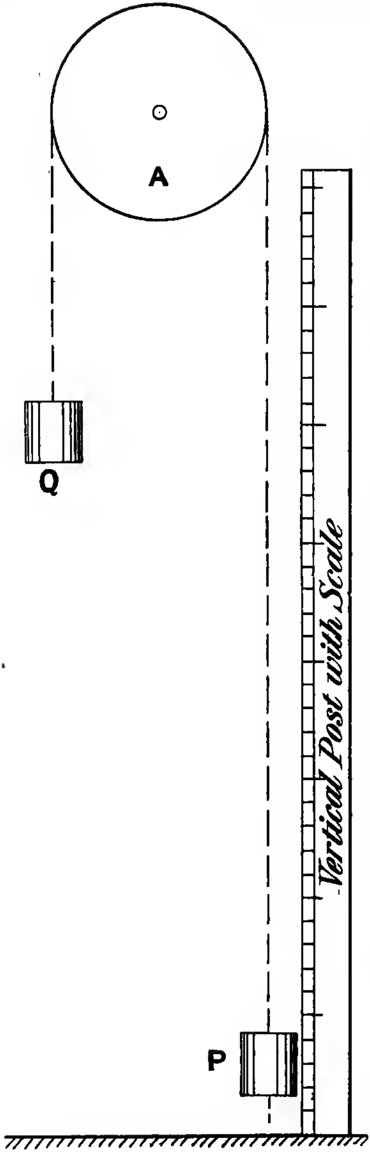


Fig. 170.—Atwood's machine.

The weight of a body is the pull of the earth upon it, while the mass of a body is the quantity of matter in it. *The former is a force, and the latter is so much matter.*

We could dispense with the term weight, if it had not such popularity with the general public, who use it in the place and sense of mass.

There ought not to be any confusion in the student's mind if he remembers that weight is a force and mass is not, though they are both often measured in units having the same name; but it is only in *name* that they are alike; just in the same way that a pound of *tea* and a pound sterling are entirely different. One measures a mass of matter, and the other measures worth or monetary value.

It is quite an accident that at any given place the masses of two bodies bear the same ratio one to the other as their weights do at that place; and hence we are able to compare masses by weighing them.

The apparatus we may

use to show that force is proportional to acceleration is shown diagrammatically in Fig. 170, and was first devised for the purpose by Atwood, but in a different form to that shown. A pulley A is supported on a ball-bearing, similar to that used in a bicycle for the purpose of reducing friction, but more especially for rendering the friction as nearly as possible constant. The pulley should not be too small in diameter, 8 inches being about the minimum. Two equal weights, P and Q, are suspended at the ends of a fine cord, such as whipcord or coarse silk fishing-line, which is passed over a groove in the pulley.

These weights should be not less than 7 lbs. each, and preferably larger. In addition to these, there should be a large number of riders of different weights, which can be superimposed on P or Q. Let P represent the fixed mass on the end P, and Q the fixed mass on the end Q, and let $R =$ the mass of the rider placed on Q. Then, if there is no friction, R would be the force which moved the total mass $(Q + P + R)$,[1] and produced in it a certain acceleration which it is our purpose to determine.

But we must eliminate the effect of friction, or rather balance it by an equal and opposite force. This is done by putting on Q a small rider just sufficient to keep the weights moving very slowly *and without increase of velocity.* This is done before R is applied to Q. Let F be the mass of the rider applied to balance friction. It should be extremely small.

The method of experiment will consist of varying one element at a time. We shall first keep the moving force and masses constant, vary the distance moved, and measure the time of movement.

The measurement of time is the operation which is least accurate and most difficult of accomplishment. The best way is to arrange a couple of electrical contacts on the graduated

[1] The mass of $(Q + P + R)$ as suggested above will not be exactly the mass moved. There is in addition the equivalent mass of the pulley supporting the weights. This should be small, but it is too difficult a matter to take it into consideration here. See experiment on the flywheel.

post at the side of the weight P (Fig. 170), and connect these contacts to a pencil recording on a revolving drum (driven by clockwork at a uniform rate) the time of motion between the contacts. This arrangement is too expensive to be generally used in a Mechanics laboratory, and in its place some cheaper form of apparatus must be adopted. This generally takes the form of a water-clock, which can be arranged in a number of different ways; the main feature always being that the amount of water flowing under constant head is caught during the motion of the masses P and Q, and that the mass of water which will flow from a given orifice under constant head is proportional to the time of flow.

In Fig. 171, A is a small tank containing a ball float-valve for maintaining the water at a constant level. A small glass or brass tube, B, is connected to it through a piece of rubber tube, C, to allow of it being switched over to deliver into the measuring vessel D, and then back to the drain E.

A loose pulley, F, is turned about a pin in the graduated post by the weight W, and the pulley is connected to the tube B by a small connecting-rod G. There are three pegs, *a*, *b*, and *c*, of different lengths in the periphery of the pulley which respectively engage with the end of the lever K centred at L. An arm, M, attached to the pulley presses on the weight P and prevents it from rising. A fine cord connects the under side of the weight P with the end T of the lever TSN, the end T being also connected to the helical spring X. The cord contains loops every few inches, which are hitched in turn to the hook at the end T of the lever TSN.

After setting the apparatus as shown in the figure, and putting a rider R lbs. on the weight Q, in addition to F lbs., which balances friction, press the lower end of the lever K lightly with the finger, to release the peg *a* in the pulley from the end of the lever K. This permits the pulley F to turn and throw over the arm M and the tube B, releasing the weights P and Q, and directing the stream of water into the measuring vessel D.

When the cord connecting the weight P and the lever T becomes taut, it lifts T and releases the lower end of K, which

is pulled to the left by the elastic band V, which releases the second peg *b*, and the tube B is thereby thrown back to its original position, delivering water to the drain again. The

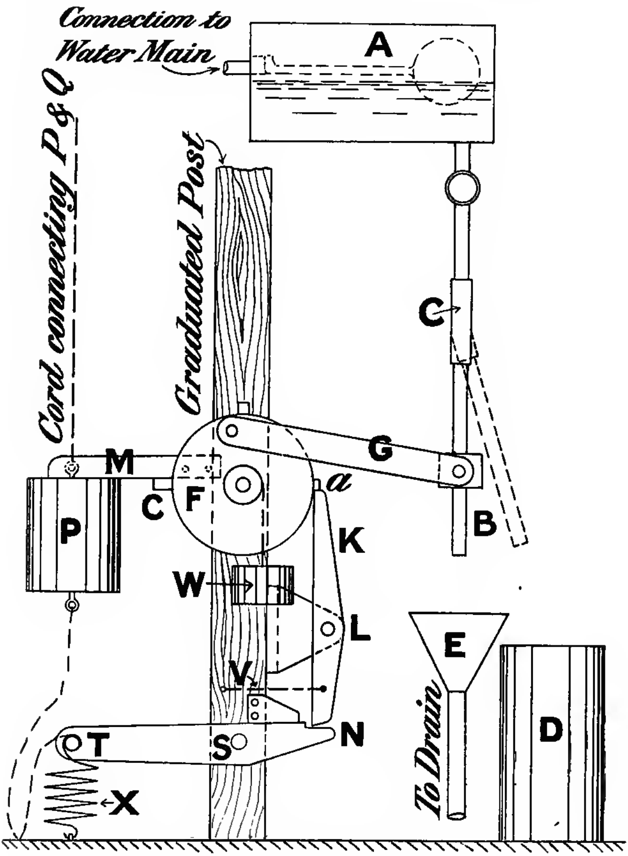


FIG. 171.

spring X checks the motion of the weights P and Q, and prevents the connecting-cord from breaking.

The quantity of water caught in the measuring vessel is

proportional to the time of motion. This should be measured to one-hundredth of a pound. The distance moved is obtained by a square with the blade touching the top of the weight, as in Fig. 172.

Repeat the experiment with other lengths of cord,

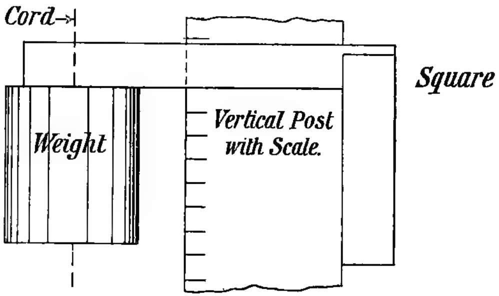


FIG. 172.

connecting T to the weight P, and tabulate the results as below.

EXPERIMENT I. ON ATWOOD'S MACHINE.

Date, . Observer, .

Object.—To determine the relation between the distance moved over and the time of motion with a constant moving force and masses moved.

Friction = F = lbs.
P = Q = lbs.
R = lbs.
Total mass moved = P + Q + F + R = lbs.

Distance moved (feet).	
Time of motion . .	

Now plot the distance moved on a time base similar to Fig. 162. This has been done in Fig. 173. Draw a smooth curve through the points so plotted.

To obtain the velocity curve, use the method shown in Fig. 163. Obtain the slope of the curve at several points by

drawing tangents, and plot these values of the slope (velocity) on the same base. In this case it happens to be a straight

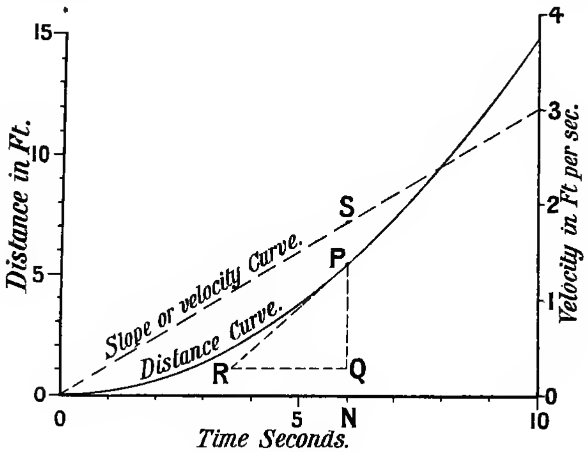


FIG. 173.

line, and therefore we can find its equation by the method given in the Appendix. It is—

$$\text{Velocity} = 0{\cdot}3 \times \text{time}$$

But this is the kind of equation we obtained with constant acceleration (page 138), which was—

$$v = V + at$$

In the present case the bodies started from rest, and consequently $V = O$.

Comparing the two equations, we see that 0·3 represents the acceleration; it is constant because the slope of the velocity curve is constant.

As a further check upon the results just obtained, see if the equation

$$S = Vt + \frac{1}{2}at^2$$

holds good, as it ought to if the acceleration is constant in the experiment.

As there is no initial velocity, we shall have—

$$S = \frac{at^2}{2}$$

Let $t^2 = x$; then—

$$S = \frac{a}{2}x$$

which is the equation to a *straight* line through the origin, the slope of the line being $\frac{a}{2}$. Hence square all the times, and plot these horizontally, and the distances s vertically. The resulting curve is sufficiently straight to confirm the above indication of constant acceleration.

A somewhat different method of procedure might have been adopted, after plotting the curve in Fig. 173.

See if the equation connecting space and time is of the type—

$$s = mt^n$$

Take logarithms of both sides, thus—

$$\log s = \log m + n \log t.$$

This is the equation to a straight line if the logarithms are plotted. Hence, if the points obtained by plotting the logarithms of s and t all lie upon a straight line, we know that the equation connecting s and t must be of the type—

$$s = mt^n$$

where m and n are constants (see Appendix).

This experiment shows that the pull of the earth on the rider R (which was constant) produced a motion of the masses P + Q + F + R, in which the acceleration was constant. Therefore we are at liberty to anticipate that a constant force produces constant acceleration when acting on a given mass of matter.

Experiment II. on Atwood's Machine.

Date, . Observer, .

Object.—To determine the relation between the moving force and the acceleration produced by it when the total mass acted upon and the distance moved over are kept constant.

Total mass moved $= P + Q + F + R =$ lbs.

Friction $F =$ lbs.

Distance over which motion takes place, ft.

Time.	Moving Force.	Acceleration produced.

As the force has to be varied, while the mass moved remains constant, a rider must be removed from one weight to the other to produce the variation of force. Hence, put a number of riders on P, and a large rider or a number of riders on Q slightly greater in weight than those on P. Arrange for the weights to have the maximum possible range of motion. Measure the time as in the previous experiment. Having obtained the observations, calculate the acceleration in each case, and enter it in column III. of the table. This we can do by means of the equation—

$$s = Vt + \tfrac{1}{2}at^2$$

because we have previously shown that this equation holds good for any set of observations taken on an Atwood's Machine. In the above, V is zero, and therefore—

$$a = \frac{2s}{t^2}$$

Now plot the force on an acceleration base.

The resulting curve is straight, and passes through the origin; hence its equation is—

$$\text{Force} = \text{slope} \times \text{acceleration}$$

Therefore, when the force is variable, the acceleration produced is proportional to the force producing it.

EXPERIMENT III. ON ATWOOD'S MACHINE.

Date, . Observer, .

Object.—To determine the relation between the mass moved and the acceleration produced, when the distance moved over and the moving force are kept constant.

Distance moved =
Moving force =

Time.	Mass moved.	Acceleration produced.

An equal number of comparatively heavy riders must now be placed on each end of the cord. Determine the time of

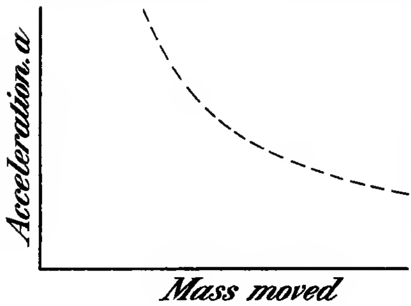


FIG. 174.

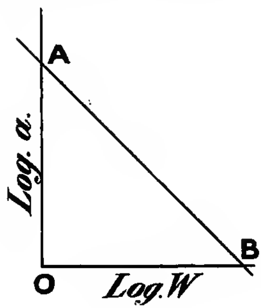


FIG. 175.

motion with different total masses, but with constant difference of weights, and calculate the acceleration produced in each case, as in the last experiment.

Plot the total mass along the base and the corresponding accelerations vertically upwards (Fig. 174). The curve not being a straight line, we must try the method given in the Appendix.

Plot the logarithms of the accelerations upwards and the logarithms of the masses horizontally (Fig. 175). We now get a series of points which lie approximately on a straight line, whose equation is—

$$\text{Log. of acceleration} = \text{OA} - \frac{\text{OA}}{\text{OB}}\text{log. of mass moved}$$

As $\frac{\text{OA}}{\text{OB}} = 1$, this equation can be written as—

$$\text{Logarithm of acceleration} + \text{logarithm of mass} = \text{OA}$$

Taking anti-logarithms, we get—

$$\text{Acceleration} \times \text{mass} = 10^{\text{OA}} = \text{a constant}$$

$$\text{or acceleration} = \frac{\text{a constant}}{\text{mass moved}}$$

Summary of Experiments on Atwood's Machine.—We have found by these experiments that—

(1) When the moving force was constant, the acceleration produced was constant (the mass acted upon being constant).

(2) When the moving force was varied, the mass moved being constant, the acceleration produced was proportional to the moving force; or—

$$a = c\text{F}$$

where c is some constant number.

(3) When the moving force was constant, and the mass moved was varied, the acceleration produced was inversely proportional to the mass moved; or—

$$a = \frac{k}{\text{W}}$$

where k is some constant number.

We now want to express these results in one equation, assuming all the quantities to be capable of variation.

Examine for a moment the expression for the area of a triangle. It is—

$$\text{area} = \tfrac{1}{2}\,\text{base} \times \text{height}$$
$$\text{or } A = \tfrac{1}{2}b \times h$$

If we keep the height constant, the areas of a series of these triangles will be proportional to their bases; or—

$$A = c \times b$$

where c is a constant multiplier or coefficient. Similarly, if the height is made to vary and the base kept constant, the area is proportional to the height; or—

$$A = k \times h$$

where k is another constant. But when both the height and the base are varied, the area is proportional to both of them, and, as we see above, is proportional to their product; and we may write the result as—

$$A = b \times h \times ck$$

where ck in this case is $\frac{1}{2}$, but in the general case may be any constant.

Reasoning from analogy, and returning to our summary, we replace the area A by the acceleration a, the base b by the force F, and the height h by the fraction $\frac{1}{W}$. We then get—

$$a = \frac{F}{W} \times ck$$

$$\text{or } a = \frac{F}{W} \times \text{a constant}$$

$$\text{or } a \times W = F \times \text{a constant}$$

that is, acceleration $\times$ mass = force $\times$ a constant

We now desire to ascertain the value of the constant. Take a case in which the force, the mass acted upon, and the acceleration produced are well known. Such a case is that of

a body near the earth's surface falling freely towards the earth due to the earth's attraction.

Let the mass of the body be (say) 10 lbs., then the force or pull of the earth upon it is 10 lbs., and the acceleration produced is approximately 32·2 feet per second per second. Putting these numbers in the above equation, we have—

$$32{\cdot}2 \times 10 = 10 \times \text{the constant}$$
$$\text{therefore the constant} = 32{\cdot}2$$

This is generally represented by g, and hence we may write—

$$\text{Moving force} \times g = \text{mass moved} \times \text{acceleration produced}$$
$$\text{or } F.g = W.a$$

This equation will hold as long as F and W are expressed in the same units; for instance, both in pounds or both in tons. For further information on the units of force, etc., see p. 205.

Example.—A body is perfectly free to move, that is, it experiences no resistance to motion of any kind. The body weighs 15 lbs., and is acted upon by a force of 6 lbs. Find how far it will move in 12 seconds, what velocity it will have acquired at the end of 4 seconds, and how long it will take to move over 100 feet.

The relation between the force acting, the mass acted upon, and the acceleration produced, is given by the equation—

$$\text{Force} \times g = \text{mass moved} \times \text{acceleration produced}$$
$$\text{or } 6 \times 32 = 15 \times \text{acceleration}$$
$$\text{hence } 12{\cdot}8 \text{ ft. per second per second} = \text{acceleration}$$

Having obtained the acceleration, it only remains to use the equations of motion with constant acceleration on page 138.

The distance in 12 seconds is given by—

$$s = Vt + \frac{1}{2}at^2$$
$$= 0 + \frac{1}{2} \times 12{\cdot}8 \times 144$$
$$= 921{\cdot}6 \text{ ft.}$$

In 4 seconds its velocity will be—

$$\begin{aligned} v &= \mathrm{V} + at \\ &= 0 + 12{\cdot}8 \times 4 \\ &= 51{\cdot}2 \text{ ft. per second} \end{aligned}$$

The time it takes to move over 100 ft. is given by—

$$s = \mathrm{V}t + \frac{1}{2}at^2$$

$$\text{or } 100 = 0 + \frac{12{\cdot}8}{2}t^2$$

$$\sqrt{\frac{100}{6{\cdot}4}} = 3{\cdot}95 = t \text{ seconds}$$

Example.—A mass of W lbs., resting on a smooth table (Fig. 176), is connected to another mass, w lbs., by a light cord. Assuming there is no friction of the pulley, find the acceleration

FIG. 176.

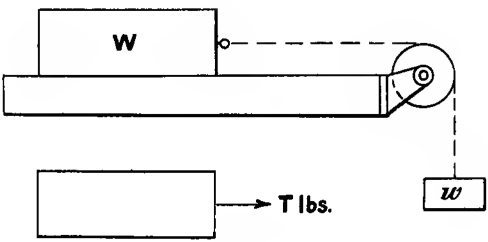


FIG. 177.

produced, the tension in the cord, and the distance moved over in 4 seconds. Take W = 10 lbs. and w = 4 lbs.

The moving force is the pull of the earth on the mass w = 4 lbs. The total mass moved = 10 + 4 = 14 lbs. Then—

$$\text{Force} \times g = \text{mass moved} \times \text{acceleration}$$

$$\text{or } 4 \times 32 = 14 \times a$$

$$\therefore a = \frac{64}{7} \text{ ft. per second per second}$$

The distance moved over in 4 seconds is—

$$s = Vt + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2} \times \frac{64}{7} \times 16$$

$$= 73 \cdot 1 \text{ ft.}$$

The tension (T) in the cord can be found thus—

The body W is acted on by a moving force T lbs. (Fig. 177). The mass moved is W lbs., hence as—

$$\text{Force} \times g = \text{mass moved} \times \text{acceleration}$$

$$T \times 32 = 10 \times \frac{64}{7}$$

$$\therefore T = \frac{10}{32} \times \frac{64}{7}$$

$$= 2\frac{6}{7} \text{ lbs.}$$

The same result would have been obtained if we had selected the body w lbs.

The force making it move is $(w - T)$ lbs., and the mass moved is 4 lbs. Hence—

$$(4 - T) \times 32 = 4 \times \frac{64}{7}$$

$$4 - T = \frac{8}{7}$$

$$4 - \frac{8}{7} = T = 2\frac{6}{7} \text{ lbs.}$$

Example.—Assume in the last example that there was friction between the mass W and the table, and that the coefficient of friction was 0·23. Find the distance moved over in 3 seconds, and the tension in the cord.

The resistance of friction to motion was $0 \cdot 23 \times 10 = 2 \cdot 3$ lbs., and hence the net moving force was $4 - 2 \cdot 3 = 1 \cdot 7$ lbs. Inserting this in the equation—

Force × g = mass moved × acceleration produced, we get—

$$1{\cdot}7 \times 32{\cdot}2 = 14 \times \text{acceleration}$$

$$\text{or acceleration} = 3{\cdot}9 \text{ ft. per second per second}$$

$$\text{Also } s = Vt + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2} \times 3{\cdot}9 \times 9$$

$$= 17{\cdot}6 \text{ ft.}$$

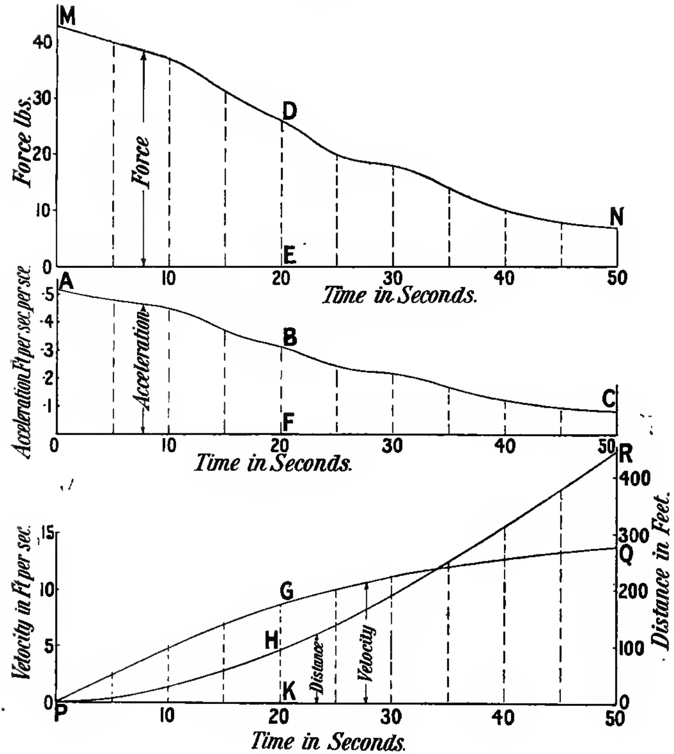


FIG. 178.

Let T represent the tension in the cord. The body w lbs. is acted upon by two forces, one of 4 lbs. downwards and one of T lbs. upwards.

Net force = (4 − T) lbs.

As—

$$\text{Force} \times g = \text{mass} \times \text{acceleration}$$

$$(4 - T) \times 32{\cdot}2 = 4 \times 3{\cdot}9$$

Solving for T, we get $T = 3{\cdot}515$ lbs.

Example.—A body weighing 1·2 tons is free to move without any resistance. It is acted upon by a force whose magnitude at the end of 5-second intervals is given by the ordinate to the curve MDN (Fig. 178). Draw a diagram on a time-base showing the distance travelled in any time from the start.

We know that force $\times\, g$ = mass moved $\times$ acceleration

$$\text{or force} \times 32{\cdot}2 = 1{\cdot}2 \times 2240 \times \text{acceleration}$$

$$\text{or acceleration} = \frac{32{\cdot}2}{1{\cdot}2 \times 2240} \times \text{force}$$

$$= 0{\cdot}01198 \times \text{force}$$

Hence, if we multiply each force by 0·01198, we shall get the corresponding acceleration in feet per second per second at that instant. Tabulate the quantities so obtained as below.

Time.	Force.	Acceleration.	Velocity.	Distance.
secs.	lbs.	feet per sec. per sec.	feet per sec.	feet.
0	43	0·515	0·0	0·0
5	40	0·479	2·48	6·2
10	37	0·443	4·78	24·2
15	31	0·370	6·91	53·4
20	26	0·311	8·61	92·2
25	20	0·240	9·99	138·7
30	18	0·216	11·13	191·5
35	14	0·168	12·09	249·5
40	10	0·120	12·81	311·8
45	8	0·096	13·35	377·3
50	7	0·084	13·80	445·3

Plot the acceleration diagram ABC on a time-base (Fig. 178).

On page 136 we found that the area of the acceleration diagram on a time-base gave us the velocity of the body. Hence, determine the area of the acceleration diagram up to the end of the respective 5-second intervals. Enter these in the fourth column of the above table.

Plot these on a time-base, as in Fig. 178 (curve PGQ).

Again, the area of the velocity diagram on a time-base gives the distance moved over. Determine these areas, and enter them in the fifth column.

Plot these numbers on a time-base, as shown by the curve PHR (Fig. 178). This curve gives the distance moved over at the end of any time.

A method of determining the quantities in the fourth and fifth columns of the above table is as follows :—

The area of the first strip in the acceleration diagram, Fig. 178, is—

$$\text{average height} \times \text{width}$$

$$= \frac{0·515 + 0·479}{2} \times 5 = 2·48 \text{ ft. per second}$$

Again, the area of the second strip—

$$= \frac{0·479 + 0·443}{2} \times 5 = 2·3 \text{ ft. per second}$$

Area of figure up to end of 10th second—

$$= 2·48 + 2·3 = 4·78 \text{ ft. per second}$$

Area of third strip—

$$= \frac{0·443 + 0·370}{2} \times 5 = 2·13$$

Area of figure up to end of 15th second—

$$= 4·78 + 2·13 = 6·91 \text{ ft. per second}$$

The same method applies to the last column in the table. The middle heights might have been *measured* and used instead of those calculated.

It is sometimes convenient to express the relation between a constant force and the motion it produces in a manner slightly different to the equation.

$$\text{Force} \times g = \text{mass moved} \times \text{acceleration produced}$$

Multiply both sides of the equation by the time t during which the force acts. Then—

$$\text{Force} \times t \times g = \text{mass} \times a \times t$$

$$\text{But } a \times t = v - V = \text{final velocity} - \text{initial velocity}$$

$$= \text{change of velocity in } t \text{ seconds}$$

hence—

Force × t × g = mass moved × change of velocity

The change of velocity is that which takes place while the force is acting, *i.e.* during t seconds.

Example.—Assuming the resistance to motion is constant, how long would a train take to reduce its speed from 50 to 30 miles per hour, the resistance to motion being 160 lbs. per ton?

The change of velocity is (50 − 30) = 20 miles per hour, *i.e.* $20 \times \frac{88}{60}$ ft. per second.

The force retarding the train is 160 lbs. for every ton; hence we may substitute 160 lbs. for the force and 2240 lbs., or one ton, for the mass in the equation; then—

$$160 \times 32{\cdot}2 \times t = 2240 \times \frac{88}{3}$$

$$\text{or } t = \frac{40 \times 88}{3 \times 160 \times 32{\cdot}2}$$

$$= 12{\cdot}7 \text{ seconds}$$

The velocity of a body falling down any curved path is the same, whatever the shape of the path.

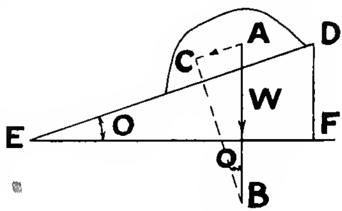


FIG. 179.

This can be shown in the following manner. In Fig. 179 let ED be an incline making an angle θ with the horizon. Resolve the weight of the body W parallel to and perpendicular to the plane. The former component AC is urging the body down the plane. Its magnitude is W sin θ, for the line CB is perpendicular to ED, and the line EF is perpendicular to AB, and, as the angle between perpendiculars equals the angle between the original lines, that at B must equal θ.

$$\text{Then AC} = \text{AB} \sin\theta = \text{W} \sin\theta$$

Also as—

$$\text{Force} \times g = \text{mass moved} \times \text{acceleration}$$
$$\text{then W} \sin\theta \times g = \text{W} \times a$$
$$\text{or } g \sin\theta = a$$

And the velocity v at the bottom of the plane is given by—

$$v^2 = \text{V}^2 + 2as \text{ (see page 138)}$$
$$= o + 2g \sin\theta \times \text{ED}$$
$$\text{but } \frac{\text{DF}}{\text{DE}} = \sin\theta \text{ and } \frac{\text{DF}}{\sin\theta} = \text{ED}$$
$$\text{hence } v^2 = 2g \sin\theta \times \frac{\text{DF}}{\sin\theta}$$
$$= 2g \,.\, \text{DF}$$
$$- 2g \times \text{vertical height of fall}$$

Hence the velocity at the foot of an incline is independent of the slope, and depends only on the vertical height through which the body falls, or, in other words, in falling from a given point to another point the shape of the path has no influence on the final velocity.

Centrifugal Force.—We have seen, in what has preceded this, that force is necessary to produce or change the motion of a body in any way whatever. As motion has direction as well as quantity, we conclude that force is necessary to change the direction of motion as well as the magnitude of the motion.

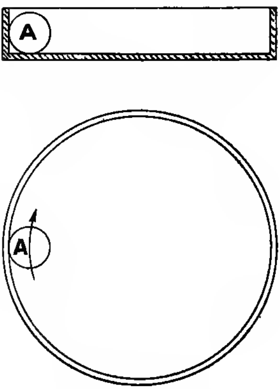


FIG. 180.

In the accompanying figure (180) a ball, A, is assumed to roll round the circular tray shown. The object of the side or fence of the tray is to compel the ball to move round in a circle parallel to the fence itself. We have previously shown that the pressure between two surfaces in contact must be perpendicular to

those surfaces at the point of contact, and hence the pressure between the ball and fence must be perpendicular to the fence, that is, in the direction of a radius of the tray through the centre of the ball. We have also just seen that a force is necessary to change the direction of motion, and the motion produced is in the direction of the force; and as the direction of the ball is continually and uniformly changing, there must be a force producing this change, and it must act *towards the centre* of the tray. This force is called the *centripetal force* on the ball. A single force cannot exist alone,[1] but must be balanced by an equal and opposite force, or the ball reacts or pushes outwards from the centre on the fence of the tray, this force being called into play by the centripetal force. This can be easily realized as follows:—

A cord cannot be strained until there is a force *at each end*, and these forces are equal and opposite in direction. We may replace the fence by a piece of cord, attaching the ball to the centre of the tray. The force or pull on the inner end of the cord is the centripetal force on the ball, while the corresponding and equal pull in the opposite direction is called the *centrifugal force* of the ball.

If the centripetal force ceased to exist at any instant, the centrifugal force would also cease, and the body would immediately move along a tangent to its previous circular path, because the cause of the change of direction of the motion had been removed.

The magnitude of the centripetal and centrifugal force is—

$$\frac{WV^2}{gR}$$

where W is the mass of the body, R the radius of the path of its centre of gravity in feet, and V the velocity of the body in feet per second.

Let the body, which is travelling round the circumference of the circle (Fig. 181) with a velocity v feet per second, be at

[1] This should have become evident to the student while studying the cases of equilibrium in Chapters III. and IV.

A at the instant under consideration. Draw MN (Fig. 182) to represent the velocity v of the body at A. Consider some new position, B, of the body near to A. The distance AB = average velocity × time = vt.

Draw MP to represent the velocity v of the body at B.

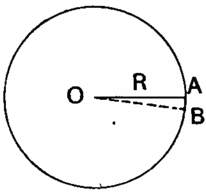


FIG. 181.

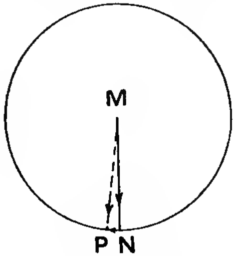


FIG. 182.—Hodograph.

Then the velocity MN = v, together with the change of velocity NP, produce a resultant velocity MP = v. Because the two triangles OAB and MNP are similar—

$$\frac{\text{OA}}{\text{AB}} = \frac{\text{MN}}{\text{NP}} \text{ or } \frac{\text{R}}{vt} = \frac{v}{\text{change of velocity NP}}$$

Therefore—

$$\frac{\text{change of velocity}}{t} = \frac{v^2}{\text{R}}$$

But—

$$\text{acceleration} = \text{change of velocity per sec.} = \frac{\text{change of velocity}}{t}$$

Hence—

$$\text{acceleration} = \frac{v^2}{\text{R}}$$

$$\text{and force} \times g = \text{W} \times \text{acceleration}$$

Therefore—

$$\text{force} = \frac{\text{W}}{g} \times \frac{v^2}{\text{R}}$$

This is the centripetal force which is equal to the centrifugal force.

The second proof of this expression given below may be somewhat difficult to many junior students, in which case it may be omitted on first reading.

Consider what would happen to a body under each of two conditions.

(1) When *no* centripetal force is acting.
(2) When centripetal force *is* acting.

In the first case, let centripetal action cease at the instant the body arrives at A (Fig. 183). It will then move along the tangent AB with a velocity V feet per second.

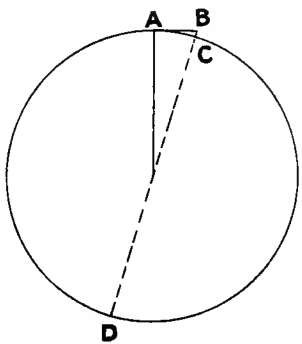


FIG. 183.

Let t be the time in seconds required to traverse AB, then—

$$Vt = AB$$

If the centripetal force had been acting, the body would have travelled along the arc to C.

The difference between the positions of the body in cases (1) and (2) is the distance or length BC, and hence the body has travelled the equivalent distance to BC under the action of centripetal force during the time t seconds while the body was travelling from A to C. But the distance moved over a body under a constant force is given by—

$$\text{initial velocity} \times t + \tfrac{1}{2}\,\text{acceleration} \times t^2$$

In the present case there was no initial velocity in the direction BC, and hence—

$$BC = \tfrac{1}{2}\,\text{acceleration} \times t^2$$

but from an equation above we have—

$$t = \frac{AB}{V}$$

$$\text{and } t^2 = \frac{AB^2}{V^2}$$

Substituting, we get—

$$BC = \frac{\text{acceleration}}{2} \times \frac{AB^2}{V^2}$$

$$\left.\begin{array}{r}\text{or acceleration towards} \\ \text{the centre}\end{array}\right\} = 2V^2 \times \frac{BC}{AB^2}$$

In Fig. 6 of the Introduction we find—

$$AB^2 = AC \times AD$$

Now let the time t seconds be exceedingly small, then the points C and B will be very close to A, and BD will become almost equal to CD. If we make the time interval *indefinitely* small, then BD ultimately becomes equal to CD = 2 × radius of circle = 2R, say. Then—

$$AB^2 = BC \times 2R$$

And transposing, we get—

$$\frac{1}{2R} = \frac{BC}{AB^2}$$

Substitute this above, and we get—

$$\text{acceleration} = 2V^2 \times \frac{1}{2R} = \frac{V^2}{R}$$

$$\text{but force} \times g = \text{mass} \times \text{acceleration}$$

$$\text{or force} = \frac{W}{g} \times \frac{V^2}{R}$$

This is the centripetal or centrifugal force spoken of.

Summary of Chapter VI.

Velocity is the slope of the distance—time curve.

Acceleration is the slope of the velocity—time curve.

Velocity is given by the area of the acceleration—time curve.

Distance is given by the area of the velocity—time curve.

The velocity of a body after falling down any curved path = $\sqrt{2gh}$.

$$\textbf{Force} \times g = \textbf{mass} \times \textbf{acceleration produced.}$$

$$\textbf{Force} \times g \times t = \textbf{mass} \times \textbf{change of velocity.}$$

$$\textbf{Centrifugal force} = \frac{\textbf{mass} \times v^2}{g \cdot r} = \frac{\textbf{mass} \times a^2 r}{g}$$

Examples on Chapter VI.

1. When is velocity said to be uniform?

A train travels at the rate of 30 miles an hour for 600 yds., then at 45 miles an hour for 800 yds., and then at 60 miles an hour for 500 yds. Find the whole time taken. *Ans.* 94·5 secs.

2. The velocity of a body is increased uniformly in each minute of its motion by 66,000 yds. a minute; by how many feet a second is its velocity increased in each second? If the acceleration of a body's velocity, due to the action of a certain force, is 55 in feet and seconds, what is it in yards and minutes, and how far will the body travel in 10 secs.?

3. Two bodies, A and B, move along the same straight line. They were initially 100 ft. apart. A's acceleration is 20 and B's 12 ft. per second in each second. If both moved in the same direction, and A started 3 secs. after B, how far would they be apart in 6 secs.? *Ans.* 126 ft.

4. Convert 20 yds. per minute to feet per second.
   ,, 1500 ft. per hour to yards per second.
   ,, $2\frac{1}{2}$ miles per second to yards per minute.
   ,, 13 miles per hour per hour to feet per second per second.
   ,, 608 ft. per minute per hour to yards per second per minute.

5. A body moves with an acceleration of 3 ft. per second per second, and starts with a velocity of 13 ft. per second. What is its velocity after having moved over 50 ft.? Also how long does it take to do the journey? *Ans.* 21·64 ft.-secs.; 2·88 secs.

6. A body is known to move with a constant acceleration of 10 ft.

per second in a second. What does this statement mean? If a represents the constant acceleration, V the initial velocity, v the final velocity, and s the distance moved over, what is the relation between these symbols, and show how it is obtained. The velocity of a body at some instant is 100 ft. secs.; it then undergoes an acceleration whose value is given above. What is the velocity after 40 ft. have been described, and how much farther must it go before the velocity is reduced to 10 ft. per second?

Ans. 96 ft. per second; 455 ft.

7. A particle moves in a straight line and for any second of its motion, the velocity at the end is 6 ft. per second greater than at the beginning of that second. What was the acceleration and the distance moved over in the 5th and 7th seconds from rest?

8. Describe some experiment to show that when a particle is moving from rest with constant acceleration the space described is proportional to the square of the time.

9. The stroke of an engine running at 120 revolutions per minute is 4 ft. What is the average speed of the crank-pin in feet per second, and what is the angular velocity of the crank expressed in radians or circular measure?

10. A bullet is fired vertically from a rifle, and leaves the muzzle of the rifle with a velocity of 1000 ft. a second. How high will it go (neglecting effect of air resistance), and in how many minutes will it reach the earth again? *Ans.* 15,500 ft., and 1 min. 2 secs.

11. A body is projected into the air with a velocity of 70 ft. per second at an angle of 30° to the horizon. Find, by drawing its path, its vertical and horizontal range.

12. A bucket of water weighing 50 lbs. falls down a well, dragging up an empty bucket weighing 20 lbs. by means of a rope which passes over a pulley. Neglecting the mass of the rope and pulley, what will be the velocity of the buckets after running from rest through 40 ft.?

13. The coupling between an engine and a train, the mass of which is 100 tons, can bear a tension equal to the weight of 15 tons. Find the shortest time in which a speed of 24 miles per hour may be attained on a level line, the resistance to the motion of the train being neglected.

14. Define acceleration.

A particle is projected horizontally with a velocity of 24 ft. per second, from a height of 64 ft.; find its distance from the point of projection when it reaches the ground.

15. A force of 20 lbs. acting on a body makes it move through 28 ft. in 3 secs. from rest. What was the weight of the body? *Ans.* 104 lbs.

16. A body weighing 55 lbs. is moved from rest by a constant force of $5\frac{1}{2}$ lbs. How far will the body move in 12 secs., and what will be its velocity at the end of 3 secs. and at the end of 12 secs.?

17. A body is projected directly up a smooth inclined plane (inclination 45° to horizon) with a velocity of 20 ft. per second. How high will it rise up the plane? *Ans.* 8·85 ft.

18. A body is projected upwards with a velocity of 250 ft. per second. How high will it rise? How long before it will be 400 ft. from the ground?

19. A train weighs 200 tons. When running on the level at 60 miles per hour, the brakes are put down, giving a constant retardation of 20 tons (neglecting the rotation of the wheels). Show how to fix the length and time of the stop.

20. An engine and tender weigh 80 tons. The train 200 tons. The average resistance to motion is 16 lbs. per ton. The train starts from rest, and in 2 minutes attains a speed of $13\frac{5}{7}$ ft. per second. What has been the average pull on the drawbar between the tender and the first carriage, also the horizontal force exerted by the engine-wheels on the rails?

Ans. drawbar $2\frac{1}{7}$ tons and 3 tons.

21. If at the end of 2 minutes the drawbar in question 20 snapped, how far would the carriages go before coming to rest, and how long would it take the engine and tender to acquire a velocity of 60 miles per hour?

Ans. 823 ft. nearly; 1 min. 2 secs.

22. State briefly the meaning of the following terms, and how is each quantity measured? Velocity (linear and angular). Acceleration (linear and angular).

If the velocity of a body decreases uniformly from 50 miles per hour to 7 yds. per second in $\frac{1}{4}$ minute, what is the value of the acceleration in foot-second units, and in yard-minute units?

Ans. $-3{\cdot}49$ ft. per second per second ($-4186\frac{2}{3}$ yard-minutes).

23. Two weights of 10 lbs. each are supported at the ends of a thin cord passing over a loose pulley. If $\frac{1}{4}$ lb. must be attached to one of the weights before it will begin to move without appreciable acceleration, what acceleration would be produced if the $\frac{1}{4}$ lb. was replaced by $\frac{1}{2}$ lb., and what would then be the tension in the cord?

Ans. 0·39 ft.-secs.^{2}; 10·122 lbs.

24. Find the time in which a particle will slide down a cord drawn through the highest point of a circle whose plane is vertical. Find the straight line down which a particle will slide in the shortest time from a given point to a given inclined plane.

25. A shot weighing $\frac{1}{2}$ oz., and moving with a velocity of 2500 ft. per second, enters a butt to the extent of 5 ft. What was the average retarding force offered by the butt, and how long was the shot in stopping? What was the retardation? *Ans.* 610 lbs., 0·004 secs., 625,000 ft.-secs.^{2}.

26. An engine and train weigh together 300 tons. Their speed is reduced by pressing brake blocks upon the wheel tyres. The maximum pressure allowed upon the blocks is 600 lbs. per ton of weight. If the coefficient of friction between block and tyre is 0·2, find how far the train will go while the brake is reducing its speed from 50 to 40 miles per hour, from 40 to 30 miles per hour, and from 30 miles per hour to rest. Also find the respective time.

27. A constant pressure upon the brake of a bicycle produces an approximately constant retardation. If a rider moving at 15 miles per

hour stops in a distance of 22 ft., what has been the retardation, and how long did it take to stop?

28. A train running at 33 miles per hour has its speed reduced in $\frac{3}{4}$ minute to 12 miles per hour. What was the acceleration, and how far did it go during that time?

29. A train, when starting from rest, has an acceleration of 0·4 ft. per second per second. Assuming it to be constant, and there is no friction or other resistance, how long before it would be moving at the rate of 40 miles per hour and 60 miles per hour, and how far would it go from rest in 3 minutes?

30. What is the relation between the force producing motion in a body, and the motion produced, and what are your reasons for believing the relation to be true?

A body falling freely acquires in 1 second a velocity of 981 c.m. per second. If a force equal to the weight of 1 gramme pull a mass of a kilogramme along a smooth level surface, find the velocity when the mass has moved 1 metre.

31. Define "force." A mass of 20 lbs. rests upon a horizontal plank, the coefficient of friction between the two being 0·3. What would be the force necessary to make the body move over 8 ft. in 2 secs., and what would the velocity be at the end of $\frac{1}{5}$ of a second from the start?

32. A train whose mass is 320 tons, is moving with a velocity of 2000 yds. per minute, and is stopped by the brake in 1400 ft.; what is the average retarding force? Also, if it were stopped in 20 secs., what would be the average force, the acceleration, and the distance moved over?

33. A weight of 30 lbs. is made to slide along a horizontal table by another weight of 15 lbs., the two being connected to a cord, and the latter weight hangs vertically, the cord being bent round a loose pulley. The coefficient of friction is 0·4. What is the velocity of the bodies after having moved 3 ft. from rest?

34. A stone, dropped over a cliff, strikes the ground in 3 secs. How high is the cliff, and where was the stone when half the time had elapsed?

Ans. 144 ft.; 36 ft.

35. A 3-ton cage, descending a shaft with a speed of 9 yds. a second, is brought to a stop by a uniform force in the space of 18 ft. What is the tension in the rope while the stoppage is occurring? *Ans.* $4\frac{115}{128}$ tons.

36. A cricket-ball thrown up is caught by the thrower in 7 secs. Draw to scale a figure showing its position at the end of every entire second since its start.

37. A ball thrown up is caught by the thrower 7 secs. afterwards. How high did it go, and with what speed was it thrown? How far below its highest point was it, 4 secs. after its start?

38. What force must be applied for one-tenth of a second to a mass of 10 tons in order to produce in it a velocity of 3840 ft. per minute? What would be the momentum of the mass so moving?

39. A ship is sailing north at the rate of 8 miles an hour through the

sea, and a man walks at the rate of 7 ft. per second straight across her level deck on a line drawn at right angles to her length; draw a diagram (as well as you can to scale) by measuring which one might find the angle the man's resultant path makes with the north, and calculate his velocity with respect to the sea.

40. A stone is thrown vertically upward with a velocity of 160 ft. a second. How high will it rise? And how long will it be before it returns to your hand?

If you let another stone drop down a well, at the instant the first is within 20 ft. of your hand on its return journey, at what distance below your hand will the two bodies meet?

*41. Distinguish between the momentum and the energy of a moving body. A 30-ton mass is moving on smooth level rails at 20 miles an hour; what steady force can stop it, (*a*) in half a minute, (*b*) in half a mile? Specify the force completely.

42. Does the rope of a colliery-hoist have to bear most strain when the cage is at the top or at the bottom of the shaft? To eliminate the weight of the rope itself, consider only the portion immediately above the cage. Explain under what circumstances the stress may be greater than the weight of the cage attached to it.

43. A sphere rolls down a slope with uniform acceleration. It is observed to move 12 ft. in 2 secs. and 20 ft. in the next 2 secs. Find the acceleration and the distance moved from rest before it is first observed.

Ans. 2 ft. per second per second and 2 ft.

*44. A man drags a mass weighing 100 lbs. along a smooth horizontal plane, working at the rate of $\frac{1}{8}$ horse-power. What is the acceleration of the mass when it is moving at the rate of 5 ft. per second?

Ans. 4·4 ft. per second per second.

45. A body, whose mass is 100 lbs., is projected along a horizontal surface with a velocity of 10 ft. per second. A constant horizontal retarding force acts on the body, and brings it to rest after the body has passed over 100 ft. Find the magnitude of the retarding force. *Ans.* 25 oz.

46. A parachute, weighing 1 cwt., falling with a uniform acceleration from rest, descends 16 ft. in the first 4 secs. Find the resultant vertical pressure of the air on the parachute. *Ans.* 105 lbs.

*47. A bullet, weighing 1 oz., is fired horizontally from a height of 16 ft. When it strikes the ground the vertical velocity is $\frac{1}{20}$th of the horizontal velocity. Find the energy, in ft.-lbs., possessed by the bullet at the instant of projection. *Ans.* 400 ft.-lbs.

48. A bird can fly in still air at 20 miles an hour. When a wind is blowing straight from the north at 10 miles an hour, in what direction must the bird aim in order that it may fly from east to west, and at what speed will it travel relative to the earth's surface?

Ans. 17·32 miles per hour.

* These examples may be worked in connection with the next chapter.

49. A locomotive draws a load of 200 tons. Find the pull needed (1) at a constant speed if the friction is 0·05 of the load; (2) if the friction is the same, and the speed rises from 30 ft. per second to 60 ft. per second in 1 minute. ($g = 32$ ft. per second^2.) *Ans.* 13·125 tons.

50. A uniform force acting on a mass of 6 oz. for 2 secs. generates a velocity of 10 ft. per second. Find the measure of the force in dynes. (1 ft. = 30·5 cms.; 1 lb. = 453·6 grms.) *Ans.* 25940·25 dynes.

51. A uniform force equal to the weight of 20 lbs. acts upon a body which is initially at rest, and causes it to move through 24 ft. in the first second. Find the mass of the body. *Ans.* $13\frac{1}{3}$ lbs.

52. What is meant by acceleration?

A meteorite burst at a height of 57,600 ft., and one of the fragments was brought instantaneously to rest by the explosion. It then descended with an acceleration of 32 ft. per second, while the sound of the explosion travelled with a velocity of 1100 ft. per second^2. Which reached the ground first, the fragment or the sound, and what time did each take?

Ans. fragment took 60 secs.; sound took $52\frac{4}{11}$ secs.

*53. State Newton's Laws of Motion.

Which of these laws are required in the proof of the relation $mv^2 = 2Fs$, where m is the mass of a body, F a uniform force acting upon it, v the velocity of the body after it has moved from rest through a space s in the direction of the force?

54. A weight of 2 lbs., attached to a string, falls vertically down a mine with uniform acceleration. Find the value of the acceleration if the tension on the string is 1 oz. ($g = 32$.)

Ans. acceleration = 31 ft. per second per second.

55. A stone thrown vertically upwards is observed to pass upwards through a point P, and, after an interval of 2 secs., to pass downwards through the same point. Find the velocity of the stone at P.

Ans. g ft. per second.

* This example may be worked in connection with the next chapter.

CHAPTER VII.

WORK, ENERGY, IMPACT. SIMPLE HARMONIC MOTION.

If a force of F lbs. act upon a body whose mass is W lbs., while it is moving through a distance s feet; and if V were its initial velocity and v its final velocity; then we have seen in the last chapter that—

$$v^2 = V^2 + 2as$$

We also saw that—

$$F \times g = W \times a$$

Substitute for a in the first equation its value derived from the second, and we get—

$$v^2 - V^2 = \frac{2\,Fg}{W} S$$

$$\text{or } \frac{W}{2g}(v^2 - V^2) = F \times S$$

The right-hand side of this equation is called the *work* done by the force F lbs. in moving the body through s feet; or it may be called the energy given to the body by another body which acts upon it with a force F lbs.

As the two sides of an equation must refer to the same *kind* of quantities, and the right-hand side represents work or energy, the left-hand side must also represent work or energy.

The quantity—

$$\frac{WV^2}{2g}$$

is called the *Kinetic Energy* of a body, and is the energy stored in or possessed by the body due to its velocity of V ft. per second and its mass W lbs.

The above equation simply states that if a body is *free to move*, and is acted upon by a force while it moves through a given distance (in the line of action of the force), that the work done by the force on the body equals the increase of kinetic energy due to the work done upon or given to it.

This is a part statement of what is known as the "Principle of the Conservation of Energy," sometimes called the "Principle of Work."[1]

We may write the equation thus—

$$\frac{WV^2}{2g} + FS = \frac{Wv^2}{2g}$$

which means that if to the kinetic energy of the body (before the force begins to act) be added the work done on the body by the force during its action, we get as a result the kinetic energy of the body at the end of the action of the force.

As energy and work are only two different names for the same thing, we may briefly state the above equation thus—

$$\left.\begin{array}{c}\text{Work stored in}\\ \text{body at start}\end{array}\right\} + \left\{\begin{array}{c}\text{Work done upon or}\\ \text{added to body}\end{array}\right\} = \left\{\begin{array}{c}\text{Work stored in}\\ \text{body at finish}\end{array}\right.$$

It must be borne in mind that in this chapter so far we have only considered a *free* body, *i.e.* one unopposed by any resistance such as friction. If friction exists, the above equation must be modified as follows :—

$$\left.\begin{array}{l}\text{Work}\\ \text{stored}\\ \text{in body}\\ \text{at start}\end{array}\right\} + \left\{\begin{array}{l}\text{Work done}\\ \text{upon or}\\ \text{added to}\\ \text{body}\end{array}\right\} = \left\{\begin{array}{l}\text{Work done by or}\\ \text{given out by body}\\ \text{in overcoming}\\ \text{friction}\end{array}\right\} + \left\{\begin{array}{l}\text{Work}\\ \text{stored}\\ \text{in body}\\ \text{at finish}\end{array}\right.$$

[1] See also Chapter VIII.

Example.—In Fig. 176 let W = 20 lbs. and w = 7 lbs. Also let the coefficient of friction be 0·21. Find the velocity attained after moving 4 ft., and the velocity after 4 seconds of motion.

The work stored in the body at the start is zero, because it has no velocity.

The work done upon the system by the pull of the earth on w lbs. through 4 ft.—

$$= 7 \times 4 = 28 \text{ ft.-lbs.}$$

Work done against friction $= 0{\cdot}21 \times 20 \times 4$

$$= 16{\cdot}8 \text{ ft.-lbs.}$$

The work stored in the system at the end of 4 ft.—

$$= \frac{(\mathrm{W} + w)}{2g} v^2$$

$$= \frac{20 + 7}{64{\cdot}4} v^2 \text{ ft.-lbs.}$$

Then, substituting these quantities, we have—

$$0 + 28 = 16{\cdot}8 + \frac{27}{64{\cdot}4} v^2$$

$$\text{or } \frac{11{\cdot}2 \times 64{\cdot}4}{27} = v^2$$

$$\text{and } v = 5{\cdot}17 \text{ ft. per second}$$

Again, because the force was constant the acceleration must also be constant, and consequently—

$$v^2 = \mathrm{V}^2 + 2as$$

$$\text{or } 26{\cdot}7 = 0 + 8a$$

$$\text{and } a = 3{\cdot}33 \text{ ft. per second per second}$$

Then the velocity at the end of 3 seconds—

$$= \mathrm{V} + at$$

$$= 0 + 3{\cdot}33 + 4$$

$$= 13{\cdot}32 \text{ ft. per second}$$

Flywheel Experiment.—A useful experiment, which

may be carried out here, is that on the flywheel. The calculations may become long and tedious if the wheel has spokes instead of a central web or plate, or only a plain disc, as in Fig. 184, and the difficulty of the experiment is enhanced by the necessity of the accurate measurement of time. A stop-watch is the simplest instrument for measuring time, but the results obtained may be far from accurate if the observer is not careful.

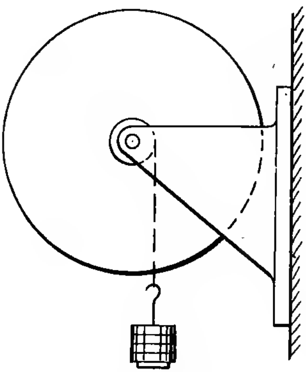


FIG. 184.

Probably some such apparatus as that mentioned in connection with the Atwood machine would give more consistent results.

The wheel is generally mounted on cone centres or a ball-bearing to minimize friction.

The Object of this Experiment is to illustrate the "Principle of Work" or the law of the "Conservation of Energy."

As energy cannot be generated or destroyed, the quantity given to a machine or appliance can be traced in its passage through the machine; and if sufficient data be at hand, the amounts of the several portions, into which the total is divided, may be calculated.

In this experiment, the amount of energy given to the machine (flywheel and attached weight) by the earth is measured by the pull of the earth on the falling weight X, multiplied by the distance through which it falls.

Some of this is stored up in the wheel as *Kinetic Energy*, some of it is used to turn the wheel against the friction of the bearings, and consequently converted into heat energy there, and some is stored in the falling weight as kinetic energy, which is again converted into heat energy on striking and indenting the floor.

We may connect these qualities thus—

$$\left.\begin{array}{l}\text{Energy given}\\ \text{to machine}\\ \text{during fall-}\\ \text{ing of weight}\end{array}\right\} = \left\{\begin{array}{c}\text{K.E.}\\ \text{stored}\\ \text{in}\\ \text{wheel}\end{array}\right\} + \left\{\begin{array}{l}\text{Energy con-}\\ \text{verted into}\\ \text{heat by}\\ \text{friction}\end{array}\right\} + \left\{\begin{array}{l}\text{K.E. stored in}\\ \text{falling weight at}\\ \text{the instant of}\\ \text{striking floor}\end{array}\right.$$

$$\text{Energy A} = \text{Energy B} + \text{Energy C} + \text{Energy D}$$

$$\text{The kinetic energy of a body} = \frac{WV^2}{2g} \text{ ft.-lbs.}$$

when W = weight in pounds and V = velocity of body in feet per second.

If the flywheel has no spokes, then the rim, the central web or disc, and the hub or boss, must be treated separately in calculating its kinetic energy; while if it has spokes, the rim, the spokes, and the boss must be treated separately.

As particles at different distances from the centre will have different velocities at the same instant, we must use some single velocity in the calculation which will represent all the particles together.

This is not the mean velocity of the wheel, but a velocity which, in the case of an ordinary flywheel, is not far different from it. Some idea of the difference may be gained from the footnote below.[1]

[1] Consider the rim only, its inside diameter being 2 ft. and its outside diameter being 2·5 ft. Let the width of the rim be 0·3 ft. Divide the radial thickness of the rim into (say) five rings (Fig. 185), the radial thickness of each being 0·05 ft. (For more accurate work a *large number* of rings should be taken.)

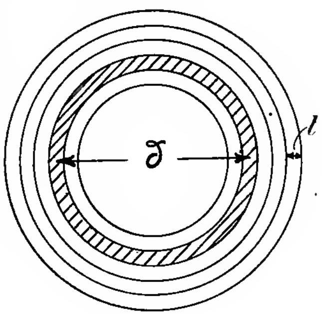


FIG. 185.

The weight of a ring

$= \text{volume} \times \text{weight per cubic foot}$

$= \text{mean length} \times \text{width} \times \text{thickness} \times 465$

$= \pi\delta \times w \times t \times 465 \text{ lbs.}$

where δ, w, and t represent mean diameter, width, and thickness

The true value for the K.E. stored in a ring is—

$$\frac{W}{2g} \times N^2 \times \frac{D^2 + d^2}{8} \times \pi^2 \text{ ft.-lbs.}$$

where D and d are the outside and inside diameters in feet, and N the number of turns per second. The approximate value given in the footnote is sufficiently accurate in most cases.

If the wheel has spokes, the K.E. of *each* spoke is—

$$\frac{W}{2g} \times N^2 \times \frac{4}{3} L^2 \times \pi^2 \text{ ft.-lbs.}$$

where W is its mass, and L its length in feet measured from the axis of the wheel.

The K.E. of the hub is so small that it can be left out of the calculation.

The work done against friction will equal the resistance of friction $\times$ fall of weight. The friction will vary slightly with different weights on the hub of the wheel. We can easily find it when the wheel alone is producing friction, by attaching to the cord just sufficient weight to keep the wheel turning *without* acceleration. This weight, w lbs., just balances the friction of

respectively of the ring. If the ring make N turns per second, its mean velocity is $\pi\delta N$ feet per second, and its kinetic energy—

$$= \frac{WV^2}{2g} = \frac{\pi\delta wt\ 465}{2g} \times (\pi\delta N)^2$$
$$= 225\ wt\delta^3 N^2$$

The width, thickness, and N of each ring is the same.

Putting in the different values for δ, we have for the K.E. of the whole rim—

$$225 \times 0{\cdot}3 \times 0{\cdot}05\ N^2\{2{\cdot}05^3 + 2{\cdot}15^3 + 2{\cdot}25^3 + 2{\cdot}35^3 + 2{\cdot}45^3\}$$
$$= 195\ N^2 \text{ ft.-lbs.}$$

If we take the whole rim as a single ring, this expression becomes—

$$192\ N^2 \text{ ft.-lbs.}$$

The greater the thickness of the rim, the greater will be the difference between the two results.

the wheel and w. If W is the weight of the wheel, and a weight of 14 lbs. is used on the hub for the purpose of the experiment for accelerating the wheel, the resistance of friction will then be—

$$w \times \frac{W + 14}{W + w} \text{ lbs.}$$

The velocity of the falling weight at the instant of striking the floor, and N, the speed of the wheel in turns per second at the same instant, may be calculated in the following manner :—

As the moving *force* is *constant*, the *acceleration* of the weight and wheel must be *constant* (force is proportional to acceleration); that is, the velocity of the weight increases uniformly from zero at the start to V feet per second at the instant of striking the ground. Similarly, the velocity of the wheel will increase uniformly from zero to N turns per second.

The average velocity of the weight is then—

$$\frac{o + V}{2} = \frac{V}{2}$$

And of the wheel—

$$\frac{o + N}{2} = \frac{N}{2}$$

And as distance moved over = average velocity × time—

$$h = \frac{V}{2} \times t$$

$$\text{therefore } V = \frac{2h}{t} \text{ feet per second}$$

Similarly, the number of turns (n) made by wheel while the weight is falling is—

$$n = \frac{N}{2} \times t$$

$$\text{therefore } N = \frac{2n}{t} \text{ turns per second}$$

To determine n, put a 7-lb. weight on the cord to strain it straight, and turn the wheel by hand to see as near as possible how many turns the wheel makes in falling a certain number of feet (the number of feet is the distance through which the weight is going to be permitted to fall in the experiment). Call the distance h feet, and the number of turns (n). The body must be made to fall through h feet during *every* experiment.

Now determine with the stop-watch how long it takes the weight 7 lbs. to fall h feet while turning the flywheel, repeating the observation three times, and determine the average time.

Tabulate thus—

Weight on cord.	Height of fall.	Turns.	Time of falling h feet	Velocity of weight as it strikes floor.	Revolutions per second of wheel as the weight strikes floor.
lbs.	h feet.	n.	secs.	feet per sec.	
7	4·5	8·1	17·5, 18·0, 17·0 } 17·5	0·514	0·926
14	4·5	8·1	11·9, 11·9, 11·8 } 11·9	0·756	1·361
21	4·5	8·1	9·6, 9·6, 9·6 } 9·6	0·9375	1·688

DIMENSIONS OF FLYWHEEL.

Width of rim	2·1 ins.
Radial thickness	1·36 ,,
Outside diameter	23·85 ,,
Inside ,,	21·13 ,,
Mean diameter of spoke	0·9 ,,
Length	10·6 ,,
Number of spokes	6
Calculated weight of rim	54·5 lbs.
,, ,, spokes	10·9 ,,
,, ,, hub	2·0 ,,
,, ,, whole wheel	67·4 ,,

It was found in the experiment that a weight of 1 lb. just balanced the friction of the wheel. As the weight of the wheel was 67·4 lbs., the friction with 7 lbs. on hub

$$= 1 \times \frac{67{\cdot}4 + 7}{67{\cdot}4 + 1}$$

$$= 1{\cdot}08$$

And with 14 and 21 lbs. it is 1·19 and 1·29 lbs. respectively.

Substitute the numerical values in the equation—

$$\text{Energy A} = \text{Energy B} + \text{Energy C} + \text{Energy D}$$

and find the error in each case; that is, the excess or deficit of the right-hand side of the equation over the left.

Then tabulate thus—

Energy put in to machine.	K.E. in wheel.	Friction energy.	K.E. in weight.	Error.	
(A) ft.-lbs.	(B) ft.-lbs.	(C) ft.-lbs.	(D) ft.-lbs.	ft.-lbs.	per cent.
31·5	26·74	4·86	0·03	− 0·13	0·4
63·0	57·76	5·36	0·12	− 0·24	0·38
94·5	88·5	5·83	0·29	− 0·162	0·17

Result of Flywheel Experiment.—The above table shows very clearly that, so far as the flywheel is concerned, the "Principle of Work" holds good, the error being in each case less than that which we should expect from such rough apparatus and methods of experiment.

The student should now be in a position to appreciate the fact (demonstrated by experiments and experience which we have not space to describe here) that *energy* or *work* is *something* in the same way that matter is something, and that neither of these things can be generated or destroyed; that is, neither of them can be produced out of nothing, nor can either of them be annihilated. The *apparent* destruction of either of them is easily explained if sufficient data is provided, and then it is always found that the portion *apparently* destroyed has merely passed from one form to another, or from one place to

another. For example, when coal is burnt in a fire it nearly all disappears *as coal*, but combines with the oxygen of the atmosphere, forming carbonic acid and steam, both of which are invisible, and consequently may, at first sight, have been thought to have disappeared, whereas every particle of matter which was coal before burning has simply changed its form and position, and is in nowise destroyed.

The same with *energy*. The energy stored up in or possessed by the coal in the chemical or potential form is, during burning, chiefly converted into the form called heat, and partly used up in moving the particles of carbonic acid and steam further apart, and is stored up in them.

The heat energy (if the coal is burnt in a boiler furnace) will be partly used in warming up the particles of carbonic acid and steam, partly passed into the water in the boiler, warming it up and converting it into steam, while the remainder leaks away by radiation to the atmosphere and surrounding objects.

The steam (really the heat energy in the steam) may then be used to drive an engine; that is, the heat energy is conveyed by the steam to a machine (an engine), by which some of it is converted to another (mechanical) form, and directed along another channel (the mechanism of the engine), while the remainder chiefly passes out through the exhaust-pipe to the atmosphere or condenser, while the balance has leaked away to the atmosphere and surrounding objects, and has raised their temperatures.

The major part of that which was converted to another form has (if there was a dynamo attached to the engine) been conveyed through the engine parts to the dynamo, and is there again changed into the electrical form of energy, while the balance has been used to overcome the friction of the engine and dynamo, and finally reappears as heat in the neighbourhood in which the friction was overcome. There is also a small amount converted into heat in the dynamo. The electrical energy which leaves the dynamo is partly converted into heat in the conducting wires, and partly converted into heat and light in the lamps which are lighted by the current.

This illustration is given to impress upon the student that,

given enough data, we can follow a quantity of energy through its many changes, and that none of it can go out of existence.

It may be likened to the goods in a carrier's van, which may be of many different kinds, and conveyed in different direction along different roads, yet exist and can be accounted for and followed to their different destinations if the waybill is filled up clearly and sufficiently.

Example.—A body weighing 3220 lbs. was lifted vertically by a rope, there being a damped spring balance to indicate the pulling force F lb. of the rope. When the body had been lifted x feet from its position of rest, the pulling force was automatically recorded as follows :—

x	0	18	43	60	74	95	111	130
F	7700	7680	7430	7130	6770	5960	5160	3970

Find approximately the work done on the body when it had risen 115 feet. How much of this was stored as potential and how much as kinetic energy? What was then the velocity of the body?

The work done in lifting the weight of 3220 lbs. through 115 ft. = 3220 × 115 = 480,300 ft.-lbs.

Plot the different values of x along one axis (Fig. 186), and the corresponding values of F along the other. Draw a curve through the points. Take strips having a width of x = 10 ft. Their middle heights can be read off upon the vertical scale at the side. Their sum is 76,850 lbs., and the corresponding work done by F must be represented by the area of the figure.

$$\text{Area of 11 strips} = \text{work done by F} = 76{,}850 \times 10 \text{ ft.-lbs.}$$

$$\text{area of half a strip} = 5000 \times 5$$
$$= 25{,}000 \text{ ft.-lbs.}$$

$$\text{Total work done} = \text{total area}$$
$$= 768{,}500 + 25{,}000$$
$$= 793{,}500 \text{ ft.-lbs.}$$

Then—

$$\left.\begin{array}{c}\text{Total work done}\\ \text{by force F}\end{array}\right\} = \left.\begin{array}{c}\text{work done in}\\ \text{lifting weight}\end{array}\right\} + \begin{array}{c}\text{K.E. in body at}\\ \text{end of lift}\end{array}$$

$$\text{or } 793{,}500 = 480{,}300 + \frac{Wv^2}{2g} .$$

$$\text{or } 313{,}200 = \frac{3220 \times v^2}{64{\cdot}4}$$

that is—

$$v = \sqrt{6280} = 79{\cdot}2 \text{ ft. per second}$$

Potential energy in body at 115 ft. high = 480,300 ft.-lbs.

Kinetic energy in body at 115 ft. high = 313,200 ft.-lbs.

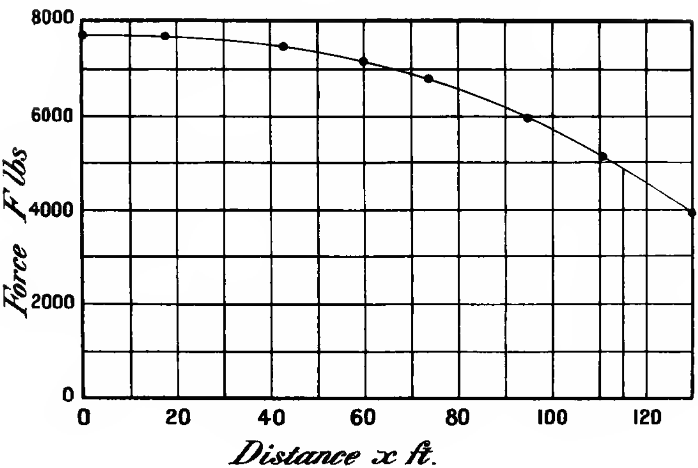


Fig. 186.

Example.—A gas-engine running at the average speed of 150 revolutions per minute has the following dimensions :—

Mean diameter of flywheel .	6 ft.
Width of rim ,, .	8 in.
Thickness of rim ,, .	6 in.
Diameter of piston . .	10 in.
Stroke	18 in.
Mean effective pressure on piston	80 lbs. per square inch.

The engine is running light, and an explosion occurs every sixteenth revolution. Calculate the change of speed which takes place between every explosion. Neglect the spokes, boss, and crank-shaft.

$$\text{Work done per explosion} = \text{total pressure on piston} \times \text{stroke}$$
$$= 80 \times \frac{\pi}{4} \times 100 \times \frac{18}{12} \text{ ft.-lbs.}$$
$$= 3000\,\pi \text{ ft.-lbs.}$$

$$\text{Weight of flywheel} = \text{volume} \times \text{weight per cubic foot}$$
$$= \text{length of rim} \times \text{sectional area} \times \text{weight of 1 cubic foot}$$
$$= \pi \times 6 \times \frac{8}{12} \times \frac{6}{12} \times 465 \text{ lbs.}$$
$$= 930\pi \text{ lbs.}$$

$$\text{Mean velocity of rim} = \text{distance moved by it per second}$$
$$= \frac{\text{distance per minute}}{60}$$
$$= \frac{\pi D \times \text{number of turns per minute}}{60}$$
$$= \frac{\pi \times 6 \times 150}{60}$$
$$= 15\pi \text{ ft. per second}$$

Let V_1 be the greatest velocity just after an explosion, and V_2 the least velocity just before the next explosion.

$$\text{The average velocity} = \frac{V_1 + V_2}{2}$$

which from above equals 15π ft. per second ;

$$\textit{i.e.}\ \frac{V_1 + V_2}{2} = 15\pi$$
$$\text{or } V_2 = 30\pi - V_1$$

$$\text{Greatest K.E. of wheel} = \frac{WV_1^2}{2g}$$
$$\text{least K.E. of wheel} = \frac{WV_2^2}{2g}$$
$$\text{change of K.E.} = \frac{W}{2g}(V_1^2 - V_2^2) \text{ ft.-lbs.}$$

Substitute $(30\pi - V_1)$ for V_2, and we get—

$$\text{Change of K.E.} = \frac{W}{2g}[V_1^2 - (30\pi - V_1)^2]$$

$$= \frac{W}{2g}(60\pi V_1 - 900\pi^2)$$

But the change of K.E. is equal to the work done on the piston to produce it, which equals 3000π ft.-lbs. Hence—

$$3000\pi = \frac{W}{2g}(60\pi V_1 - 900\pi^2)$$

and as $W = 930\pi$ lbs.

$$3000\pi = \frac{930\pi}{64 \cdot 4}(60\pi V_1 - 900\pi^2)$$

Solving for V_1, we get—

$$V_1 = 48 \cdot 3 \text{ ft. per second}$$

Also from above—

$$V_2 = 30\pi - V_1$$

$$= 94 \cdot 4 - 48 \cdot 3$$

$$= 46 \cdot 1 \text{-ft.-seconds}$$

$$\left.\begin{array}{r}\text{change in velocity}\\ \text{between explosions}\end{array}\right\} = 48 \cdot 3 - 46 \cdot 1$$

$$= 2 \cdot 2 \text{ ft. per second}$$

$$\text{percentage change} = \frac{2 \cdot 2}{\text{mean speed}} \times 100$$

$$= \frac{2 \cdot 2}{15\pi} \times 100 = 4 \cdot 6$$

Example.—A flywheel possesses 120,000 ft.-lbs. of energy when its speed is 120 turns per minute. How much does it part with in slowing down to 117 turns per minute? If its mean radius is 3 ft., find its weight. What force applied to the surface of the rim (radius 3 ft. 3 in.) would abstract 70 per cent. of the energy of the wheel at 117 revolutions while it made 10 turns, and how long would it take to do this?

The kinetic energy stored in the wheel is represented by $\frac{WV^2}{2g}$ ft.-lbs.

Now, V = velocity of rim in feet per second

$$= \text{length of circumference} \times \text{number of turns per second}$$

$$= 2\pi 3 \times \frac{120}{60}$$

$$= 12\pi \text{ ft. per second}$$

$$\therefore\ 120{,}000 = \text{K.E.} = \frac{WV^2}{2g} = \frac{W(12\pi)^2}{64{\cdot}4}$$

$$\text{and } W = \frac{120{,}000 \times 64{\cdot}4}{144\pi^2}$$

$$= 5410 \text{ lbs.}$$

Also, as the K.E. of a body is proportional to the square of its velocity—

$$\frac{\text{K.E. at 117 turns per minute}}{\text{K.E. at 120 turns per minute}} = \frac{117^2}{120^2}$$

$$\text{or K.E. at 117 turns} = \left(\frac{117}{120}\right)^2 \times 120{,}000$$

$$= 114{,}000 \text{ ft.-lbs.}$$

Hence, in slowing down from 120 to 117 turns per minute the wheel loses—

$$120{,}000 - 114{,}000 = 6000 \text{ ft.-lbs.}$$

Again, let F be the force in pounds applied to the rim in the direction of a tangent trying to stop the wheel, and the wheel makes 10 turns, while 70 per cent. of 114,000 ft.-lbs. is abstracted from the wheel.

$$\left.\begin{array}{c}\text{The work done by the force}\\ \text{during this time}\end{array}\right\} = \left\{\begin{array}{l}\text{force} \times \text{distance moved through}\\ \text{by point of application of force}\end{array}\right.$$

$$= F \times 2\pi \times 3\tfrac{1}{4} \times 10 \text{ ft.-lbs.}$$

$$\text{hence } 0{\cdot}7 \times 114{,}000 = F \times 2\pi \times \frac{13}{4} \times 10$$

$$\text{and } F = 388 \text{ lbs.}$$

Let N be the number of turns per minute after the abstraction of $0{\cdot}7 \times 114{,}000 = 79{,}800$ ft.-lbs.

$$\left.\begin{array}{c}\text{Then the energy}\\ \text{left in the wheel}\end{array}\right\} = 114{,}000 - 79{,}800 = 34{,}200 \text{ ft.-lbs.}$$

$$\text{and } \frac{34{,}200}{120{,}000} = \frac{\text{K.E. at N revolutions per minute}}{\text{K.E. at 120 revolutions per minute}} = \frac{N^2}{120^2}$$

$$\text{hence } N = 41 \text{ revolutions per minute}$$

and as force × time of action = $\dfrac{\text{mass moved}}{g}$ × change of velocity

$$388 \times t = \frac{5410}{32{\cdot}2} \times \frac{2\pi 3}{60}(117 - 41)$$

$$\text{and } t = 10{\cdot}3 \text{ seconds}$$

The Radius of Gyration of a body is the distance from the axis of rotation at which the mass of the body can be supposed to be concentrated to produce the same result. For

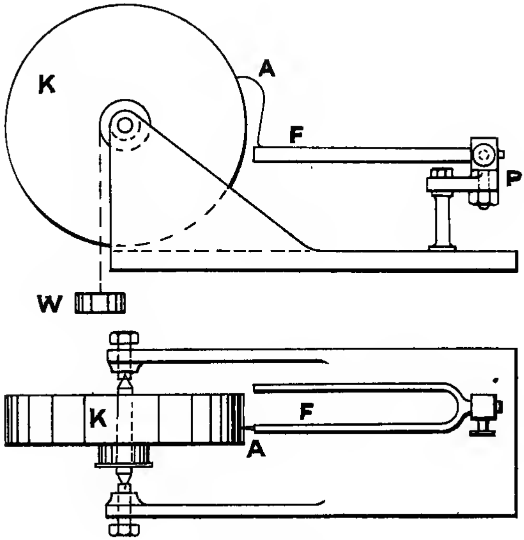


FIG. 187.

example, a particle whose mass is W lbs. situated at a distance R ft. from an axis of rotation, and rotating with an angular velocity α radians per second, possesses—

$$\text{kinetic energy} = \frac{WV^2}{2g} \text{ ft.-lbs.} = \frac{W\alpha^2R^2}{2g} \text{ ft.-lbs.}$$

But if, as is always the case, the body under consideration is

not a particle, but a very large number of particles grouped near together, these particles will not *all* be at the same distance, R, from the axis; and hence we want to find a value of R which can be placed in the above expression to give the kinetic energy of the body. It can be easily found by the aid of higher mathematics, but that is altogether outside the scope of this book.

In Fig. 187 is given an elevation of a plan of a heavy iron disc K, capable of rotation between cone centres so as to reduce friction, and turned by a weight, W, suspended from a small pulley on the axle by a fine cord. The surface of the disc K is blackened by smoking it, or a strip of paper wrapped round it is smoked, while a tuning-fork, F, has a light aluminium stilus which scratches away the soot, and produces an undulating line as the disc rotates.

The number of vibrations per second of the tuning-fork being known, the velocity of the disc can be determined at any instant by counting the number of vibrations on the smoked paper for a certain length of circumference.

As the fork F can swivel round the support P, it can be moved by hand while the disc is rotating, so as not to spoil the line scratched on the paper during the previous revolution.

The method of experiment is the same as with the flywheel (page 178), except that a watch is not used for measuring time, the tuning-fork doing it very accurately.

A table should be constructed like that on page 183, except that the last two columns may be omitted, but in their place one should be substituted, headed "Radius of Gyration," R.

The value of the radius of gyration is obtained by equating the K.E. in Column II. to—

$$\frac{W\alpha^2R^2}{2g}$$

and solving for R.

Impact.—It was shown on page 53 that when a body is in equilibrium there must be a pair of resultant forces acting on the body in opposite directions along the same line. It was pointed out, in connection with Fig. 143, that each member

acted upon the pins at its ends with equal and opposite forces, and from the statement made at the beginning of this paragraph, we see that these internal forces are each balanced by an external force acting along the same line. If a 7-lb. weight rests on the floor, it exerts a force of 7 lbs. on the floor downwards. The floor at the same time acts on the weight in the same line, but upwards in direction, with a force which must be 7 lbs., otherwise the weight would immediately begin to move under the action of the resultant force.

These are illustrations of the fact that every force, however exerted, must be balanced at its point of application, otherwise it could not exist. This is well illustrated by a skater starting off suddenly to walk on his skates, which immediately slip from under him. He cannot exert the usual horizontal force on the ice, because the ice is too smooth to offer any appreciable balancing force or resistance.

This resistance was called by Newton a "reaction," while he called the force which it balanced the "action." Hence the law—

Action and Reaction are equal and opposite.

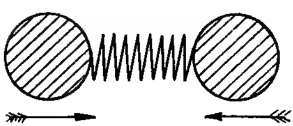


FIG. 188.

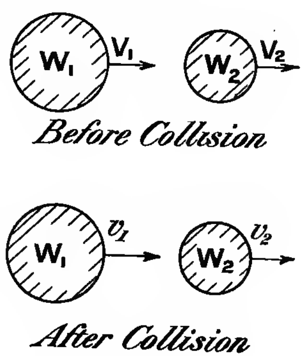


FIG. 189.

When two bodies collide, they each act upon the other with an equal force, just as if a helical spring (Fig. 188) were interposed between them, the action and reaction being

represented by the force exerted by the spring on each of the bodies.

Let the average action and reaction be each represented by F lbs., and the velocity of the bodies (along the line of action which joins their centres) be V and V_2 respectively (see Fig. 189) before impact. After impact let their velocities be v_1 and v_2 respectively. The change of velocity which has been produced in W_2 lbs. by the force F, Fig. 190 (spring in Fig. 188), during impact must be given by the equation—

Average force, F lbs. $\times$ time of action $\times\, g = W_2 \times$ change of velocity produced by F lbs. in the direction of F

In the direction of F (from left to right) the final velocity

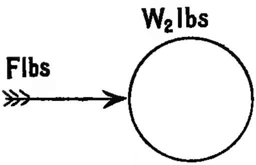


FIG. 190.

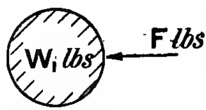


FIG. 191.

is v_2 and the initial velocity V_2, hence the change of velocity must be $v_2 - V_2$, and therefore—

$$Fg \times t = W_2\,(v_2 - V_2)$$

The force $-F$ lbs.[1] acting on W_1 lbs. for t seconds will change its velocity from V_1 to v_1, hence we must have—

$$-\,Fgt = W_1\,(v_1 - V_1)$$

Adding this to the previous equation, we get—

$$0 = W_1\,(v_1 - V_1) + W_2\,(v_2 - V_2)$$
$$\text{or } W_1V_1 + W_2V_2 = W_1v_1 + W_2v_2$$

[1] The negative sign indicates the negative direction of F in Fig. 191.

The mass of a body multiplied by its velocity is called the *momentum* of a body. Hence we may interpret the above equation as—

The total momentum of both bodies before impact	=	The total momentum of both bodies after impact

This is sometimes known as the law of *conservation of momentum.*

There is another relation necessary for the solution of problems on impact, but it is not easy to arrange an experiment to illustrate it well. It is usually stated thus :—

The relative velocity of the colliding bodies after impact $= -e \times$ their relative velocity before impact.

The quantity e is a constant, and called the *coefficient of restitution.* It is different for different materials. The relative velocity of two bodies moving in the same line is the difference of their velocities, hence the above equation may be written as—

$$v_1 - v_2 = -e\,(V_1 - V_2)$$

The sequence of the suffixes in this and the previous equation should be carefully noted.

Below are a few values of e :—

Glass	0·94	Hard steel	0·79
Cast iron	0·73	Soft steel	0·67
Brass	0·41	Lead	0·20

The coefficient of restitution may be found approximately in the following manner :—

Take two spheres of equal magnitude, and suspend them pendulumwise, as in Fig. 192. These spheres can be held in the dotted positions A and D, and allowed to fall simultaneously. They will meet in the positions B and C, and will then rebound again to a height h (above their lowest positions), which will always be less than H. It is difficult to determine h, and hence the results obtained will be only approximate.

The velocity of each body at the instant of impact is due to falling through the height H,[1] and will equal $\sqrt{2gH} = V$ say.

The velocity of rebound $v = \sqrt{2gh}$

The relative velocity of approach $= V - (-V) = 2V$

The relative velocity of separation $= -v - v = -2v$

The relative velocity after impact $= -e \times$ relative velocity before impact

$$\text{or } -2v = -e \times 2V$$

$$\text{and } e = \frac{v}{V} = \frac{\sqrt{2gh}}{\sqrt{2gH}} = \sqrt{\frac{h}{H}}$$

If one of the bodies is at rest, and so large that it will not move, for instance a wall, then the ratio—

$$\frac{\text{velocity of rebound}}{\text{velocity of approach}} = \text{coefficient of restitution} = e$$

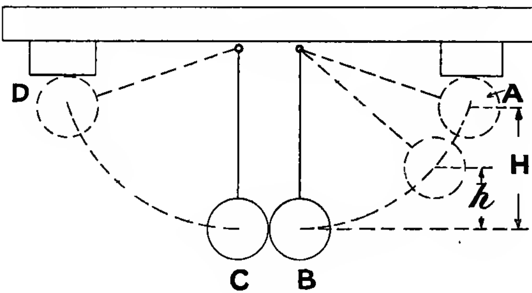


FIG. 192.

Example.—A body A, whose velocity is 20 ft. per second, weighs 10 lbs., and is moving from left to right, and collides with another body of the same material ($e = 0{\cdot}5$) weighing 15 lbs., which is moving from right to left with a velocity of 25 ft. per second. How will they move after impact?

In solving these problems, a student should make a sketch of the bodies, indicating their directions and velocities by arrows.

[1] See page 163.

Substitute in the equation—

$$v_1 - v_2 = -e(V_1 - V_2)$$

and we have—

$$v_1 - v_2 = -\tfrac{1}{2}[20 - (-25)]$$

or—

$$v_1 = v_2 - 22{\cdot}5$$

the negative sign inside the small bracket indicating that the velocity of the second body is in the negative direction.

Again, substitute in the first equation—

$$W_1V_1 + W_2V_2 = W_1v_1 + W_2v_2$$

and we get—

$$(10 \times 20) - (15 \times 25) = 10\,v_1 + 15\,v_2$$

Substituting in this the value of v_1 found above, we get $v_2 = 16$ ft.-secs., and $v_1 = -6{\cdot}5$; the negative sign indicating that the body moves in the negative direction after impact.

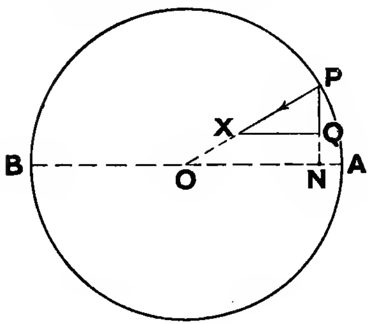


FIG. 193.

Simple Harmonic Motion.—Let a particle move round the circle, Fig. 193, with the constant velocity v ft. per second. Project the position of P on to the diameter at N, then the point N moves backwards and forwards along the diameter AB, and its motion is called *simple harmonic.*

The particle at P has an acceleration towards the centre of the circle [1] of

$$\frac{v^2}{\mathrm{OP}} \text{ ft. per second per second}$$

[1] See page 166.

Resolve this acceleration in the direction of the diameter, thus: Set off PX to represent the acceleration $\frac{v^2}{\mathrm{OP}}$, then QX is the component in the direction of the diameter. But from similar triangles—

$$\frac{\mathrm{QX}}{\mathrm{ON}} = \frac{\mathrm{PX}}{\mathrm{OP}}$$

Substituting for PX its value $\frac{v^2}{\mathrm{OP}}$, we get—

$$\frac{\mathrm{QX}}{\mathrm{ON}} = \frac{v^2}{\mathrm{OP}^2} = \frac{v^2}{\mathrm{R}^2}$$

where R = radius of circle

But QX is the acceleration of P parallel to the diameter; that is, the acceleration of N along the diameter; hence, *with simple harmonic motion, the acceleration of a particle is proportional to its distance from the middle point of its path*,

$$\text{and } \frac{\text{acceleration}}{\text{displacement}} = \frac{v^2}{\mathrm{R}^2}$$

Let the particle P make n revolutions per second, then the time taken for one revolution $= \frac{1}{n}$ second $= t$ seconds (say).

$$\begin{aligned} \text{But } v &= \text{distance moved by P per second} \\ &= \text{length of circumference} \times n \\ &= 2\pi \mathrm{R} n \end{aligned}$$

$$\text{Therefore } n = \frac{v}{2\pi\mathrm{R}}$$

$$\begin{aligned} \text{and } t = \frac{1}{n} &= \frac{2\pi\mathrm{R}}{v} \\ &= 2\pi\sqrt{\frac{\mathrm{R}^2}{v^2}} \\ &= 2\pi\sqrt{\frac{\text{displacement N}}{\text{acceleration of N}}} \end{aligned}$$

But t = time of one revolution of particle

= time in which N makes a complete to and fro journey from A to B and back to A

= *time of one complete oscillation*

Hence, if a body's motion is simple harmonic, the time t seconds of one complete oscillation

$$= 2\pi\sqrt{\frac{\text{its displacement at any instant}}{\text{its acceleration at that instant}}}$$

The Pendulum.—Let OA, Fig. 194, be a simple pendulum in which the point A oscillates along the circular path ABC. While at the position A, the forces acting on the bob of the pendulum are W, its weight downwards, and the tension along AO. Resolve W parallel and perpendicular to AO. The latter component, DE, is that which causes motion, and it equals W sin θ.

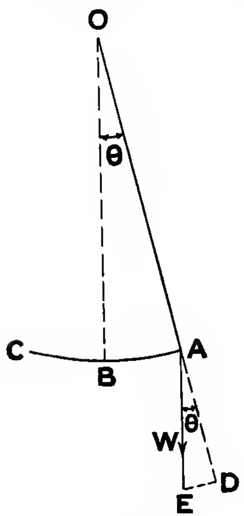


FIG. 194.

But force $\times g$ = mass moved $\times$ acceleration produced

or W sin $\theta \times g$ = W $\times$ acceleration,

that is g sin θ = acceleration along the curve at A

Now, the arc ABC is always *very small* compared with OB, and when the arc is small the angle θ must be correspondingly small. Where an angle is small, its sine is approximately equal to its circular measure. Call its circular measure θ, then $g\theta$ = acceleration along the curve, *i.e.* in the direction of motion.

But length of arc = radius $\times$ circular measure of the angle

therefore AB = OB $\times$ θ

$$\text{and } \theta = \frac{AB}{OB}$$

Substituting in the equation—

$$g \,.\, \theta = \text{acceleration}$$

$$\text{we get } g \cdot \frac{AB}{OB} = \text{acceleration} = \frac{g}{l} \times AB$$

where l the length of the pendulum is substituted for OB.

But AB is the displacement of the bob from the middle point of its path, and g and l are constant, hence by the previous article the motion is simple harmonic, because the displacement is proportional to the acceleration in the direction of motion. This being so, we must have—

$$\left.\begin{array}{l}\text{The time } t \text{ secs. of one} \\ \text{complete oscillation}\end{array}\right\} = 2\pi\sqrt{\frac{\text{displacement at any instant}}{\text{acceleration at that instant}}}$$

$$= 2\pi\sqrt{\frac{AB}{\frac{g}{l} \,.\, AB}}$$

$$= 2\pi\sqrt{\frac{l}{g}}$$

This should be verified by experiment. Suspend a weight of about 4 lbs. by as fine a cord as possible, its maximum length being about 6 ft. Observe the time of say 100 complete oscillations with a stop-watch. Enter the result in the table below.

Shorten the cord and again get the time of 100 oscillations. Repeat with different lengths.

RECORD OF EXPERIMENT ON A SIMPLE PENDULUM.

l.	Time of 100 nscillations.	Time of 1 oscillation, t secs.	t^2.

Plot the numbers in the first column along the base and those in the fourth column vertically. The result is a straight line if the experiment has been accurately carried out, and the slope of the line will be found to be as nearly as possible 1·22, which is $\frac{4^2\pi}{g}$.

Another illustration of simple harmonic motion can be carried out as follows:—The deflection of a helical spring was shown on page 10 to be proportional to the force producing it, hence the acceleration due to this force must be proportional to the displacement because force is proportional to the acceleration produced by it. But in the article on simple harmonic motion we find that if the acceleration of a body is proportional to its displacement, its motion is simple harmonic, and the time of one complete oscillation is—

$$2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

Fix a weight on the end of a long helical spring, and set it oscillating vertically. Observe the time of 100 oscillations, and hence deduce the time of one oscillation.

Now find the static deflection, D ft., produced by the weight W lbs.

$$\text{As force} \times g = \text{mass} \times \text{acceleration}$$

$$\frac{\text{force} \times g}{\text{mass}} = \text{acceleration}$$

But with a helical spring, load = a constant × deflection, that is, force = D × a constant = D × C (say). Therefore—

$$\frac{\text{D} \times \text{C} \times g}{\text{mass}} = \text{acceleration}$$

$$\text{or } \frac{\text{mass}}{\text{C} \times g} = \frac{\text{displacement D}}{\text{acceleration}}$$

Substituting in the above equation, we get—

$$t = 2\pi\sqrt{\frac{\text{mass moved}}{\text{C} \times g}}$$

The mass moved is that on the end of the spring, together with the equivalent mass of the spring. This latter will be small if the spring is not too stiff.

The equivalent mass of the spring is about one-third of its mass.

The pendulum and the expression—

$$t = 2\pi\sqrt{\frac{l}{g}}$$

can be used for the purpose of finding g—in fact, the accurate values of g are determined this way. Observe the time of a number of oscillations, and so get the time of one oscillation (called its period or periodic time). Measure the length of the pendulum and calculate g.

A simple piece of apparatus by which g can be obtained is shown in Fig. 195. A thin lath, P, is arranged to oscillate about a knife-edge at its upper end. It can be set swinging, and its periodic time determined, and hence the time required for it to move from the position shown in the figure to the dotted position. This will be one quarter of its periodic time.

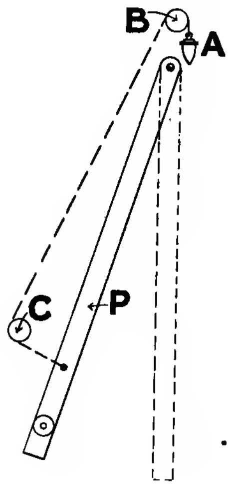


FIG. 195.

A weight, A, is suspended by a thread round loose pulleys, B and C, the other end being used to hold the pendulum in the displaced position. The flange on the weight A is a piece of felt which can be saturated with ink. If the thread be burnt or cut with a scissors, the pendulum and weight A begin to move together, and as the former is swinging into the dotted position, it strikes the weight A, and the inked felt marks the distance it has fallen from rest.

Let this be s feet, and the time of one complete oscillation be t secs. Then—

$$s = \tfrac{1}{2}g\left(\frac{t}{4}\right)^2 = \frac{gt^2}{32} \text{ or } g = \frac{32s}{t_2}$$

Newton's Laws of Motion.—The portion of Mechanics dealing with the motion of a body and its cause is outlined in Chapters VI. and VII. In them the different results and equations have been derived from experiment. We are now in a position to review the matter in these chapters, and, after having done so, we shall find the whole embodied in the three fundamental laws of motion which are called after Newton, who formulated them.

The first states what a force is, the second states how it is measured, the third states how one body acts upon another.

Law 1.—*Every body continues in its state of rest or uniform motion in a straight line until it is made to change that state by a force.*

This simply states that if no force act on a body it may be at rest, or it may move in a straight line with constant velocity. A force is required to increase or decrease the velocity of the body, or to change its direction of motion.

Law 2.—*The change of momentum of a body is proportional to the force producing it, and the change takes place in the line of action of the force.*

This is another way of stating—

$$\begin{aligned} \text{Force} \times \text{time} \times g &= \text{mass} \times \text{change of velocity} \\ &= \text{change of momentum} \\ \text{or force} \times g &= \text{mass} \times \text{acceleration} \end{aligned}$$

Law 3.—*To every action there is always an equal and opposite reaction.*

The term action in this law is used to indicate two different quantities.

Putting aside chemical action, the way in which one body invariably acts upon another is by means of force. A body

resting upon a table urges the table vertically downwards, and the body would certainly move downwards if the table did not exert an upward force (reaction) to balance the downward force (action). It must be evident that in every statical case action and reaction must be equal and opposite, because there is no motion, and consequently, by the first law of motion, there can be no resultant force acting.

If action and reaction are equal and opposite, their effects, such as the momentum produced by them, must be equal and opposite, as in impact.

There is another way in which Newton considered the equality of action and reaction. The following sentence in italics is Newton's scholium to his third law :—

If the action of an agent be measured by the product of its force and velocity ; and similarly, if the reaction of the resistance be measured by the velocities of its several parts and their several forces, whether they arise from friction, cohesion, weight, or acceleration ;—action and reaction, in all combinations of machines, will be equal and opposite.

The product of force and velocity is force $\times \frac{\text{space}}{\text{time}} = \frac{\text{work}}{\text{time}}$ = rate of doing work per unit of time, or what we call "power."

Putting the above scholium into modern language, we have—

Rate at which work or energy is given to a machine = rate at which work is done against friction + rate at which work is done against gravity + rate at which work is done in accelerating the different parts + rate at which work is done or given out by the machine for some useful purpose.

If we multiplied each term by the time of action, we get—

Work put into machine = work done in overcoming friction + work done against gravity + kinetic energy stored in the machine + useful work done by the machine.

This is the "Principle of Work."

It is not easy to arrange ocular demonstrations of the

dynamical equality of action and reaction, besides experiments in collision.

Let two boats near the shore of a lake each contain a man, and each man holds in his hand one end of a rope. If both men haul in the rope simultaneously, the boats will advance towards each other, and the momentum of each must be the same, because the action on one has been the reaction on the other, namely, the tension in the rope.

It is a common experience, especially noticeable to onlookers, that as a person walks along a light pleasure-boat, the boat also moves in a direction opposite to that of the individual. The person in the act of walking forward cannot help pushing the boat backward; in fact, he could not move forward if the boat did not move backward.

It should be noticed that as action and reaction are equal and opposite, there can be no external resultant force acting on the person and boat considered together, and therefore, as there is no resultant force, their common centre of gravity cannot move. Similarly with the two boats; their common centre of gravity could not move, because there was no external resultant force acting on them.

A common question asked in examinations is the following:—

A horse pulls a cart forward, and by the third law of motion the cart pulls the horse backward with an equal force; then why does the horse move?

The question at first sight seems paradoxical, but it is only apparent and not real. In questions of this sort it is necessary to consider *all* the bodies on which there is an action and reaction. In the problem above, the horse is attached to the cart by its harness, and where the horse goes the cart must go also.

There is another body concerned besides the horse and cart, namely, the earth. The horse urges itself and the cart forwards, and in doing so pushes on the earth in the opposite direction with its hoofs, and whatever the resulting motion happens to be, the centre of gravity of all three bodies is unaffected by the motion; that is, as the horse and cart move

forward, the earth must move backward; but of course the latter motion is infinitesimal, because the mass of the earth is so very large compared with that of the horse and cart. At the points of contact between the horse's hoofs and the earth, action and reaction are equal and opposite; the former being the horizontal component of the pressure of the hoofs on the earth, and the latter being the horizontal component of the pressure of the earth on the hoofs. As the cart is attached to the horse, it must move with it, and the action (pull of horse on cart) and reaction (pull of cart on horse) together is of the nature of a stress, and is exactly of the same character as the stress at the ends of one of the members of the bridge in Fig. 143, or the same as the stress in the drawbar between a locomotive and the train. In the latter case the action and reaction at the ends of the drawbar are equal and opposite, and by this means the locomotive and train do not separate, but the force which makes the whole train and locomotive move is the reaction of the rail on the driving-wheels in the forward direction; the corresponding action being the horizontal force exerted by the wheels on the rails, and which urges the earth backwards.

Absolute Units of Force.—It was found that if a force of F lbs. acted upon a body of mass M lbs., the acceleration (a) produced was given by the equation—

$$\mathrm{F} \times g = \mathrm{M} \times a.$$

If a smaller unit of force be selected such that its measure is [F (lbs.) $\times g$] units, each of which is called a poundal; the above equation can then be written as—

$$\text{Force in poundals} = \text{mass (lbs.)} \times \text{acceleration.}$$

This new unit of force called the **poundal** is $\frac{1}{g}$th part of a pound.

The above equation enables force to be measured irrespective of position or locality, and hence the term *absolute unit of force*, which is that force required to produce unit acceleration of unit mass in unit time.

In the metric system, the absolute unit of force is the *dyne*, which is the force required to produce an acceleration of 1 centimetre per second per second in a mass of 1 gramme during 1 second.

The corresponding unit of work is the centimetre—dyne, which is called an *erg*.

The student should try and remember that a force of g dynes is equivalent to a force of 1 gramme, and a force of g poundals is equivalent to a force of 1 lb.

Summary of Chapter VII.

Initial K.E. of a body + energy received by body = work done by body in overcoming resistances + final K.E. of body.

Total momentum before impact = total momentum after impact.

Relative velocity after impact = − e × relative velocity before impact.

In simple harmonic motion, time of one complete oscillation.

$$= 2\pi\sqrt{\frac{\textbf{displacement}}{\textbf{acceleration}}}$$

Examples on Chapter VII.

1. Give two illustrations of the equality of *action* and *reaction*.

Two men, A and B, while skating, come into direct collision, and are brought to rest. A weighs 10 stone and B 12 stone. A's speed is 6 miles per hour; what is B's speed? *Ans.* 5 miles per hour.

2. A gun weighing 3 kilogrammes fires a bullet weighing 30 grammes, and the latter has a velocity of 60,000 cm. per second. Compare the kinetic energy of the gun with that of the bullet (neglecting the powder).

Ans. As 1 is to 100.

3. A railway train, exclusive of engine, weighs 150 tons, and in starting along a level line from rest it attains a speed of 30 miles an hour in 5 minutes. What has been the mean pull between the engine and train, the resistance being taken at 10 lbs. per ton? *Ans.* 3040 lbs.

4. Define kinetic energy. If a velocity of 300 ft. per second is impressed

on a weight of 10 lbs., what is the measure of the energy now imparted to the weight? *Ans.* 14,062·5 lbs.

5. Write down the formula for the amount of energy stored up in a given weight when moving with a given velocity. Describe, with a sketch, the action of a fly-press. If each ball of the press weighs 50 lbs., and the work stored up in the balls is 400 ft.-lbs., find the velocity with which they are moving. *Ans.* 16 ft. per second.

6. The rim of a fly-wheel weighs 9 tons, and the mean linear velocity of its mass is assumed to be 40 ft. per second; how many foot-tons of work are stored up in it? If it be required to store the additional work of 9 ft.-tons, what should be the increase of velocity?

Ans. 225 ft.-tons; 0·79 ft. per second.

7. The height and length of an inclined plane are 20 ft. and 100 ft. respectively; a body weighing 100 lbs. is placed at the top of the plane and allowed to slide along its whole length; the coefficient of friction between the plane and the body is 0·15; how many units of work (foot-pounds) are accumulated in the body, and what is its velocity when it reaches the foot of the plane?

8. What meaning do you attach to the phrase *horse-power?* A fire-engine pump is provided with a nozzle, the sectional area of which is 1 sq. in., and the water is projected through the nozzle with a velocity of 130 ft. per second; find the horse-power of the engine required to drive the pump, irrespective of the loss by resistance of the working parts. The weight of a cubic foot of water is $62\frac{1}{2}$ lbs.

9. A flywheel weighs $2\frac{1}{2}$ tons, and its mean rim has a velocity of 40 ft. per second. If the wheel gives out 10,000 ft.-lbs. of energy, how much is its velocity diminished? *Ans.* 1·445 ft. per second.

10. State the rule for finding the amount of work stored up in a given weight when moving with a given velocity. A weight of 6 cwt. moves with a velocity of 20 ft. per second; how many units of work are stored up in it? *Ans.* 4200 ft.-lbs.

11. The head of a steam-hammer weighs 10 cwt., and has a fall of 8 ft. If it indent the iron on which it falls by 1 in., find the mean force exerted on the iron during compression. *Ans.* 97 cwt.

12. A cast-iron flywheel has a rim 5 ft. mean diameter, 8 in. wide, and 3 in. thick. The boss and spokes can be neglected. How many foot-pounds of energy are stored in it when running at 160 revolutions per minute? The shaft upon which it is keyed is 4 in. diameter, and weighs 400 lbs. If the coefficient of friction between shaft and bearing is 0·05, how many turns will it make before coming to rest (neglect resistance of the air)? What was the time in coming to rest, and the angular acceleration?

If a brake-strap had been put on the wheel with a load of 200 lbs. on one end and 30 lbs. on the other, how many turns would it then make before stopping?

13. Two bodies, A and B, collide. A weighs 17 lbs. and moves originally with a velocity + 10 ft. per second, while its velocity after

collision is $-\frac{83}{27}$ ft. per second. Find the weight of B if its velocity after collision was $\frac{370}{108}$ ft. per second, and the coefficient of restitution was 0·25. Find also the original velocity of B.

14. A weight of 10 lbs., moving with a velocity of 55 ft. per second, is brought to rest in 0·02 second. Find the average force exerted on the weight.

15. A car weighs 10 tons. It is drawn by a pull of P lbs., varying in the following way, t being the time in seconds from the start.

P	1020	980	882	720	702	650	713	722	805
t	0	2	5	8	10	13	16	19	22

The retarding force of friction equals 410 lbs. Plot (P − 410) and the time t, and find the time-average of this force. What does this represent when multiplied by 22 seconds?

Read your notes on acceleration and velocity diagrams, and describe how you would find the distance travelled by the car in the first 17 seconds.

16. The flywheel of a machine has 150,000 ft-lbs. stored in it when its speed is 250 revolutions per minute. How much energy does it part with during a reduction of speed to 200 revolutions per minute?

17. State the Third Law of Motion, and explain clearly its application to the case of a horse starting a cart into motion. If the pull back of the cart be exactly equal to the pull forward of the horse, why do they begin moving?

18. A gun delivers 400 bullets per minute, each weighing $\frac{1}{2}$ oz., with 2000 ft. per second horizontal velocity. What is the average force exerted on the bullets in the gun? *Ans.* 781 lbs.

19. A ship, weighing 8000 tons, makes the passage from Liverpool to New York at the rate of 20 knots (a knot is 6080 ft.) per hour. If the efficiency of the engines and propeller is 0·4, and the I.H.P. of the engine is 10,000, how much is the resistance to the ship's motion per ton of weight? *Ans.* 8·2 lbs.

20. The resistance of a train on the level is 17·3 lbs. per ton, the speed being 48 miles per hour. If the weight of engine and train = 190 tons, find the H.P. of the engine. If the train is brought to a standstill by the brake in 14·5 seconds, what was the average resistance of the brake? *Ans.* 14·1 tons; 253 H.P.

CHAPTER VIII.

MACHINES.

A Machine is an assemblage of parts connected together, each of which can only move in *one* definite manner.

This is equivalent to saying that the motion of each part of a machine is controlled, or that we have control over the different parts of a machine, and therefore they may be made to serve some useful purpose. If we had not control over the machine, it would, of course, be useless, and we should not be able to predict by calculation the movements of its different parts.

When the object of a machine is to do work or transmit force—such, for example, as a crane—we generally speak of it as a machine; but if we are only considering the geometry of the machine, or if we are considering an assemblage of parts, such as a clock, in which the parts are simply used to transmit motion and not to do useful work, we often speak of the contrivance as a *mechanism.*

Velocity Ratio.—The part to which the motive force is applied is called the *driving end*, and the end at which the resistance is overcome or the useful work done is called the *following* end, and sometimes called the *load* end. The ratio—

$$\frac{\text{Movement of driving end}}{\text{Movement of following end}}$$

is called the *velocity ratio* of the machine. This quantity we can calculate or determine by experiment.

Take the wheel and axle in Fig. 196, and let it make one turn. The cord supporting the load, W, will be wound up to

the extent of the circumference of the axle, which equals πd inches. The driving end will in the same time move through

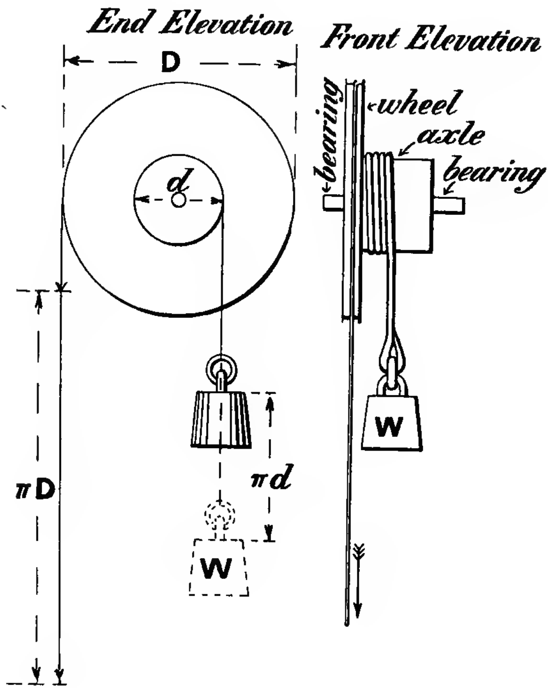


FIG. 196.—Wheel and axle.

a distance equal to the length of cord unwound from the wheel in one turn, which is πD inches.

We then have—

$$\text{Velocity ratio} = \frac{\text{movement of driving end}}{\text{movement of following end}}$$

$$= \frac{\pi D}{\pi d} = \frac{D}{d}$$

We may also find this quantity by experiment. Determine

the simultaneous movement of each end by actual measurement. This can be easily done by first placing weights on the following and driving ends, so that they will balance or remain

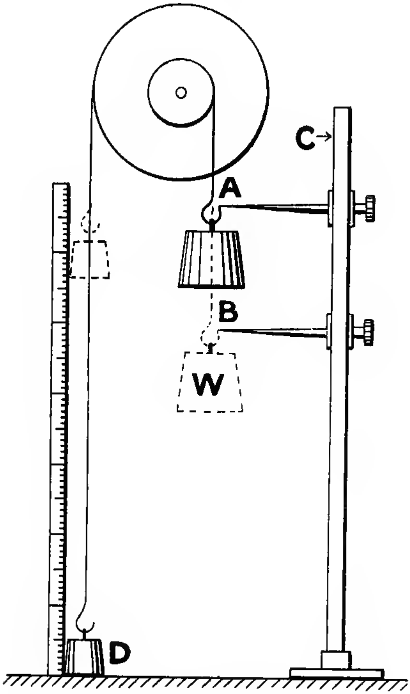


FIG. 197.

stationary in any position in which they may be placed, and at the same time keep the cords straight.

Place the weight[1] on the driving end in contact with the floor at D (Fig. 197), and set the pointer at A opposite some point on the following end. The pillar C contains a long slot in which the pointers can be adjusted. Now move

[1] Ring weights should be used, as in the figure.

the weights to some new position, shown dotted, and set the pointer B to the same point on the following end of the machine that A was previously set to. The distance between A and B is the movement of the following end. The corresponding distance moved by the driving end can be measured as shown by one or more yard-scales.

Repeat these measurements for different positions of the weights, and tabulate as below.

Movement of driving end.	Movement AB of following end.
12·8	4·1
19·0	6·1
23·4	7·5
25·0	8·0
34·8	11·3
46·0	15·0

Plot the first column vertically upwards and the second

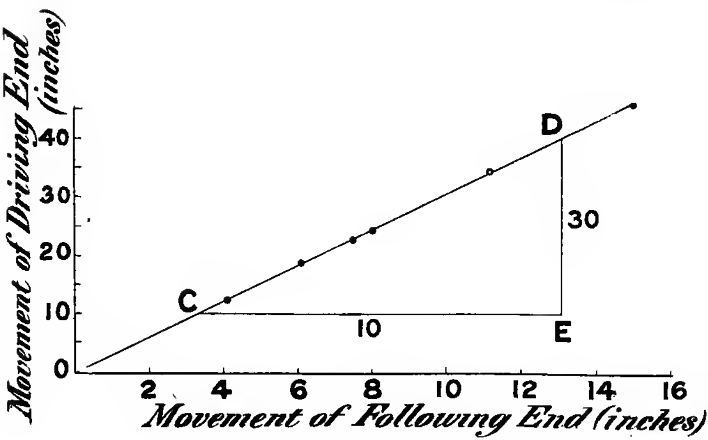


FIG. 198.

column along the base, as in Fig. 198. Drawing the nearest

line through all the points, we have for the slope of that line—

$$\frac{DE}{CE} = \frac{30}{10} = 3$$

$$= \frac{\text{movement of driving end}}{\text{movement of following end}}$$

The same method may be applied to all the simple machines.

We will now calculate the velocity ratio of a number of simple machines.

The Rope Pulley Tackle is shown in Fig. 199. To

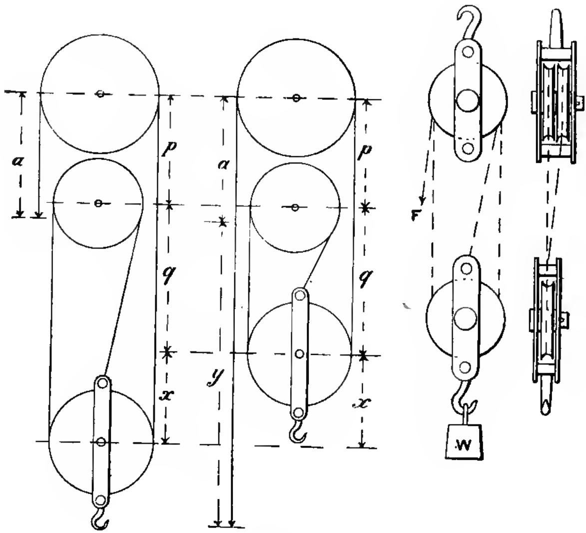


Fig. 199.

save confusion, the different pulley sheaves have been separated so as to be able to identify each of them.

On the left of the figure the arrangement is shown in one

position, and next on the right the tackle is shown displaced. The movements of the two ends are denoted by y and x.

We want to find the value of the ratio—

$$\frac{y}{x}$$

This is really a geometrical problem, and has little to do with mechanics proper. In all such problems there must be some geometrical fact upon which the solution must be built. Here the fact is that the rope is *always the same length ;* hence put down the length of the rope in each position and equate one to the other.

Let c represent the length of the curved portions of the rope in each position; then the length of the rope in the first position is—

$$a + c + p + 3q + 3x$$

And in the second position it is—

$$y + a + c + p + 3q$$

Therefore—

$$a + c + p + 3q + 3x = y + a + c + p + 3q$$

$$\text{or } 3x = y$$

$$\text{and } \frac{y}{x} = 3$$

This is the number of ropes supporting the lower block, hence the velocity ratio can be found direct from inspection.

The Chinese Wheel and Axle is shown in Fig. 200. Let the contrivance make one turn. The straight parts of the ropes are shown on the left in Fig. 201 before movement, and on the right after movement. Let D. = diameter of wheel, d_1 and d_2 the diameters of the axle.

During one turn the larger axle winds up a length equal to its circumference = πd_1, and simultaneously the smaller axle

unwinds a length equal to its circumference $= \pi d_2$, and hence the hanging part of the rope is shorter by the amount—

$$\pi\ (d_1 - d_2)$$

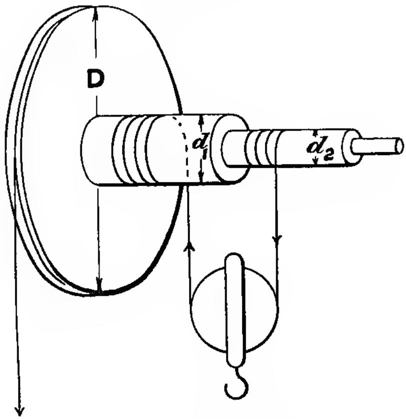


FIG. 200.—Chinese wheel and axle.

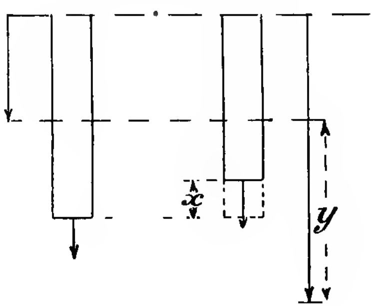


FIG. 201.

But the hanging part of the rope is shorter by $2x$, as shown in Fig. 201. Hence—

$$2x = \pi\ (d_1 - d_2)$$

$$\text{and } x = \frac{\pi}{2}\ (d_1 - d_2)$$

The part of the rope unwound from the wheel $= \pi D = y$. Hence—

$$\frac{y}{x} = \frac{\pi D}{\frac{\pi}{2}(d_1 - d_2)}$$

$$= \frac{2D}{d_1 - d_2}$$

This is the velocity ratio of this machine.

A very useful form of the Chinese wheel and axle is called Weston's Differential Pulley Block. It is shown diagrammatically in Fig. 202. The wheel and the larger part of the axle are one and the same, and the chain used is endless. The grooves in the upper pulleys in which the chain runs have little pockets cast in them, into which the links of the chain fit easily. These prevent the chain from *slipping* over the wheel. They are equivalent to the fastening of the ends of the ropes to the wheel and axles in Fig. 200.

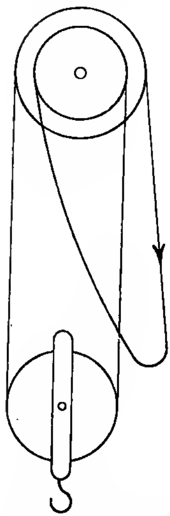


FIG. 202.—Weston's differential pulley lifting tackle.

As the wheel and the larger part of the axle in Fig. 200 are of the same diameter in Weston's tackle, the formula for the velocity ratio given above now becomes—

$$\text{Velocity ratio} = \frac{2d_1}{d_1 - d_2}$$

The Screw-jack.—We can find the velocity ratio in the ordinary screw-jack thus:—

Let the screw make one turn. During that time the end of the lever where the motive force is applied moves round in the circle ABC (Fig. 203), while the load is lifted through a distance equal to the vertical movement of the screw (called the pitch of the screw).

Let p = pitch and l = length of lever, then—

$$\text{velocity ratio} = \frac{\text{movement of driving end}}{\text{movement of following end}} = \frac{2\pi l}{p}$$

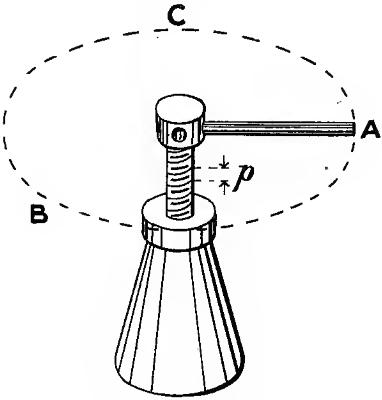


FIG. 203.—Screw-jack.

The Worm and Worm-wheel.—In Fig. 204 is shown the worm and worm-wheel, sometimes called the endless screw.

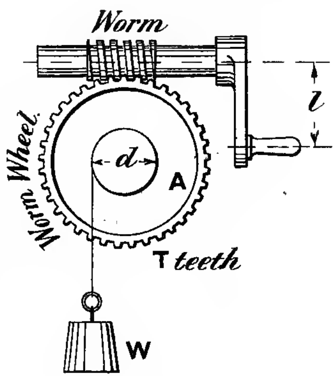


FIG. 204.—Worm and worm-wheel.

Let the worm-wheel, A, make one turn. As the screw or worm turns, it forces the teeth of the worm-wheel to move with it, and hence each turn of the screw will move the worm-wheel round to the extent of one tooth, and therefore the screw must make as many turns as there are teeth in the wheel, to rotate the wheel once. If there are T teeth in the wheel, the screw, and consequently the handle (length l), must

make T turns for one of the wheel A and the drum supporting the load.

$$\text{Movement of load during one turn of wheel} = \pi d$$
$$\text{Movement of end of handle} = 2\pi l \times \text{T}$$
$$\text{Velocity ratio} = \frac{2\pi l \times \text{T}}{\pi d}$$
$$= \frac{2l\text{T}}{d}$$

Wheel-trains.—When dealing with trains of wheels, we generally use for the velocity ratio—

$$\frac{\text{number of turns of driving-wheel}}{\text{number of turns of following-wheel}}$$

The circles in Fig. 205 represent what are termed the *pitch circles* of two toothed or spur wheels in gear with each other. The pitch circle is a circle which passes, approximately, through the middle of the elevation of the teeth, and which appears to roll upon the pitch circle of the companion wheel, just as if the pitch circles were the outlines of discs which rolled together by the friction at their circumferences.

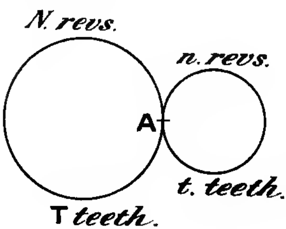


FIG. 205.

Let capital letters refer to the left wheel and italic letters to the other wheel. Also let N represent the number of turns per minute or revolutions per minute, while T represents the number of teeth, and D the diameter of a wheel.

The teeth prevent one pitch circle from slipping over the other.

As there is no slipping, the same length of circumference of each pitch circle must pass the point of contact during any given interval of time.

$$\left.\begin{array}{l}\text{Length of circumference of left wheel pass-}\\ \quad\text{ing point of contact per minute}\end{array}\right\} = \pi \mathrm{DN}$$

$$\left.\begin{array}{l}\text{Length of circumference of the other wheel}\\ \quad\text{passing point of contact per minute}\end{array}\right\} = \pi dn$$

As these must be the same—

$$\pi \mathrm{DN} = \pi dn$$

$$\text{or DN} = dn$$

As the size of the teeth is the same on both wheels, the number on each wheel must be proportional to the diameters of the wheels, and hence we may write—

$$\mathrm{TN} = tn$$

The circumferential *pitch of the teeth* of a wheel is the distance measured along the circumference of the pitch circle from the centre of one tooth to the centre of the next.

The same expressions hold for belt pulleys as for toothed wheels, for the belt does the same as the teeth—it compels the circumferences of both pulleys to move over the same distance in the same time; because the belt and each pulley move together without slipping, therefore each pulley circumference must move over the same distance as the belt; that is, the same distance as the other to which it is connected by the belt.

In Fig. 206 we have what is generally called a clock-train

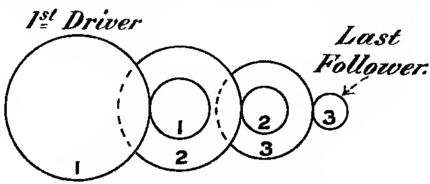


FIG. 206.—Clock-train of wheels.

of wheels. It is made up of a number of pairs of wheels, and we have numbered these pairs 1, 2, etc., these numbers being the suffixes of the symbols below.

As in the previous paragraphs, the capital letters refer to the drivers and the italic letters to the followers.

Using the equation for each pair previously obtained, we have—

$$N_1T_1 = n_1t_1$$
$$N_2T_2 = n_2t_2$$
$$N_3T_3 = n_3t_3$$

Multiplying all the left sides of these equations together, and then the right sides, and equating them, we get—

$$N_1 . N_2 . N_3 . T_1 . T_2 . T_3 = n_1 . n_2 . n_2 . t_1 . t_2 . t_3$$

But the first follower and the second driver are made fast upon the same spindle, and hence they make the same number of turns per minute; that is—

$$n_1 = N_2$$

Similarly—

$$n_2 = N_3$$

Cancelling these equal quantities, we get—

$$N_1 \times T_1 . T_2 . T_3 = n_3 \times t_1 . t_2 . t_3$$

which may be stated thus—

$$\left.\begin{array}{c}\textbf{Number of}\\ \textbf{turns of}\\ \textbf{first}\\ \textbf{driver}\end{array}\right\} \times \left\{\begin{array}{c}\textbf{Teeth of}\\ \textbf{each}\\ \textbf{driver}\end{array}\right\} = \left\{\begin{array}{c}\textbf{Number of}\\ \textbf{turns of}\\ \textbf{last}\\ \textbf{follower}\end{array}\right\} \times \left\{\begin{array}{c}\textbf{Teeth of}\\ \textbf{each}\\ \textbf{follower}\end{array}\right.$$

In the case of belts, the diameters must be substituted for the numbers of teeth.

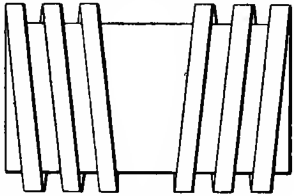
Example.—If in Fig. 206 the numbers were as follows: $N_1 = 18$, $T_1 = 72$, $T_2 = 45$, $T_3 = 40$, $t_1 = 20$, $t_2 = 20$, $t_3 = 24$. Find n_3.

Substituting in the general equation above, we have—

$$18 \times 72 \times 45 \times 40 = n_3 \times 20 \times 20 \times 24$$
$$\text{or } n_3 = 243 \text{ revolutions per minute}$$

Screw-cutting.—A screw may be either cut with stock and dies, or more accurately in a lathe.

In Fig. 207 the thread on the left is called a *right-hand* thread, because if the screw is turned in its nut in clockwise direction it will move *forward* axially. The distance it moves axially during one turn is called its *pitch*. The screw on the right of the figure is left hand.



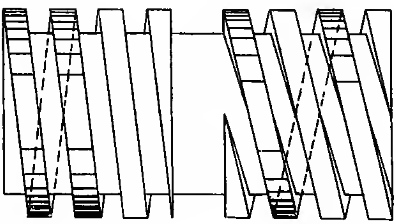
Right-hand thread. Left-hand thread.

FIG. 207.

In Fig. 208 the thread on the left is *single*, but that on the right is *double*; that is, there are two distinct but similar threads.

The general idea of cutting threads with a lathe can be obtained from the accompanying diagrammatic sketch (Fig. 209), which represents a plan view.

The mandril or spindle D of the lathe is supported in



Single thread. Double thread.

FIG. 208.

bearings C, and driven by a belt on one of the pulleys of the cone M. The bar W, on which the screw has to be cut, is supported by two cone centres, and is made to rotate with the mandril.

The other end of the mandril carries a spur-wheel, A, which is connected to another spur-wheel, B, by intermediate wheels. The wheel B is fixed on the leading screw-shaft E, the screwed part of which passes through a nut in the slide-rest N, which holds the cutting-tool T. The bar W rotates, and the screw E

simultaneously causes the tool T to move from right to left, and traces out the spiral shown on the bar W. It not only traces out the spiral, but removes a certain amount of metal, and thus produces the gaps between the threads.

The leading screw is generally made with 2 threads to the inch, and hence it will have to turn twice while the tool moves 1 in. parallel to the axis of W. Let it be required to cut a screw of 12 threads to 1 in. The mandril must make 12 turns while the tool moves 1 in., that is, while the

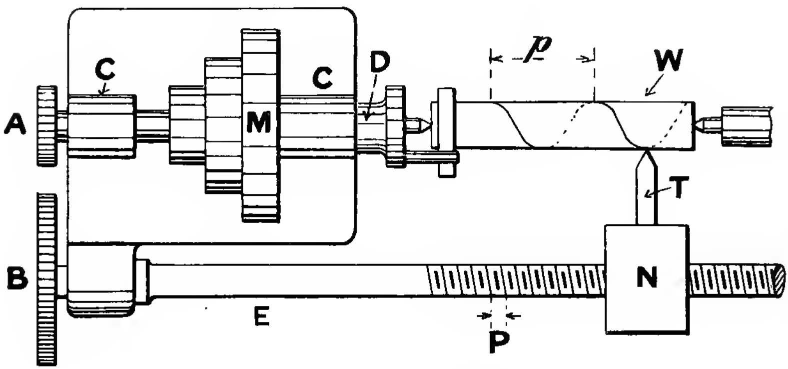


FIG. 209.—Diagram of screw-cutting arrangement.

leading screw makes 2 turns. Then A makes 12 turns, while B makes 2 turns. Also the mandril must turn in the direction of the hands of a clock as looked at from the left end.

Similarly, as the leading screw is right-handed, it must be turned in the *same direction*. Our problem is now reduced to selecting wheels for A and B with one or more intermediate wheels, so that A makes 12 revolutions while B makes 2.

To ensure the direction of both being the same, there must be at least one intermediate wheel (Fig. 210); but if there is only one intermediate wheel, it must be an *idle* one, and therefore does not come into the calculation. We then have—

$$\text{A's teeth} \times \text{A's revolutions} = \text{B's teeth} \times \text{B's revolutions}$$

$$\text{or A's teeth} = \text{B's teeth} \times \frac{\text{B's revolutions}}{\text{A's revolutions}}$$

$$= \text{B's teeth} \times \frac{2}{12}$$

Therefore A's teeth must be one-sixth of B's teeth. If B has 120 teeth, then A must must have 20 teeth.

If there should not be a wheel available with so great a number of teeth as 120, or less than 20, a clock-train must be

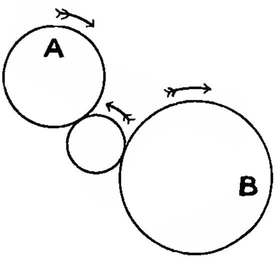


FIG. 210.

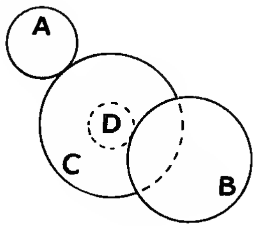


FIG. 211.

arranged, as shown in Fig. 211. Consider A the first driver and B the last follower. The wheels C and D are both keyed to the same spindle and rotate together. Then—

$$\text{A's turns} \times \frac{\text{teeth of each driver}}{\text{teeth of each follower}} = \text{B's turns}$$

$$\text{or } \frac{\text{teeth in A} \times \text{teeth in D}}{\text{teeth in C} \times \text{teeth in B}} = \frac{2}{12} \quad \ldots \ldots \ldots \quad \text{(X)}$$

Now select two of the wheels, say A with 20 teeth, and B with 80 teeth. Then—

$$\frac{20 \times \text{teeth in D}}{\text{teeth in C} \times 80} = \frac{1}{6}$$

$$\text{or } \frac{\text{teeth in D}}{\text{teeth in C}} = \frac{4}{6} = \frac{2}{3}$$

Now select two more wheels for C and D, such that the teeth in $D = \frac{2}{3}$ the teeth in C.

If we take C with 60 teeth, then D must have 40 teeth; or if C has 48, D must have 32 teeth.

Looking at the right-hand side of the equation (X) above, we see that—

$$\frac{2}{12} = \frac{\text{number of threads per inch on leading screw}}{\text{number of threads per inch on screw to be cut}}$$

Hence, we may make the general statement—

$$\frac{\textbf{product of number of teeth in driving-wheels}}{\textbf{product of number of teeth in following-wheels}}$$
$$= \frac{\textbf{number of threads to the inch on leading screw}}{\textbf{number of threads to the inch on screw to be cut}}$$

when the mandril-wheel is considered the first driver. The bar must revolve in the same direction as the leading screw for a right-hand thread.

If a left-hand thread has to be cut, the tool must be made to move from left to right, and this necessitates an extra spur-wheel in the train merely to change the direction of motion.

Hydraulic Machinery.—The fundamental parts of most hydraulic machines are shown diagrammatically in Fig. 212. A plunger, whose diameter is d, is forced down into its cylinder through a distance s, and drives the water into the larger cylinder (diameter D) forcing its piston upward through a distance S. As water is approximately incompressible, the volume forced out of the small cylinder must enter the large cylinder.

$$\text{Volume displaced by plunger} = s \times \frac{\pi}{4} d^2 \text{ cubic inches.}$$

$$\text{Volume displaced by piston} = S \times \frac{\pi}{4} D^2 \text{ cubic inches.}$$

As these are equal—

$$sd^2 = SD^2$$

$$\text{or velocity ratio} = \frac{s}{S} = \frac{D^2}{d^2}$$

Mechanical Advantage.—We generally use the simple machines to procure some advantage which cannot be so easily obtained without them.

For example, by the use of a lifting appliance, a man may

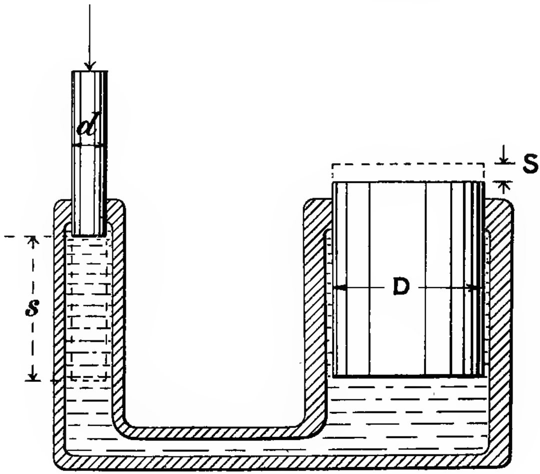


FIG. 212.

apply a force of 20 lbs. to one end of a machine and thereby lift a load of, say, 140 lbs. at the other. The advantage gained here is 7, or, in other words, by the use of the machine, a man is able to lift a load equal in weight to seven times the force he exerts. We may then define *mechanical advantage* as—

$$\frac{\text{load}}{\text{driving force}}$$

$$\text{or } \frac{\text{force at following end}}{\text{force at driving end}}$$

If we experiment on a machine such as a five-rope lifting tackle, similar to Fig. 199, we shall find that the mechanical advantage is not a constant quantity.

The following table gives the values of the load and driving force [1] in the first two columns derived from experiment, and the third column gives the calculated values of the mechanical advantage.

Load.	Driving Force.	Mechanical advantage.	Mechanical efficiency.
lbs.	lbs.		
7	2·9	2·33	0·47
14	5·0	2·86	0·57
21	6·75	3·13	0·62
28	8·7	3·27	0·65
35	10·7	3·35	0·67
42	12·8	3·41	0·681
49	14·7	3·45	0·69
56	16·4	3·50	0·70
63	17·9	3·51	0·701
70	19·7	3·56	0·71
77	21·4	3·55	0·70
84	23·7	3·57	0·71
91	25·7	3·58	0·711
98	27·5	3·58	0·711
105	28·6	3·60	0·714
112	30·8	3·60	0·714

To discover the reason for the variation in the mechanical advantage, we must inquire into the corresponding variation in the driving force, because—

$$\text{mechanical advantage} = \frac{\text{load}}{\text{driving force}}$$

Plot the numbers in Column I. along the base in Fig. 213, and the numbers in Column II. vertically upwards. We find the points lie on a straight line, AB, whose equation is—

$$\text{Driving force or effort} = 1{\cdot}3 + 0{\cdot}263 \text{ load.}$$

We know that the driving effort has not only to lift the

[1] The driving force is found by attaching the driving rope to a spring balance, and reading the balance while it is steadily pulled by the hand.

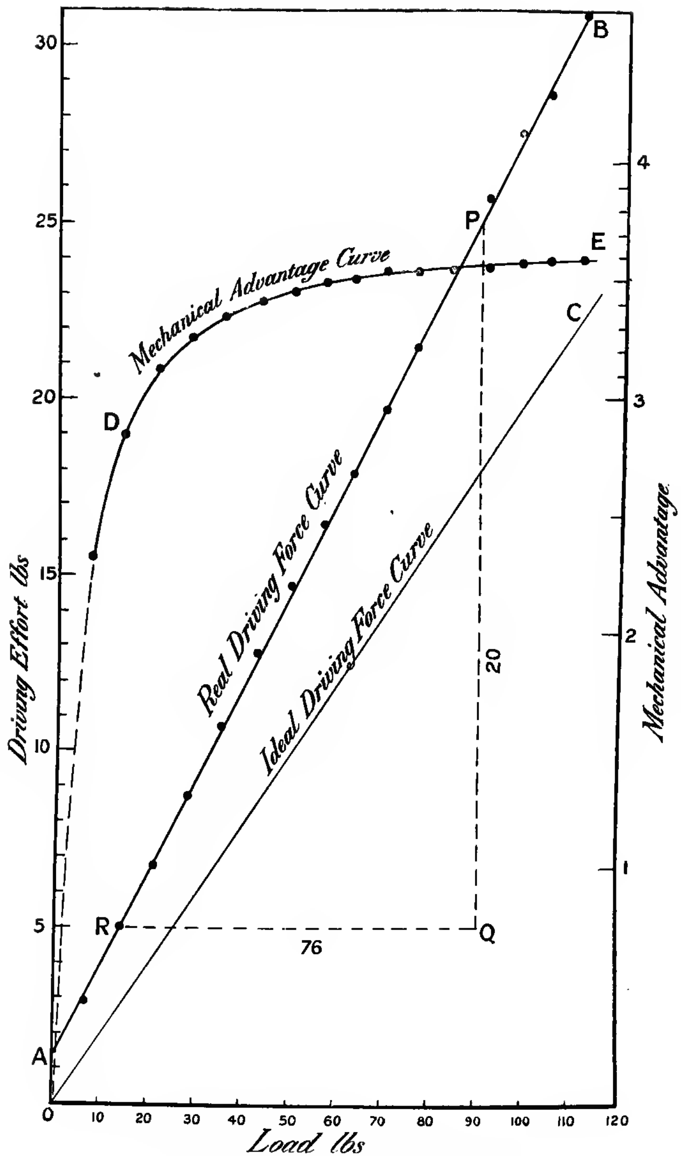


FIG. 213.—Observations on a five-rope lifting tackle.

load, but it has also to overcome the friction of the machine. The latter is caused by the weight of the load, together with the weight of the parts of the machine. This latter is constant, and as it is the only constant force or resistance, we may be led to expect that the constant 1·3 in the above equation refers to the friction due to the weight of the machine parts. If we find that this is a fact, we shall have discovered the reason for the variation in the mechanical advantage with different loads, for the fraction—

$$\frac{\text{load}}{0{\cdot}263\ \text{load}} = \frac{1}{0{\cdot}263} = 0{\cdot}38$$

is constant; but the fraction—

$$\frac{\text{load}}{1{\cdot}3 + 0{\cdot}263\ \text{load}}$$

is not constant, and varies continually as the load is increased.

We have now to discover the amount of friction with different loads.

Take a machine with as little friction as possible; a lever balanced on knife-edges (Fig. 215) will do admirably. Here the friction is so small as to be unmeasurable with ordinary apparatus. Suspend a 7-lb. weight from one arm, and a 21-lb. weight from the other. When they exactly balance one another, the mechanical advantage is—

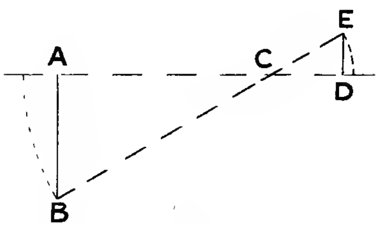


FIG. 214.

$$\frac{\text{load}}{\text{driving effort}} = \frac{21}{7} = 3$$

The distance moved by the driving end in the direction of the driving force is AB (Fig. 214), and the distance moved by the load is ED. But the triangles ABC and CED are similar

(see Introduction), hence their corresponding sides are proportional, and therefore—

$$\frac{AB}{BC} = \frac{ED}{EC}$$

or, rearranging, we get—

$$\frac{AB}{ED} = \frac{BC}{EC} = \frac{3}{1}$$

$$\text{But } \frac{AB}{ED} = \text{velocity ratio} = 3$$

Hence, the velocity ratio of a machine without friction

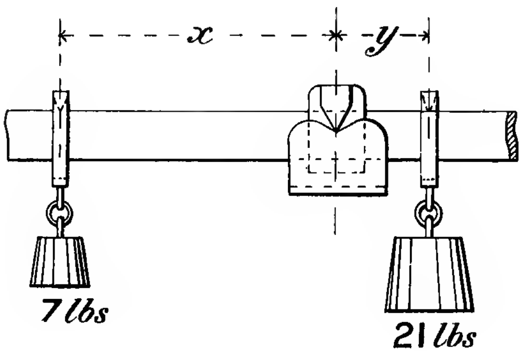


Fig. 215.

equals the mechanical advantage. Then, in the absence of any friction, we must have—

$$\frac{\text{load}}{\text{driving force}} = \text{mechanical advantage} = \text{velocity ratio}$$

And—

$$\text{driving force} = \frac{\text{load}}{\text{velocity ratio}}$$

We are now in a position to draw the driving-force curve

without friction (OC) in Fig. 213. Its slope is $\frac{1}{\text{velocity ratio}}$ and it passes through the origin. Draw it in, and then the part of the ordinate between this curve and the driving-force curve *with* friction must be *that part of the driving force required to overcome friction.* We can determine it algebraically thus—

$$\text{driving force with friction} \quad = c + m \text{ load}$$

$$\text{driving force without friction} = \frac{\text{load}}{\text{V}}$$

Subtracting, we get—

$$\left.\begin{array}{c}\text{The part of driving force required}\\ \text{to overcome friction only}\end{array}\right\} = c + \left(m - \frac{1}{\text{V}}\right)\text{load}$$

In the case in question (Fig. 213), $c = 1{\cdot}3$, $m = 0{\cdot}263$, and $\text{V} = 5$.

The left side of the last equation is generally termed simply the *friction* of the machine. We can then write for the five-rope pulley-tackle experimented on—

$$\text{friction} = 1{\cdot}3 + 0{\cdot}063 \text{ load}$$

This result is what was anticipated above, namely, a constant due to the weight of the parts of the machine, and another term which increased with the load. The number 0·067 is the coefficient of friction of the machine as a whole.

Work.—*The work done by a force equals the magnitude of the force* × *the distance through which a body is moved by the force in the direction of the force.* The work done by the driving force = driving force × distance moved by driving end, and the work done at the following end in lifting the load = load × distance moved by following end.

The fraction or ratio—

$$\frac{\text{work done at following end}}{\text{work done at driving end}}$$

is called the *mechanical efficiency* of the machine, and it equals—

$$\frac{\text{load} \times \text{distance moved by load}}{\text{driving force} \times \text{distance moved by driving force}}$$

which equals—

$$\frac{\text{mechanical advantage}^{1}}{\text{velocity ratio}}$$

$$\text{since } \frac{\text{load}}{\text{driving force}} = \text{mechanical advantage}$$

$$\text{and } \frac{\text{distance moved by driving force}}{\text{distance moved by load}} = \text{velocity ratio}$$

If there is no friction, the efficiency is 1; but in all other cases it is less than 1.

As the velocity ratio is constant, the mechanical efficiency curve will be similar to the mechanical advantage curve (OBE, Fig. 213), and hence, if a different scale be taken for the former, these curves can be made to coincide.

The last column in the previous table has been calculated by dividing the numbers in the last column but one by the velocity ratio 5.

Equations may also be found for the mechanical advantage and efficiency. Thus—

[1] Efficiency may sometimes be conveniently expressed in another way. We have above—

$$\text{efficiency} = \frac{\text{mechanical advantage}}{\text{velocity ratio}}$$
$$= \frac{\text{load}}{\text{effort} \times \text{velocity ratio}}$$

But $\frac{\text{load}}{\text{velocity ratio}}$ = the ideal effort when there is no friction, therefore we get after substitution—

$$\text{efficiency} = \frac{\text{ideal effort}}{\text{real effort}}$$

Efficiency is often expressed as a percentage, thus—if an efficiency is 0·9, we say that it is 90 per cent. (= 0·9 × 100), meaning, of course, that we are getting a return of 90 per cent. of the work which was spent.

$$\text{mechanical advantage} = \frac{\text{load}}{\text{driving force}}$$

$$= \frac{\text{load}}{1{\cdot}3 + 0{\cdot}263 \text{ load}}$$

after substituting from the equation to the driving force.

Similarly—

$$\text{mechanical efficiency} = \frac{\text{mechanical advantage}}{\text{velocity ratio}}$$

$$= \frac{\text{load}}{(1{\cdot}3 + 0{\cdot}263 \text{ load})[5]}$$

We may now summarize the results obtained with the simple machine used above—

(1) Velocity ratio $= 5$

(2) Driving force[1] $= 1{\cdot}3 + 0{\cdot}263$ load

(3) Friction[1] $= 1{\cdot}3 + 0{\cdot}063$ load

(4) Mechanical advantage $= \dfrac{\text{load}}{1{\cdot}3 + 0{\cdot}263 \text{ load}}$

(5) Mechanical efficiency $= \dfrac{\text{load}}{\text{velocity ratio } (1{\cdot}3 + 263 \text{ load})}$

Principle of Work.—When considering the lever in which the friction was so small as to be unmeasurable with ordinary apparatus, we found that—

$$\text{mechanical advantage} = \text{velocity ratio}$$

$$\text{or} \quad \frac{\text{load}}{\text{driving force}} = \frac{\text{movement of driving end}}{\text{movement of following end}}$$

Rearranging these quantities, we have load $\times$ distance moved by load $=$ driving force $\times$ distance moved by driving force.

[1] Note that the driving force and friction curves on a load-base are *straight*.

But from the previous article we learn that the force acting × distance moved over = work done by force; hence the above equation is simply another way of stating that the work put in at the driving end of a machine must equal the work derived from the machine at the following end, *when there is no friction or other resistance in the machine.*

We see here that work behaves in a way similar to matter. For example, it is similar to water flowing through a pipe, the pipe playing the part of the machine above. A certain quantity of water is poured into the pipe at one end, and it or an equal quantity of water must come out of the other end of the pipe if there is no leakage.

In the case of our machine (the lever), we poured a certain quantity of work (sometimes called energy) into one end of the machine, and an equal quantity was given out by the machine at the other end. If there had been leakage from the pipe, then—

$$\text{Water put in} = \text{leakage} + \text{water received at the other end}$$

The counterpart of leakage in the pipe is the work done in the machine in overcoming various resistances, principally friction, all of which work is eventually converted into heat and dissipated by radiation. Then the corresponding equation for the machine is—

$$\text{Work put in} = \text{lost or useless work} + \text{work received from machine}$$

The work received from the machine is employed in doing something useful, and is often called the useful work.

The fraction of the whole work put in which is useful is called the efficiency, as described in a previous article, or—

$$\text{efficiency} = \frac{\text{useful work derived from a machine}}{\text{total work put into the machine}}$$

Example.—It was found that an effort or driving force of 40 lbs. was required at one end of a machine to overcome a force or load of 210 lbs. at the following end, and again that an effort of 66 lbs. was required to overcome a resistance or load of 510 lbs.

If the velocity ratio was 20, deduce the general relations between the load and driving effort, mechanical advantage, mechanical efficiency, and friction.

We know from page 232 that if we experiment on any machine working under normal conditions, the relation between the driving effort and the load can be represented by a straight line, when the load is plotted horizontally and the driving effort vertically.

In Fig. 216 the quantities given in the question have been

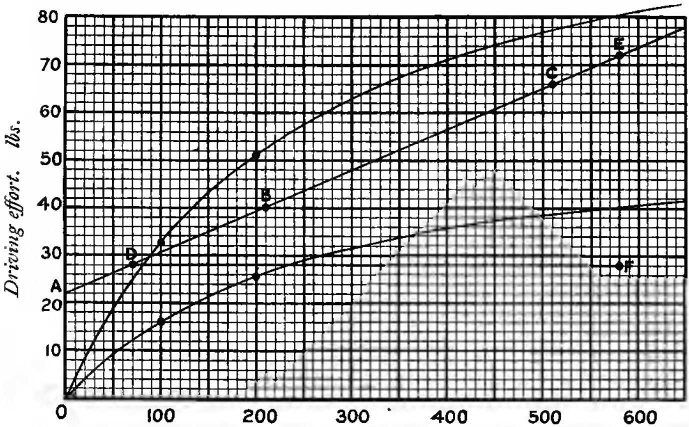


Fig. 216.

plotted, giving the points B and C. Joining B and C, and determining the equation to the line BC,[1] we get—

driving force = 22 + ·0863 load

The number 22 is the length of OA, and 0·0863 is $\frac{\text{EF}}{\text{DF}}$

If there were no friction, we saw on page 229 that the driving force would then be—

$$\frac{\text{load}}{\text{velocity ratio}} = \frac{\text{load}}{20} = 0{\cdot}05 \text{ load}$$

[1] See Appendix for method.

Writing these two results together thus—

$$\text{(with friction) driving force} = 22 + 0{\cdot}0863 \text{ load}$$
$$\text{(without friction) driving force} = 0{\cdot}05 \text{ load}$$

and subtracting the latter from the former, we get—

$$\left.\begin{array}{l}\text{that part of the driving force required}\\ \quad\text{to overcome friction}\end{array}\right\} = 22 + 0{\cdot}0363 \text{ load}$$

$$\text{the mechanical advantage} = \frac{\text{load}}{\text{driving force}}$$
$$= \frac{\text{load}}{22 + 0{\cdot}863 \text{ load}}$$

the value of the driving force being substituted from the equation on the previous page.

If we insert different values for the load, we get corresponding values for the mechanical advantage.

Thus with a load of 100 lbs. the mechanical advantage is—

$$\frac{100}{22 + 8{\cdot}63} = 3{\cdot}26$$

Plot this over the load of 100 lbs. in Fig. 216, the vertical scale being ten times as great as that used for the driving force. Repeating this calculation for a large number of loads, we get the upper curve.

$$\text{The mechanical efficiency} = \frac{\text{mechanical advantage}}{\text{velocity ratio}}$$

Hence, divide each mechanical advantage by the velocity ratio, and plot these above their corresponding loads. In this way we get the lower curve, fig. 216, the vertical scale being one hundred times as large as that for the effort.

Example.—A screw or bottle jack is operated through a lever 5 ft. long. What force applied at the end of the bar is necessary to lift 5 tons, if the pitch of the screw was $\frac{5}{8}$ in., and the efficiency of the machine 35 per cent.?

Let F be the force which must be applied at the end of the lever and at right angles to it. Let the screw make one turn; then the load W will be lifted through $\frac{5}{8}$ in., while the end of the lever will move through $2 \times \pi \times 60$ ins. The work done in each case will be—

$$W \times \tfrac{5}{8} \text{ in.-lbs. and } F \times 2\pi \times 60 \text{ in. lbs.}$$

As the efficiency is only 35 per cent., $(100 - 35) = 65$ per cent. of the work put in is lost in overcoming friction. Substituting in the equation—

$$\left.\begin{matrix}\text{work put in by}\\ \text{driving force}\end{matrix}\right\} = \text{lost work} + \text{work derived from machine}$$

we have—

$$F \times 2\pi \times 60 = 0{\cdot}65F \times 2\pi \times 60 + W \times \tfrac{5}{8}$$

$$\text{or } 0{\cdot}35F \times 2\pi \times 60 = W \times \tfrac{5}{8} = 5 \times 2240 \times \tfrac{5}{8}$$

$$\text{and } F = \frac{5 \times 2240 \times 5}{0{\cdot}35 \times 2\pi \times 60 \times 8}$$

$$= 53 \text{ lbs.}$$

Reversibility.—While experimenting on different machines, it must have been noticed that some of them, such as

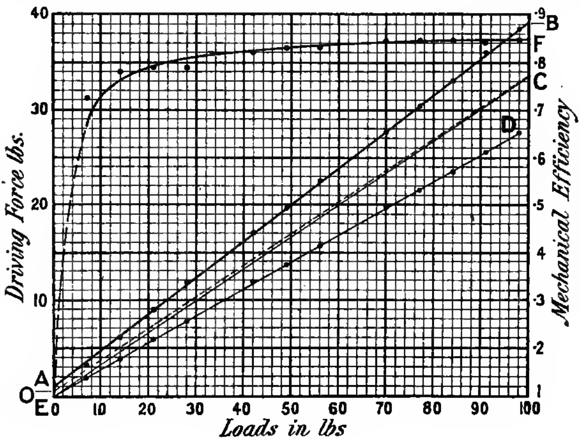


FIG. 217.

the rope pulley-tackle, would run back (with the load on) when some of the driving force was removed. The weight (previously lifted) acted as the driving force for the backward motion, and the old driving force acted the part of load in the backward

motion. The machine in this case was *reversible*, that is, it was capable of reversing its motion with all the forces acting in the *same directions* as during the forward motion, but with one or more of the forces changed in magnitude.

It will be found that the Weston's differential pulley-block will not run backwards even when all the driving force is removed, and the same occurs with the worm and worm-wheel

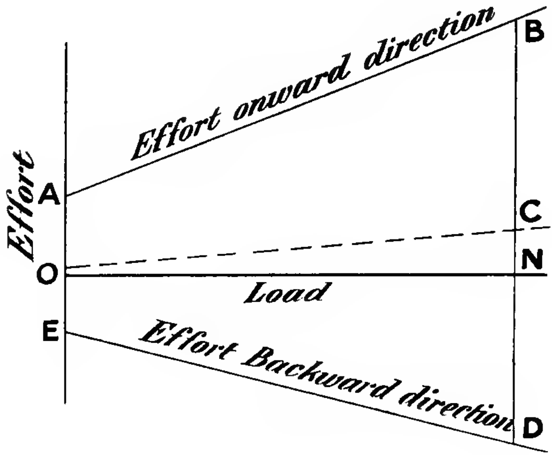


Fig. 218.

and screw-jack. These machines are not reversible according to our definition above. It is possible to make them work backwards, but only when the driving force has its direction changed from positive to negative.

Fig. 217 refers to a three-rope pulley-tackle, and Fig. 218 to a screw-jack. The similar points in the two figures are similarly lettered. AB is the driving-force curve on a load-base when the machine is working in the forward direction or while lifting, and ED is the same while working backward or lowering. In the case of the pulley-tackle, the driving force while lifting and lowering was exerted in the *same* direction

shown by the backward curve being on the same side of the base-line as the forward curve. But in the case of the screw-jack, a negative force had to be applied to make the machine move in the reverse direction, which is shown by the backward direction curve being on the opposite side of the base to the forward curve.

In Fig. 217 the ideal driving-force curve EC has been drawn, and another line, OC (shown dotted), has been drawn through the points of bisection of the vertical intercepts between AB and ED.

It was shown in Fig. 23 that if the pressure producing friction were constant, the line representing the driving effort with no friction bisected the intercepts between the driving force for backward and forward motion. It will be seen in Fig. 217 that the lines OC and EC are very close together, and would coincide, but for the fact that the total pressure producing friction (load + driving force + weight of parts) is not quite the same for forward as for backward motion; as, during the latter, the driving effort is less than during forward motion. The difference is small, and for the purpose of the following argument may be neglected.

When the backward driving-force curve is on the base-line the friction $BC = \frac{BN}{2}$ (Fig. 218); and consequently, if the backward driving-force curve is below the base, BC is greater than $\frac{BN}{2}$, or the friction is greater than half the forward driving force, and therefore the efficiency of the machine is less than 0·5. But if the line CD falls below the base, a negative driving force is required for reversed motion; *therefore, when the mechanical efficiency of a machine is less than 50 per cent., it will not run backwards of its own accord, even when the driving effort is removed.*

Experiment to illustrate the "Principle of Work."—The apparatus used is the skeleton mechanism of an engine (Fig. 219), in which OP is a line on a disc representing the crank, PQ a cord or wire representing the connecting-rod. Q is a roller, and W a weight, to represent the total pressure

on the piston. This latter is connected to the crosshead Q by a cord.

Another cord passes round the disc and is connected to a spring balance, E, the balance being connected to a cord which

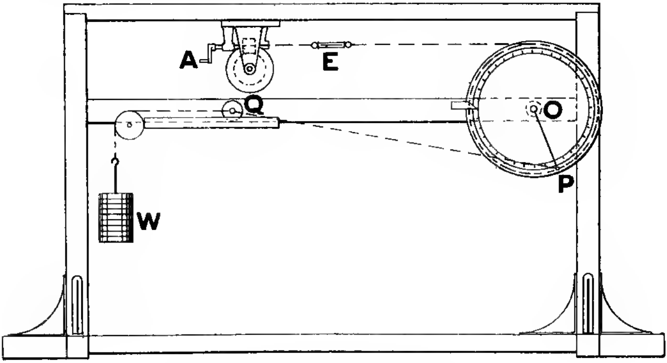


FIG. 219.—Skeleton engine mechanism.

is passed round a winding-drum worked through a worm and worm-wheel by the handle A.

Method of Experiment.—Make the weight W equal to about 10 lbs.

Take observations of the balance E for every 10 degrees in the crank-pin circle between 10° and 170° inclusive. The construction of the apparatus interferes with the reading at 0° and 180°.

The friction of the apparatus must be got rid of in this experiment, and hence we must adopt the method described on page 28, of turning the handle A in the forward direction, so as to bring the required point in the disc under the pointer C, and reading the balance E, and then turning the handle A in the opposite direction to bring the same point up to C in the opposite direction, reading the balance E a second time. The mean of these two readings gives the reading which would have been obtained if there had been no friction.

Enter the observations in a table as below :—

RECORD OF EXPERIMENT ON ENGINE MECHANISM.

Date, June 6, 1902. Observer, W. Wade.

Length of crank = 7·25 in.

Length of connecting-rod = 25·5 in.

Load on piston = 10·5 lbs.

Crank angle.	Crank effort.		
	Forward.	Backward.	Mean.
degrees.	lbs.	lbs.	lbs.
10	2·5	2·0	2·25
20	4·75	4·2	4·5
30	7·1	6·1	6·6
40	8·5	7·75	8·13
50	10·3	9·1	9·7
60	11·0	10·0	10·5
70	11·3	10·4	10·85
80	11·2	10·4	10·8
90	10·8	10·1	10·5
100	10·3	9·5	9·9
110	9·5	8·5	9·0
120	8·6	7·4	8·0
130	7·2	6·2	6·7
140	5·7	5·0	5·35
150	4·2	3·7	3·95
160	2·9	2·6	2·75
170	1·5	1·2	1·35

Average crank effort = 6·7 lbs.

Plot your results as follows :—

On a base OX, Fig. 220, plot upwards a line OY to represent 10·5 lbs., and draw OX to represent the stroke of the piston = 2 × OP = 2 × 7·25. Then the area of the rectangle represents the work done on the piston = 10·5 × 2 × 7·25 = 150 in.-lbs.

Now draw a line MN (Fig. 220) to represent (to the same scale as in the previous diagram) the length of the path of the crank-pin P, during half a turn of the crank, and divide it into 18 equal parts corresponding to the 18 intervals of 10° each. Label these points of division, and at the points of division plot

upwards the numbers in the fourth column of the previous table, and draw in the outline carefully with a French curve.

The area of this diagram represents the work done on the

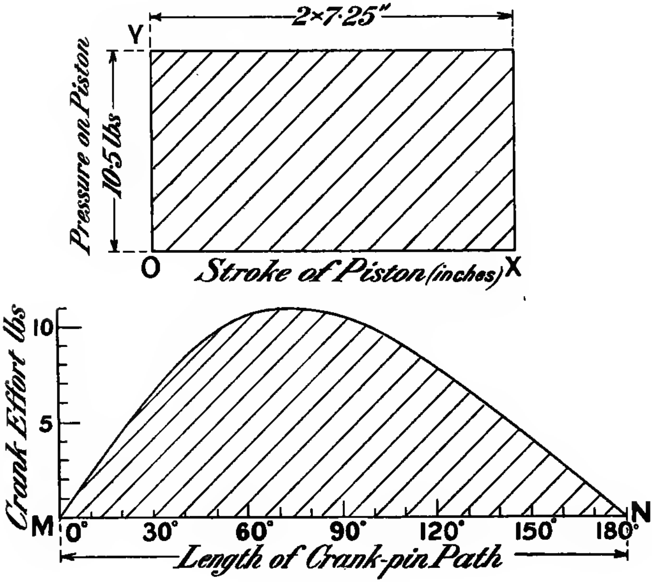


FIG. 220.

crank-pin during the half-turn of the crank, and it should, as nearly as possible, equal the work done on the piston. In the case in question, the crank = 7·25 inches, and therefore the work done on the piston = 150 in.-lbs.

The mean crank effort was 6·7 lbs., and therefore the work done (area of the diagram) = $6{\cdot}7 \times \pi \times 7{\cdot}25 = 149{\cdot}5$ in.-lbs., or the work put into the machine at the piston end, equals the work derived from the machine (at the crank end) within the limits of error of experiment.

It was stated on page 229 that, in the absence of friction, the velocity ratio was *numerically* equal to the mechanical advantage.

If we consider the piston effort to be the driving force—

$$\left.\begin{array}{r}\text{The average me-}\\\text{chanical advantage}\end{array}\right\} = \frac{\text{mean load on crank-pin}}{\text{mean effort on piston}} = \frac{6{\cdot}7}{10{\cdot}5} = \frac{0{\cdot}64}{1}$$

$$\left.\begin{array}{r}\text{the average}\\\text{velocity ratio}\end{array}\right\} = \frac{\text{distance moved by driving end}}{\text{distance moved by load end}}$$

$$= \frac{\text{stroke}}{\text{crank-pin half-circle}}$$

$$= \frac{2}{\pi} = \frac{0{\cdot}64}{1}$$

and for any point in the stroke the same must hold.

The method of determining the load on the crank-pin geometrically was given on page 102. The experimental and geometrical quantities should be compared and the difference noted.

Summary of Chapter VIII.

$$\textbf{Velocity ratio} = \frac{\textbf{movement of driving end of a machine}}{\textbf{movement of following or load end}}$$

With any pair of wheels gearing together :—

Revolutions of driver × diameter of driver = revolution of follower × diameter of follower.

With a clock train :—

$$\textbf{Revolutions of first driver} \times \frac{\textbf{diameter of each driver}}{\textbf{diameter of each follower}}$$

$$= \textbf{revolutions of last follower}$$

$$\textbf{Mechanical advantage} = \frac{\textbf{load}}{\textbf{driving force}}$$

$$\textbf{Driving force} = \textbf{intercept} + \textbf{slope} \times \textbf{load}$$

$$\textbf{Mechanical efficiency} = \frac{\textbf{useful work done by machine}}{\textbf{total work supplied to machine}}$$

$$= \frac{\textbf{mechanical advantage}}{\textbf{velocity ratio}}$$

$$= \frac{\textbf{ideal driving force}}{\textbf{real driving force}}$$

Principle of work :—

Work put into a machine =
useless work + useful work derived from machine.

EXAMPLES ON CHAPTER VIII.

1. Calculate the velocity ratio in a wheel and axle in which the wheel is 3 ft. 1 in. diameter and the axle 4 in. diameter, and neglect the size of the ropes. If the rope on the wheel had been ½ in. diameter and that on the axle ¾ in. diameter, what would then have been the velocity ratio?
Ans. (1) 9·25 ; (2) 7·9.

2. Describe carefully and accurately how the velocity ratio is found by experiment.

3. A rope pulley lifting tackle has three sheaves in the top and bottom blocks, and the end of the rope is made fast to the top block. Calculate from first principles the velocity ratio. *Ans.* 6.

4. In a Chinese wheel and axle the wheel was 2 ft. 8 in. in diameter, and the parts of the axle 8 in. and 9 in. respectively. What is the velocity ratio of the machine, and what would it be if the rope on the wheel were ½ in. in diameter and that on the axles 1 in. in diameter?

5. What modern useful appliance is derived from the Chinese wheel and axle? If the pulley sheaves are 8 and 8½ in. in diameter, what is the velocity ratio?

6. If the lever of a screw-jack is 3 ft. 9 in. in length, and the pitch of the screw ½ in., calculate the velocity ratio. *Ans.* 180π.

7. A screw-jack has a double thread, in which the thickness of the thread is ¼ in. If the lever is 2 ft. 8 in. long, what is the velocity ratio?
Ans. 66π.

8. A worm and worm-wheel similar to that in Fig. 204 has a single thread for a worm driven by a handle 18 in. long. The worm-wheel has 98 teeth, and the drum on which the rope is coiled is 8 in. in diameter. Calculate the velocity ratio.

9. A Chinese wheel and axle has a wheel 22 in. diameter, and the parts of the axle 7½ and 8 in. diameter. Calculate from first principles the velocity ratio. *Ans.* 88.

10. Two spur or toothed wheels in gear have 27 and 115 teeth respectively. If the former made 230 revolutions in 3 minutes, how many turns will the latter make in a quarter of an hour?

11. Two pulleys, 15 in. and 33 in. in diameter respectively, are connected by a belt. If the former makes 500 revolutions in 9 minutes, how many turns will be made by the other in 25 minutes?

12. A 24-in. pulley on an engine-shaft makes 214 revolutions per minute, and drives by a belt a 34-in. pulley on another spindle. This

second spindle has keyed upon it a double worm, which gears with a worm-wheel having 108 teeth on a third spindle. Calculate the velocity ratio.

13. Define "velocity ratio." Calculate it in the following cases:—*Screw-jack:* screw pitch, $\frac{1}{2}$ in.; length of handle, 59 in. *Wheel and axle:* wheel, 25 in. diameter; axle, 3 in. radius. *Ans.* 741·6 and 4·16.

14. Three spindles, A, B, and C, are arranged parallel to one another. On A there is a spur-wheel with 52 teeth, which gears with another spur-wheel having 19 teeth on the spindle B. On this spindle a spur-wheel having 81 teeth, gears with another of 21 teeth on spindle C. Calculate the velocity ratio, and if the spindle A made 15 turns, how many would B make in the same time. *Ans.* $\frac{\text{Revs. of C}}{\text{Revs. of A}} = \frac{10·6}{1}$; Revs. of B = 41·2.

15. Prove the relation between the sizes of the wheels in a clock-train and the speed of the first and last spindles.

16. With a leading screw of $\frac{1}{2}$ in. pitch, and change-wheels of 20, 24, 30, 40, 55, 60, 80, and 100, show how you would select wheels to cut screws of 6, 11, and 16 threads to the inch.

17. Given the change-wheels with 18, 30, 40, 50, 88 teeth, show how you would arrange them to cut a screw thread of 11 threads to the inch if the leading screw has 3 threads to the inch. If the leading screw had been $\frac{1}{2}$ in. pitch, how would you then have arranged them?

18. The available change-wheels of a lathe have 20, 30, 40, 55, and 100 teeth respectively. Describe how and why you would select particular wheels from this set to cut a single right-hand screw of 11 threads to the inch. The leading screw has 3 threads to the inch.

19. A, B, C are three spur-wheels attached to parallel axes. A gears with B, and B with C. A has 20 teeth, B has 10, and C 40. A makes 5 turns per minute. What is the speed of C? And what kind of wheel is B?
Ans. 2·5 revs.; B is an idle wheel.

20. When an ordinary train of wheels is employed for obtaining an increased speed of rotation, how are the wheels arranged? Sketch an arrangement of four pulleys with bands for driving a fan at 1500 revolutions per minute from a shaft making 100 revolutions per minute.

21. Investigate a formula for the velocity ratio between the first and last wheels in a train.

An engine-shaft which runs at n revolutions per minute, carries a pulley, A, of 56 in. in diameter, which drives by a belt a pulley, B, on the line-shaft of 36 in. in diameter. The line-shaft carries a second pulley, C, of 42 in. diameter, which is connected by a belt with a pulley, D, of 24 in. diameter, carried on a counter-shaft. Another pulley, E, on the counter-shaft is 48 in. in diameter, and drives on to a 14-in. pulley on the spindle of a dynamo. Find the velocity ratio between the spindle of the dynamo and the engine-shaft. *Ans.* $\frac{n}{N} = \frac{28}{3}$.

22. A planing machine for working metals is driven by a pinion which

gears into a rack fixed to the underside of the table of the machine. At what rate per minute would the table advance if the pinion had 12 teeth of $1\frac{1}{4}$ in. pitch, and was driven at the rate of 12 revolutions per minute? Sketch and describe the mechanism employed for driving the pinion in opposite directions, but so that the return movement is made 50 per cent. faster than the forward motion. (A pinion is a small spur-wheel.)

23. A driving shaft at 100 revolutions per minute, carries a pulley 22 in. in diameter, from which a belt communicates motion to a pulley 12 in. in diameter, carried upon a counter-shaft. On the counter-shaft is also a cone pulley having steps 8, 6, and 4 in. in diameter respectively, which gives motion to another cone pulley with the same steps on a lathe spindle. Sketch the arrangement in front and end elevation, and find the greatest and least speeds at which the lathe spindle can revolve.

Ans. 366·6 and 93·3 revs.

24. What are cone or speed pulleys? Describe the use of such pulleys in any machine with which you are acquainted. (See last chapter.)

The spindle of a wood-turning lathe can, by moving the belt on its cone pulleys, be driven at the rate of 400 revolutions per minute when running at its lowest speed. If the revolutions of the driving shaft are kept constant throughout, and the largest diameter of the speed cones is 20 in., what must be the diameter of the smallest steps on the pulleys, the speed pulleys on the two shafts being of the same size? Sketch the pulleys in position. *Ans.* Highest speed 2000 and 5 steps.

25. The axes of two parallel shafts are as nearly as possible 2 ft. apart, and the shafts are to be connected by a pair of spur-wheels, the pitch of which is 1·5 in. What will be the numbers of teeth on each wheel, when the velocity ratio of the shafts is 3 to 1; and what will be the exact distance apart of their axes? *Ans.* $23\frac{5}{8}$ in.

26. A shaft making 100 revolutions per minute carries a driving pulley $2\frac{1}{2}$ ft. in diameter, and communicates motion by means of a belt to a parallel shaft at a distance of 6 ft., carrying a pulley 1 ft. in diameter. Find the speed of the belt and an expression for its length when crossed. Find also the number of revolutions per minute of the driven shaft, allowing a slip of $1\frac{1}{2}$ per cent.

27. A lathe is driven by belting from a counter-shaft, and the back gear is in use. Find how many revolutions the mandril will make with the following proportion of gearing if the counter-shafting is running at 150 revolutions per minute: diameter of pulley on counter-shaft, $4\frac{1}{4}$ in.; diameter of pulley on lathe, $8\frac{3}{4}$ in.; spur-wheel on mandril axis, 58 teeth; pinion on back shaft, 18 teeth; spur-wheel on back shaft, 58 teeth; pinion on spindle of mandril, 18 teeth. Sketch the arrangement. *Ans.* 7·1 revs.

28. The table of a drilling machine is raised by a worm and wheel in combination with a rack and pinion. Sketch the arrangement, and find what weight would be balanced on the table if a pressure of 12 lbs. were applied to the end of the handle, which is 12 in. long, the worm being single-threaded, while the worm-wheel has 30 teeth, and the pitch circle of

the rack-pinion is 4 in. in diameter. Suppose the table and accessories to weigh 500 lbs., and that 45 per cent. of the work applied is lost by friction.

29. Find the mechanical advantage in a wheel and axle.

A man raises 3 cwt. by turning a windlass whose barrel is 5 in. in diameter. If he pushes at a point 2 ft. from the axis, what force must he exert, and what work, in foot-pounds, is done in 12 revolutions of the handle?

30. In a pulley-block tackle with a velocity ratio of 6 to 1, experiment shows that 30 lbs. is just lifted by 10 lbs., and 169 lbs. by 50 lbs. Find the linear law connecting power[1] and load; and also the mechanical efficiency of the tackle for each of the above loads.

31. What do you understand by the efficiency of a machine? How would you test the efficiency of a pair of differential chain pulley-blocks, and how would you plot a curve showing the efficiency of the apparatus at different loads?

32. With a hydraulic jack it was found that driving forces of 21 lbs. and 33·5 lbs. were required to lift 2100 lbs. and 4000 lbs. respectively. The velocity ratio was 200. What would be the mechanical efficiency with a load of 5000 lbs., the mechanical advantage with a load of 3000 lbs., and the friction with a load of 6000 lbs.?

Ans. 62·5 per cent.; 112; 16·5 lbs.

33. Define the pitch of a screw-thread. In an ordinary screw-cutting lathe, the leading screw is single, and of $\frac{1}{2}$ in. pitch. The wheel on the end of the mandril has 20 teeth. Show, by the aid of an outline sketch, what wheels must be used with the above to cut a screw of 11 threads to the inch, and state your reasons for your answer. Is there any difference in the method of cutting a screw with an even or odd number of threads to the inch? Has the leading screw anything to do with it? What wheels would you use in the above lathe to cut a treble left-hand screw-thread of $1\frac{1}{2}$ in. pitch? What is the thickness of each thread (square)?

Ans. 110 teeth and $\frac{1}{2}$ in.

34. The leading screw on a lathe has 2 threads to 1 in., the thread being single. The wheels at your disposal have 20, 24, 30, 44, 50, 60, 80, and 100 teeth respectively. Select wheels to cut single screw-threads of 11, 12, and 16 threads to the inch. *Ans.* 24 teeth on mandril drives 60.

35. The leading screw of a lathe is $\frac{1}{2}$-in. pitch and right-handed. Show clearly, by the aid of sketches, how the screw is put in and out of gear with the saddle of the lathe. Determine the number of teeth in each of the wheels of a suitable set of change-wheels required for cutting a left-handed screw of $\frac{3}{8}$-in. pitch, in such a lathe, and sketch their arrangement. No wheel is to have less than 25 teeth or more than 100. *Ans.* $t_1 = \frac{3}{4}$ T.

---

[1] It is wrong to call these forces by the name of *power*. It will be seen in the next chapter that power is a different kind of thing altogether. This question was taken from a public examination paper.

36. Describe and show by sketches the means by which the slide-rest of the lathe may be connected with the leading screw.

If the slide-rest traverses the bed at the rate of $1\frac{1}{6}$ ft. when the leading screw makes 56 revolutions, what is the pitch of the screw-thread?

37. Sketch the screw-coupling commonly used to connect together the carriages of a railway train, and explain its action. With what force would the carriages be drawn together by the exercise of a pressure of 50 lbs. applied to the ball at the end of the arm, supposing the pitch of the screw to be $\frac{3}{8}$ in., and that the centre of the ball measures 1 ft. from the axis of the screw? Friction is to be neglected.

38. Pulleys A, B, C, and D, Fig. 221 (diameters 36 in., 8 in., 30 in., and 5 in. respectively), are connected by belts whose thickness can be

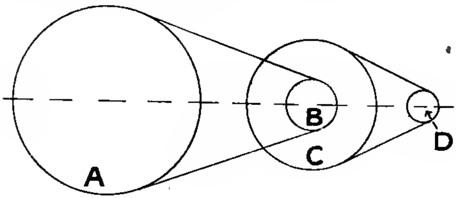


FIG. 221.

neglected. Pulleys B and C are keyed to the same shaft. If D makes 1600 revolutions per minute, how many revolutions does A make in the same time; also how many does B make per minute?

39. A tackle, consisting of an ordinary double and treble block, is employed for lifting a weight of 600 lbs. attached to the double block. What force is required, neglecting friction?

If the tackle is reversed, so that the weight is attached to the treble block, the free end of the rope being pulled upwards, what force would now be required to lift the weight? *Ans.* 120 lbs. and 100 lbs.

40. Sketch in vertical section the common screw or bottle lifting jack. The lever in such a jack is single-ended, and measures 24 in. in length; the pitch of the screw is $\frac{3}{8}$ in. What force applied at the end of the lever would be required to raise a load of 22 cwt., the effect of friction being neglected? *Ans.* $F = 6\frac{1}{8}$ lbs.

41. A common screw-jack, with a lever 16 in. in length, has a worm-wheel of 20 teeth, and a screw of $1\frac{1}{4}$ in. pitch. Sketch the arrangement, and calculate the weight lifted by the application of a constant pressure of 30 lbs. at the end of the handle, friction being neglected.

42. The efficiency of a block and tackle is 65 per cent. Assuming that a load of 1 ton is being raised by it at a steady speed of 2 in. a second, how many foot-pounds of work per minute must be done by the driving effort?

43. State the principle of work applied to a machine. In a particular machine the driving force was found to be 54 lbs., and the motion of

its point of application 32 ft. The useful work done by the machine in the same time was $\frac{1}{2}$ ft.-ton. Apply the principle of work to find the resistance of friction, supposing it to be applied at the same point as the driving force.

44. Distinguish between the velocity ratio and the mechanical advantage of a machine.

In a hydraulic lifting jack the ram is 6 in. in diameter, the pump plunger is $\frac{7}{8}$ in. diameter, the leverage for working the pump is 10 to 1. What is the velocity ratio of the machine? Experimentally we find that a force of 20 lbs. applied at the end of the lever lifts a weight of 8500 lbs. on the end of the ram. What is the mechanical advantage and efficiency of the machine at this load? *Ans.* VR = 470; ME = 0·905.

45. Describe any machine, workable by hand, for lifting weights. Describe carefully how you would make tests to find its mechanical advantage under various loads.

46. Given a Weston's pulley-block, show how we find the velocity ratio. How would you experimentally find the mechanical advantage? Does the mechanical advantage depend on the load which is being lifted, and if so, in what way?

47. The leading screw on a lathe has 2 threads to 1 in., the thread being single. The wheels at your disposal have 20, 24, 30, 44, 50, 60, 80, and 100 teeth respectively. Select wheels to cut single screw-threads of 11, 12, and 16 threads to the inch.

48. What do you understand by the efficiency of a machine, and how is it measured? In a single-purchase crab, the pinion has 12 teeth and the wheel has 78 teeth, the diameter of the barrel being 7 in. and the length of the lever handle 14 in. It is found that the application of a force of 15 lbs. at the end of the handle suffices to raise a weight of 280 lbs. Find the efficiency of the machine at this load.

49. Describe either a screw-jack (pitch of screw $\frac{1}{2}$ in., handle 19 in. long) or a simple winch for lifting weights up to 1 ton by one man. What is the mechanical advantage, neglecting friction? Describe what sort of trial you would make to find its mechanical advantage under various loads, and what sort of result would you expect to find?

50. In a machine, the load moves through one forty-fifth the distance moved through by the driving effort. If the load is 200 and 500 lbs. respectively when the corresponding effort is 30 and 56 lbs., find the general relation between friction and load.

51. A double-purchase winch has the following wheels:—first driver 10 teeth, first follower 35 teeth, second driver 10 teeth, second follower 100 teeth. The diameter of the drum is 6 in., and the diameter of the cord-pulley is 20 in. Calculate the velocity ratio of the winch, and find the efficiency when 30 lbs. placed in the scale-pan on the cord-pulley will just lift 1050 lbs. on the drum. *Ans.* VR = 116·6; efficiency = 0·3.

52. Using the data in question number 51, and in addition, knowing that a driving force of 12 lbs. is necessary to run the machine without any

load, find the equation showing the relation between driving force and load, and also the relation between the friction and load. Write down from these equations the friction when lifting a load of 1 ton and the necessary driving force, also for half a ton.

Ans. DF = 0·04 load + 12; friction = 0·0085 load + 12.

53. A lifting machine, when experimented upon, gave the following results:—With a load of 109 lbs. the driving force was 51 lbs.; and with a load of 568 lbs. the driving force was 154 lbs. Deduce the equations connecting the load with (1) driving force, (2) mechanical advantage, (3) mechanical efficiency, (4) friction, and use these equations to obtain the quantities with a load of 1000 lbs.

Ans. DF = 0·224 load + 28.

$$\text{MA} = \frac{\text{load}}{0{\cdot}224\ \text{load} + 28}; \text{ friction} = 0{\cdot}024\ \text{load} + 28.$$

54. In a lifting tackle it is found from experiment that 16 lbs. will lift 100 lbs., and that 36 lbs. will lift 300 lbs. Find the relation between the lifting force and the load lifted. Also find the efficiency when 500 lbs. is being lifted if the velocity ratio were 12 to 1.

55. Describe Weston's differential pulley-block. If the weight is to be raised at the rate of 5 ft. per minute, and the diameters of the pulleys of the compound sheave are 7 and 8 in. respectively, at what rate must the chain be handled?

56. Two hydraulic cylinders $1\frac{1}{2}$ in. and 7 in. radius respectively are connected by a pipe. If the smaller piston is depressed through 4 ft., how much will the larger piston be raised, and why?

57. The following observations were taken in an experiment on a lifting apparatus (such, for instance, as a screw-jack or pulley-tackle). Plot these observations on squared paper, and deduce as accurately as you can an equation showing the relation of driving force to load lifted.

Driving force in lbs.	12·5	18	30	40·5	50·9	70	79·8	92	108	122
Load lifted in lbs.	10	40	100	150	200	300	350	410	490	560

58. In a test of a Weston differential pulley the following results were obtained:—

Load (in lbs.).	Power [1] (in lbs.).
40	6·5
80	11·0
120	15·3
160	19·4
200	23·3
240	27·4
280	31·4

[1] See footnote to Question 30.

Obtain an expression for the law of friction for the pulley, and determine its maximum mechanical efficiency if the velocity ratio is 23·5 to 1.

59. By experiment with a Weston's differential pulley-block, it was found that a pull of 15 lbs. on the leading side of the chain was required to lift a weight of 60 lbs. (including weight of lower pulley and hook). The radius of the larger pulley was 2 in., and of the smaller pulley 1·75 in. Find the mechanical advantage with and without friction, and the efficiency of the apparatus. Why does the weight remain suspended when there is no pull on the chain? *Ans.* 16, 4, and 0·25.

60. In a Weston's pulley-block the weight of the chain, etc., is 120 lbs., and it is assumed its weight acts as a dead load upon the bearing supporting the double sheave. The diameters of the double sheave are $7\frac{1}{2}$ and 8 in., and that of the lower pulley 8 in. Diameters of spindles 1 in. What must be the coefficient of axle friction when the tackle just supports a load of W lbs. so that it will just NOT run down?

61. Express the work done when a moment M has rotated n times. If a force equal to the weight of 10 lbs. revolve three times tangentially round a circle of 5 ft. radius, find the work it would do.

If the energy thus generated were imparted to a free stationary mass of 12 lbs., how fast would it move?

62. Draw to scale a wheel and axle by which a man, sitting in a loop at the end of a rope wound round the axle, can haul himself up by pulling at a rope round the wheel with a force only one-fifth of his weight. What weight is sustained by the pivots?

63. A steady force applied to a mass of 75 tons, initially moving at the rate of 3 miles an hour, accelerates it 4 ft. a second every second. Calculate (*a*) the applied force, in pounds weight; (*b*) the speed of the body after the lapse of 1·5 minutes; (*c*) its kinetic energy at the same time, expressing it in "foot-tons."

64. The drum of a windlass is 4 in. in diameter, and the power is applied to the handle 20 in. from the axis. Find the force necessary to sustain the weight of 100 lbs., and the work done in turning the handle 10 times. *Ans.* Force = 10 lbs.; work = 1047 ft.-lbs.

65. Draw a system of pulleys with a single string going twice round each of the blocks, and find the power [1] needed to sustain a weight of 1 ton. *Ans.* 560 lbs.

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[1] See footnote on page 246.

CHAPTER IX.

WORK, POWER, AND MISCELLANEOUS PROBLEMS.

It was stated in the last chapter (p. 230) that the work done by a force was measured by the product of the force, and the distance through which the body moved while under the action of the force.

This can often be conveniently indicated by the area of a figure, the base of which represents distance, and the corresponding force being represented by an ordinate; thus, if a constant force is represented by OA, and the distance moved

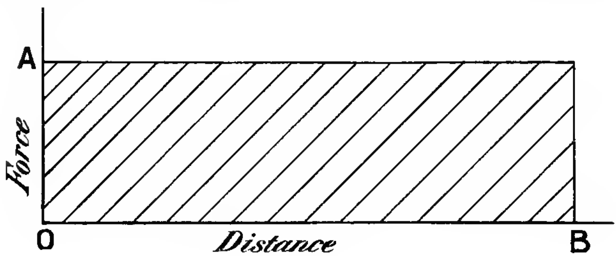

FIG. 222.

over while under the action of the force is represented by OB, then the shaded area = OA × OB = force × distance = work done by force.

When the force is not constant the area still represents the work done, and it enables the mean or average force to be calculated.

Example.—A piece of apparatus in the nature of a huge spring balance is interposed between a tender and the train it is hauling.

Assume that the motion of the pointer or pull indicator is communicated to a pencil, so that it can record on a sheet of paper (Fig. 223) a continuous curve, the ordinate to which represents the pull on the train.

Find the work done in foot-tons over a distance of 2 miles.

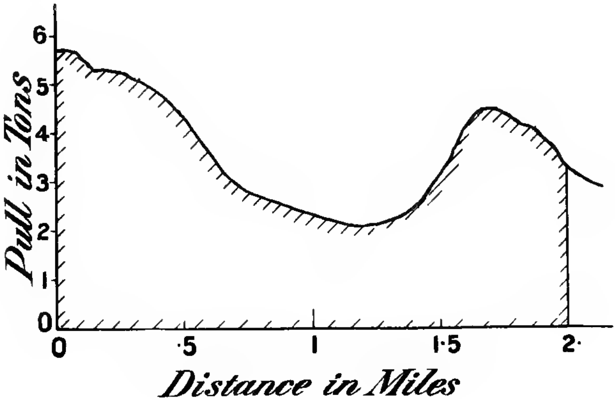

FIG. 223.

The area of the figure shown shaded, *i.e.* for a base of 2 miles, represents the work done, and = mean height × base.

Find the mean height by the method given in the Appendix. It comes out 3·67 tons.

$$\text{Work done in 2 miles} = 3{\cdot}67 \times 2 \times 5280 \text{ ft.-tons}$$

$$\text{the work done per foot} = \frac{\text{work done in 2 miles}}{2 \times 5280}$$

$$= 3{\cdot}67 \text{ ft.-tons}$$

This is *numerically* the same as the mean force or pull on the train; hence the definition of force in the summary, as—

The rate of doing work per unit of distance.

We have so far not considered how long it took the locomotive to haul the train over 2 miles of track in the last example.

If it had done the journey in 15 minutes, the rate at which it was doing work per minute was—

$$\frac{3{\cdot}67 \times 2 \times 5280}{15} = 2600 \text{ ft.-tons per minute}$$

If it had taken 2 minutes, its rate of doing work per unit of time would have been—

$$\frac{39{,}000}{2} = 19{,}500 \text{ ft.-tons per minute}$$

These numbers indicating the rate of doing work are called the *power* at which the locomotive was working under the respective conditions.

Example.—The diameter of an engine cylinder is 42 in., and the piston travels through a stroke of 5 ft. in each direction 84 times in each minute. The average pressure of the steam on each square inch of piston was 22 lbs. in the forward direction and 24·4 lbs. during the back stroke. Calculate the power of the engine expressed in ft.-lbs. per minute. Diameter of piston-rod 5 min.

Take each face of the piston separately.

Forward stroke.

$$\text{Area of piston face} = \frac{\pi}{4} \times 42^2 \text{ sq. ins.}$$

Each of these square inches sustains a pressure of 22 lbs.; hence the total pressure or force, urging the piston forward, is $\frac{\pi}{4} \times 42^2 \times 22$ lbs.

$$\left.\begin{array}{r}\text{Work done during one}\\ \text{forward stroke}\end{array}\right\} = \text{force} \times \text{distance moved through}$$

$$= \frac{\pi}{4} \times 42^2 \times 22 \times 5 \text{ ft.-lbs.}$$

$$\text{work done per minute} = \frac{\pi}{4} \times 42^2 \times 22 \times 5 \times 84$$

$$= 12{,}806{,}640 \text{ ft.-lbs.}$$

Return or back stroke.

$$\text{Area of piston face} = \frac{\pi}{4} \times 42^2 - \frac{\pi}{4} \times 5^2$$

$$= \frac{\pi}{4} \times 1739 \text{ sq. in.}$$

$$\text{total force urging piston} = \frac{\pi}{4} \times 1739 \times 24{\cdot}4 \text{ lbs.}$$

$$\text{work done during one stroke} = \frac{\pi}{4} \times 1739 \times 24{\cdot}4 \times 5 \text{ ft.-lbs.}$$

$$\text{work done per minute} = \frac{\pi}{4} \times 1739 \times 24{\cdot}4 \times 5 \times 84$$
$$= 14{,}002{,}428 \text{ ft.-lbs.}$$
$$\text{total power of engine} = 12{,}806{,}640 + 14{,}002{,}428$$
$$= 26{,}809{,}068 \text{ ft.-lbs. per minute}$$

This number is much too large to be convenient, and so a larger quantity than 1 ft.-lb. per minute has been chosen as the unit of power.

It is called the *horse-power*, and equals 33,000 ft.-lbs. per minute.

$$\text{Power of the above engine expressed in horse-power} = \frac{26{,}809{,}068}{33{,}000}$$
$$= 814$$

The mean pressures assumed in the above example are found by experiment in the following way. To each end of the cylinder D (Fig. 224) is connected an indicator, which

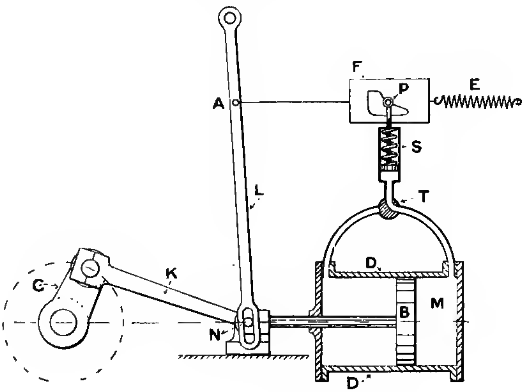

FIG. 224.—Steam-engine indicator arrangement.

consists of a small cylinder, in which slides a nicely fitting piston, above which is fitted a helical spring, S.

The piston is pressed upwards by the steam in the cylinder, thus compressing the spiral spring S above it, and the amount of movement of the piston is proportional to the pressure in

the cylinder, because the compression of a spiral spring is proportional to the force producing it (see page 10).

By connecting the piston-rod to a pencil, P, the pressure is automatically recorded on a piece of paper secured to the

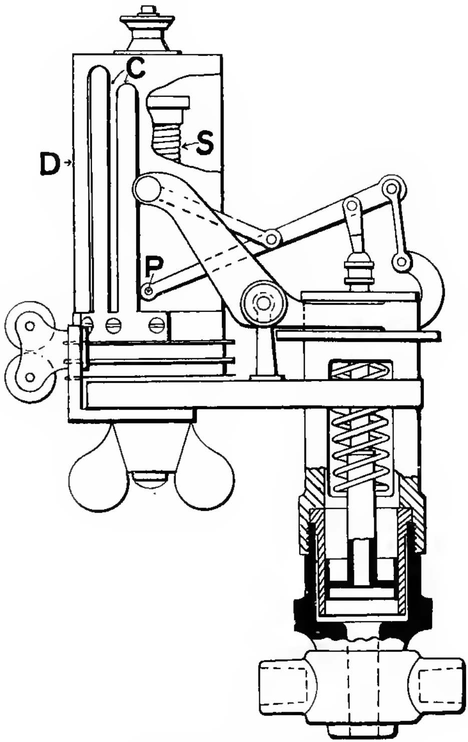

FIG. 225.—Steam-engine indicator.

board F, the amount of pressure being proportional to the height of the pencil above its zero position.

The board or frame F is made to partake of the motion of the engine piston B by coupling it to the lever L by a cord at A. The lever swings about its upper end, and consequently

the motion of A and the frame F is a duplicate of that of the pin N or the piston B, but to a reduced scale. The spring E keeps the cord tight.

The result is the same, whether the pencil is moved horizontally over the paper or the paper is moved under the pencil, and hence the diagram drawn on the paper is the same as if the paper were stationary and the pencil were moved vertically *and* horizontally over the paper, the horizontal movement being proportional to the movement of the piston B.

Any vertical ordinate bounded by the outline of the diagram represents useful or effective pressure.

The above is a very old type of indicator. In modern

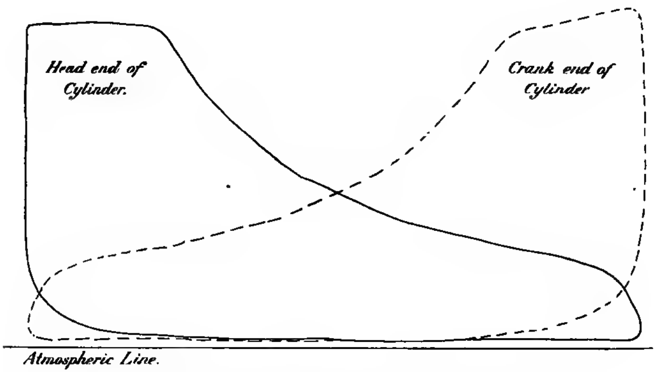

FIG. 226.—Indicator diagrams from a steam-engine.

instruments, one of which, made by Schaffer and Budenberg, is shown in Fig. 225, the movement of the piston is multiplied six times by the arrangement of links shown in the figure, the pencil being at P.

The arrangement of links is also for the purpose of making the pencil move in a vertical line, so that the amount of its motion shall in every respect represent the pressure under the indicator piston. The sliding frame in the previous figure has given place to the cylindrical drum D (Fig. 225), round which

the paper is secured by the clips C. The right end of the cord in the previous figure is now wound round the groove at the base of the paper drum, and the spring E in Fig. 224 is replaced by the spring S in Fig. 225.

The average pressure is obtained by finding the mean height of the diagram in inches, and multiplying the result by the scale of pressure (so many pounds per square inch to one inch of height).

Sometimes, for the sake of convenience, the diagrams from

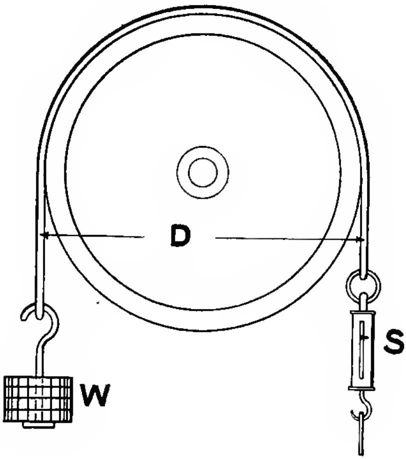

FIG. 227.

the two ends of the cylinder are drawn on the same sheet of paper. In Fig. 226 a pair of these are shown, but one is shown dotted for the purpose of easily distinguishing one from the other.

The horse-power of an engine found by means of this instrument (the indicator) is called the *indicated horse-power*, and is a measure of the rate at which work is done on the engine piston by the steam.

Brake Horse-power.—The rate at which an engine or motor does work, or the rate at which work is got out of an

engine at the flywheel, is called its brake horse-power, because the resistance is produced by a brake of some description.

A rope thrown over a pulley, as in Fig. 227, serves the purpose, if a weight, W, is suspended from one end and the other is made fast to a spring balance, S, which will indicate the pull at that end.

The resistance offered by the rope to the motion of the pulley is (W − S) lbs.

The work done in one turn is—

$$(W - S) \times \text{distance moved in 1 turn} = (W - S) \times 2\pi R$$

where R is the effective radius of the brake, *i.e.* the distance from the wheel centre to the centre line of the rope (the pull in the rope is exerted along its centre line).

If there are N revolutions per minute, the work done per minute $= (W - S) \times 2\pi RN$ ft.-lbs.

$$\left.\begin{array}{l}\text{And the brake horse-}\\ \quad\text{power}\end{array}\right\} = \frac{\text{work done per minute in ft.-lbs.}}{33{,}000}$$

$$= \frac{(W - S) \times 2\pi RN}{33{,}000}$$

As the horse-power of an engine is seldom quite constant for many seconds together, it will be necessary to take a number of observations periodically, and calculate the *average* H.P.

The number of turns per minute of the crank-shaft can be read off on the dial of a tachometer (speed indicator), or can be found from a counter, or the number can be counter if the speed is not too great.

The horse-power should be calculated for each observation, and the average obtained from the results as shown below.

RECORD OF EXPERIMENT TO DETERMINE BRAKE HORSE-POWER.

Date, May 29, 1900. Observer, J. W. Hall.

Kind of Motor—Steam Engine.

Diameter of brake-wheel = 3 ft.

Diameter of rope = ½ in.

Load W on end of brake-rope = 115 lbs.

Time.	Revolutions per minute.	Spring-balance readings.	Effective resistance of friction. R = W − S	B.H.P.
	N.	S lbs.		
7·45	265	4·5	110·5	8·50
7·55	268	4·5	110·5	8·50
8·5	272	4·0	111·0	8·75
8·15	273	3·7	111·3	8·75
8·25	266	4·0	111·0	8·53
8·35	269	4·0	111·0	8·65
8·45	270	5·5	109·5	8·56

Results.—Average speed = 269 revolutions per minute.
Average B.H.P. = 8·6.

If we determine both the indicated and brake horse-power simultaneously, we can at once calculate the mechanical efficiency of the engine; for the I.H.P. measures the rate at which work or energy enters the engine, and the B.H.P. measures the rate at which the engine gives out energy to or does work in driving some other machine. Then—

$$\text{The mechanical efficiency} = \frac{\text{work derived from a machine}}{\text{work put into the machine}}$$

$$= \frac{\text{B.H.P.} \times 33{,}000}{\text{I.H.P.} \times 33{,}000}$$

$$= \frac{\text{B.H.P.}}{\text{I.H.P.}}$$

Electrical Horse-power.—It is sometimes very convenient to drive a machine by means of an electric motor. In that case it is necessary to have a volt-meter and an ampere-

meter to measure the electrical pressure and current supplied to the motor.

Then the work done per minute by the current, or the work put into the motor per minute by the current, measured in watts

$$= \text{the pressure in volts} \times \text{current in amperes}$$

and the rate at which the motor receives work measured in horse-power

$$= \frac{\text{volts} \times \text{amperes}}{746}$$

$$\text{Also } \frac{\text{B.H.P.}}{\text{electrical H.P.}} = \text{efficiency}$$

Hence, the rate at which the motor can supply work or energy to another machine for the purpose of driving it = B.H.P.

$$= \text{electrical H.P.} \times \text{efficiency}$$

Hence, if an efficiency curve for the motor has been previously found, it can be used under the same conditions for determining the rate at which work is supplied to a machine by that electric motor.

A Transmission Dynamometer is used for the same purpose as the electric motor above, namely, for measuring the rate at which work is put into a machine. Instead of the machine whose H.P. is required being driven direct from a main shaft by a belt, the main shaft drives on to the dynamometer and the dynamometer on to the machine.

A simple form of transmission dynamometer is shown in Fig. 228. The spindle of the machine whose power is required is shown at B. The driving-spindle is C. Fixed on the spindle B is a toothed wheel, G, which gears with another, H, which runs loose upon a spindle, E, firmly fixed in the end of the vertical part of the three-armed lever M, which turns loosely upon the spindle B. Let the wheel K be urged in clockwise direction. Let P be the pressure of the teeth of K on the teeth of H.

This is shown in Fig. 229 by the arrow P. The diameter QET of the wheel H (Fig. 229) acts as a lever, turning about

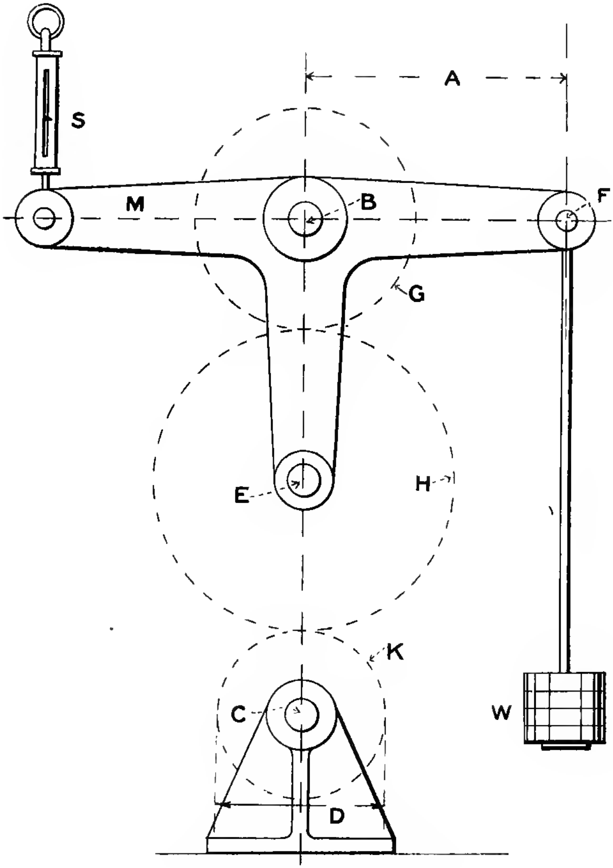

FIG. 228.—Transmission dynamometer.

T, and consequently the pressure on the spindle at E must be 2P. As the spindle is fixed in the arm, this pressure is transmitted to the arm as in Fig. 230. This is balanced by a weight

W, together with a spring balance S (Fig. 228). Either of these, or both, may be used, as suggested in the sketch.

Taking moments round the centre of the spindle B, we have—

$$2P \times a - WA - SA = 0$$

$$\text{or } P = \frac{A(W + S)}{2a}$$

If D = diameter of wheel K, and N the number of turns it makes per minute, the horse-power transmitted must be—

$$\frac{\text{force (lbs.)} \times \text{distance moved per minute (feet)}}{33{,}000}$$

$$= \frac{P \times \pi DN}{33{,}000}$$

Substituting for P, we have—

$$HP = \frac{A(W + S)}{2a} \times \frac{\pi DN}{33{,}000}$$

Another transmission dynamometer is shown in Fig. 231. It is not very dissimilar to that just described. If the wheel H in Fig. 228 had its plane turned through a right angle, so that it turned about the arm E as an axis, and still geared with the wheel G; and, further, if the wheel K were the same size as G, and placed loose upon the same shaft, B, so as to gear with H at the opposite end of the diameter to G, then we should get the dynamometer, Fig. 231.

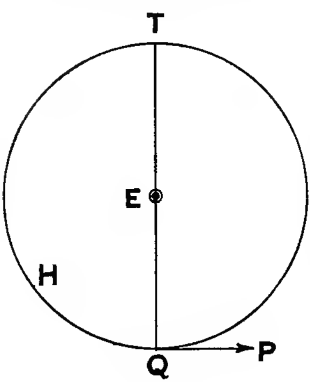

FIG. 229.

In Fig. 231 the pulley A is connected by a belt to a main shaft or motor by which it is driven, while the pulley B is connected to the machine to be driven. The pulley A is fixed

upon the shaft J, together with the bevel-wheel C. The bevel-wheel E and the pulley B are fixed upon a loose sleeve or hollow shaft K, which turns loosely on the shaft J. The two

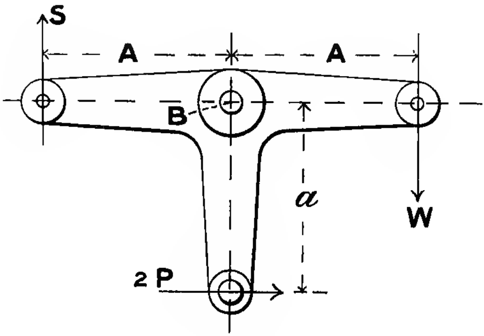

FIG. 230.

bevel-wheels D and F simply turn loosely upon the arm H, which itself turns loosely upon the shaft J. The bevel-wheels

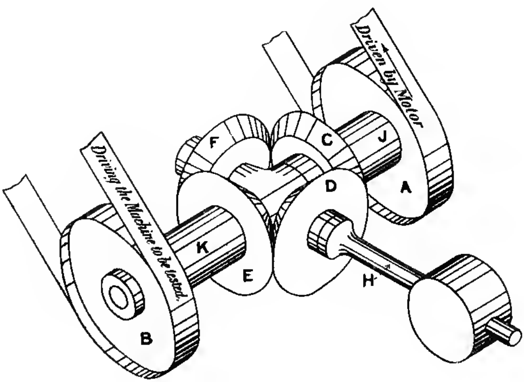

FIG. 231.—Transmission dynamometer.

C, D, E, and F are all the same size, and hence all make the same number of revolutions per minute.

Consider the arm H, and the bevel-wheel D upon it.

Let T (Fig. 232) be the pressure exerted by the wheel C upon the wheel D. The wheel D will press at O upon the

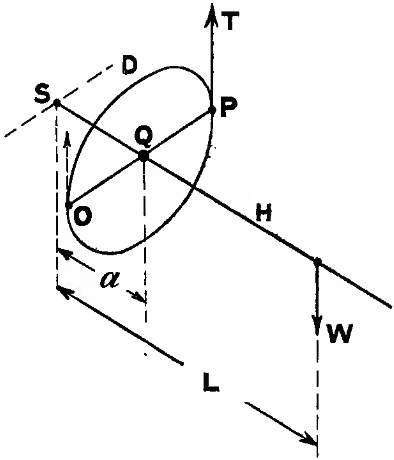

FIG. 232.

wheel E, and consequently, as action and reaction are equal and opposite (see page 192), the wheel E must react or press upwards on the wheel D with a force T, as shown at O. These two forces, T and T, will produce a resultant of 2T acting upwards on the wheel D at Q, and consequently on the arm H at Q.

Take moments of the forces on the arm H round S. We then get for the equilibrium of the arm—

$$W \times L = 2T \times a$$

Now, the horse-power transmitted through the wheel D equals—

$$\frac{\text{force T lbs. at P} \times \text{distance moved by P per minute}}{33{,}000}$$

$$= \frac{T \times \pi \times OP \times N}{33{,}000}$$

where N represents the number of revolutions of either wheel per minute. Also OP is the diameter of each bevel-wheel, and this must equal $2a$, as a is the radius of the bevel-wheels C and E. Therefore—

$$\text{the horse-power transmitted} = \frac{T \, . \, \pi \times 2a \times N}{33{,}000}$$

Substituting for $2Ta$ from the equation—

$$WL = 2Ta$$

we get—

$$H.P. = \frac{WL \, . \, \pi N}{33{,}000}$$

Note that L must be measured in feet, because we are dealing with foot-pounds per minute.

It may be worth noticing here that $T \times QP$ = turning-moment on the wheel D round its axis $= T \times a$. We can then write—

$$H.P. = \frac{\text{turning-moment} \times 2\pi N}{33{,}000}$$

But $2\pi N$ is the circular measure of the angle turned through by the wheel in one minute; hence—

$$H.P. = \frac{\text{turning-moment in lbs.-ft.} \times \text{angle in radians per minute}}{33{,}000}$$

Transmission of Energy by Belts.—Two pulleys, A and B, are connected by a belt, as shown in Fig. 233. The belt, which is generally made of leather, is somewhat elastic, and is made slightly less in length than that which it has to assume when in position on the pulleys; hence it is stretched in the process of putting on, and remains stretched until it is taken off again. This stretching places the belt in tension, and maintains it thus until it is removed from the pulleys.[1] This

[1] By constant use over a long period a belt often becomes *permanently* stretched, and then has to be shortened to produce the right amount of tension to cause sufficient friction.

tension produces a large amount of friction between the belt and pulley (for the amount, see page 267), which prevents the belt from slipping over the pulley. With large belts it is desirable, if possible, to arrange the pulleys at some horizontal

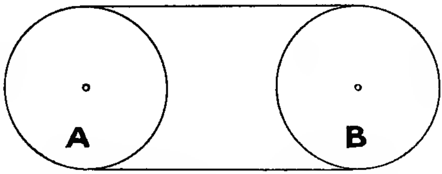

FIG. 233.

distance apart, so that the *mere weight* of the belt is sufficient to put a considerable amount of tension on it.

Assume for the moment that the pulley A in Fig. 233 is held stationary, and that an attempt is made to turn B in the clockwise direction.

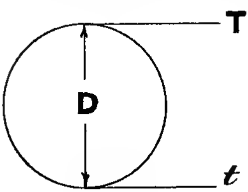

FIG. 234.

By so doing, extra tension will be placed on the top half of the belt, and the lower half will be relieved of a corresponding amount of tension, the sum of the two tensions remaining the same throughout.

Let D = the effective diameter of the pulley, Fig. 234,
= diameter of pulley + thickness of belt;
also let T = tension on the tight side,
and t = ,, ,, slack ,,

The tension T is tending to turn the pulley in one direction, while t is trying to turn it in the opposite direction; hence the effective tension which is doing work in turning the pulley is $T - t$.

The distance moved by the belt is the same as that moved by the pulley rim, which equals the length of one circumference

× number of turns per minute N. Hence the work transmitted per minute is—

$$(T - t)\pi DN \text{ ft.-lbs.}$$

$$\text{and H.P.} = \frac{(T - t)\pi DN}{33{,}000}$$

Example.—What width of single belt is required to transmit 4 H.P. when running over a 36-in. pulley, making 200 revolutions per minute, the thickness of the belt being $\frac{1}{4}$ in., and 70 lbs. per inch of width being the maximum tension allowed, and the tension on the slack side being assumed to be half that on the tight side?

$$\text{From the question, } T = 2t$$

$$\text{and the effective tension } (T - t) = T - \frac{T}{2}$$

$$= \frac{T}{2}$$

Let w = width of belt in inches.

$$\text{Then maximum tension} = 70w = T$$

$$\text{or } 35w = T = \text{effective tension} = (T - t)$$

Substituting in the above expression for the H.P. transmitted, we get—

$$4 = \text{H.P.} = \frac{35w \times \pi \times 3\tfrac{1}{24} \times 200}{33{,}000}$$

$$\text{hence } w = \frac{4 \times 33{,}000 \times 24}{35\pi \times 73 \times 200}$$

$$= 1{\cdot}97 \text{ in.}$$

A 2-in. belt would be used.

The Friction of a Belt.—It was mentioned in the above example that the tension on the tight side was twice that on the slack side. It is desirable to discover by experiment how near this assumption is to the truth. By far the best apparatus is a speed cone pulley fixed to a shaft and driven at a definite speed, with a flexible cotton rope acting like a brake. Let the pulley in Fig. 235 turn in the direction of the arrow. Four hooks or staples are arranged at A, B, C,

and D for hitching the spring balance to. When hitched in the staple A, the angle of contact between the cord and pulley is 90°. Similarly, when hitched at B, C, and D, the respective angles are 180°, 270°, and 360°, or multiples of these.

The surface of the pulley should be well polished and *quite*

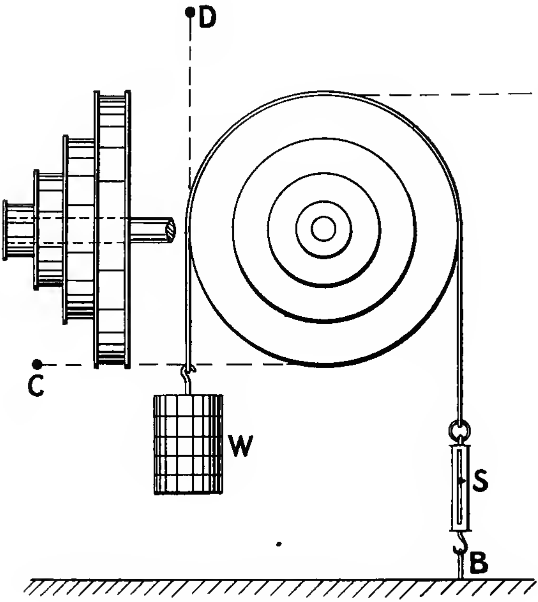

FIG. 235.—Rope friction apparatus.

dry, no oil or grease being allowed to come near the pulley. The cleaner the rope, the more consistent the results; hence the rope should never be thrown on the floor or allowed to get wet.

The load W is maintained constant throughout the experiment, a convenient amount being 56 lbs.

The following results were obtained by experiment :—

RECORD OF EXPERIMENT ON ROPE FRICTION.

Date, March 20, 1901. Observer, C. J. Feeny.

Rope used (new) cotton = $\frac{3}{8}$-in. diameter.
Diameter of pulley = 7·3 in.
Weight W = 50 lbs.
Speed of pulley = 161 turns per minute.

Turns of the rope.	Angle of contact radians.	Spring balance reading S lbs.	$\log_e W - \log_e S$.
$\frac{1}{4}$	$\frac{\pi}{2}$	34·0	0·38
$\frac{1}{2}$	π	24·5	0·71
$\frac{3}{4}$	$\frac{3\pi}{2}$	18·75	0·99
1	2π	13·5	1·32
$1\frac{1}{4}$	$\frac{5\pi}{2}$	9·0	1·70
$1\frac{1}{2}$	3π	6·5	2·04
$1\frac{3}{4}$	$\frac{7\pi}{2}$	4·5	2·40
2	4π	3·5	2·67
$2\frac{1}{4}$	$\frac{9\pi}{2}$	2·5	3·01
$2\frac{1}{2}$	5π	2·0	3·30

Plotting the third column vertically on the second column as base, we get a curve. Doing the same with W − S, we still get a curve. Trying $\frac{W}{S}$, the result is a curve. If we plot the logarithms of $\frac{W}{S}$ vertically and the logarithms of the angles of contact horizontally, we still get a curve.

Next try plotting the hyperbolic logarithms of $\frac{W}{S}$ (*i.e.* log W − log S) vertically and the angles of contact horizontally, and we get the straight line OA in Fig. 236.

The equation to this line is—

$$\log \frac{W}{S} = \text{slope} \times \text{angle of contact}$$
$$= 0{\cdot}213\theta$$

where θ represents the angle of contact in radians.

This equation can be written—

$$\frac{W}{S} = e^{0{\cdot}213\theta}$$

The number 0·213 is the coefficient of friction between the rope and the pulley.

If the angle of contact is maintained constant, the right-hand side of the above equation is constant, and equal to 1·95, when $\theta = \pi$ radians, as in Fig. 234.

$$\text{Then } W = 1{\cdot}95S$$

The coefficient of S is very nearly the 2 spoken of in the last example; in fact, much nearer than would be the case in

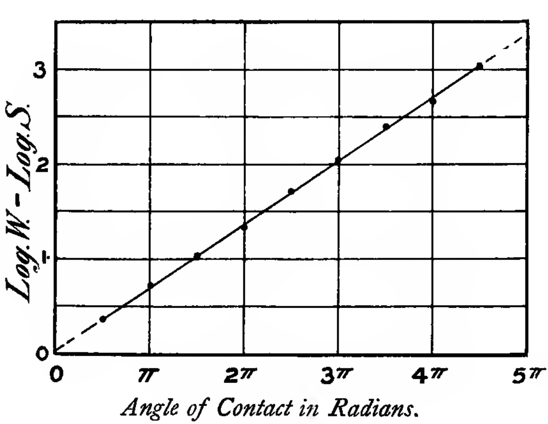

FIG. 236.

actual practice, because here we have slipping, and in actual practice it is the function of a belt *not* to slip.

The last equation suggests another short experiment. The coefficient of S, namely, 1·95, is $e^{\mu\theta}$, where μ represents the coefficient of friction. Replace 1·95 by C, then—

$$\log_e C = \mu\theta$$

$$\text{and } \mu = \frac{\log_e C}{\theta}$$

If an experiment is carried out with varying load W, but constant angle of contact, the equation—

$$W = CS$$

must hold as above. This is the equation to a straight line through the origin, if W is plotted vertically, and S horizontally, *the slope of the line being C.*

Insert the hyperbolic logarithm of C and the value of θ radians in the above equation, and we get μ, the coefficient of friction.

Next find the influence of the diameter of the pulley on the resistance of friction. Under ideal conditions the diameter has no influence, but under real conditions it certainly has.

The following were obtained from experiment:—

ANGLE OF CONTACT 180° OR π RADIANS.

Diameter of pulley in inches.	Resistance of friction (W — S) lbs.
7·3	30
5·75	28·4
4·2	26·5
2·7	25

Plotting these, we get Fig. 237, which shows that the diameter of pulley has something to do with the friction. This is, of course, because the belt does not fulfil the ideal conditions under which the expression on page 270 was constructed.

The Bending of a Beam can be studied experimentally by means of the apparatus in Fig. 238. On a stiff bed, B, rests a couple of supports, S, which carry the beam, and which can be placed anywhere along the bed.

These supports terminate in knife-edges, with a little plate

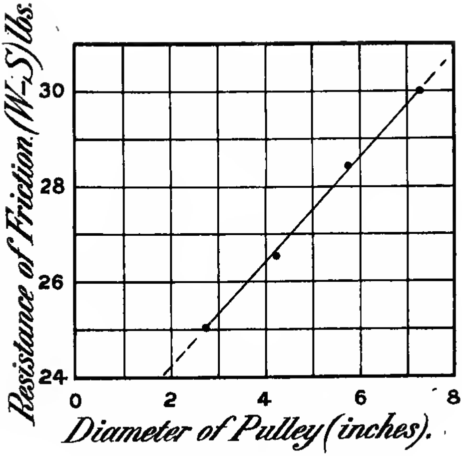

FIG. 237.

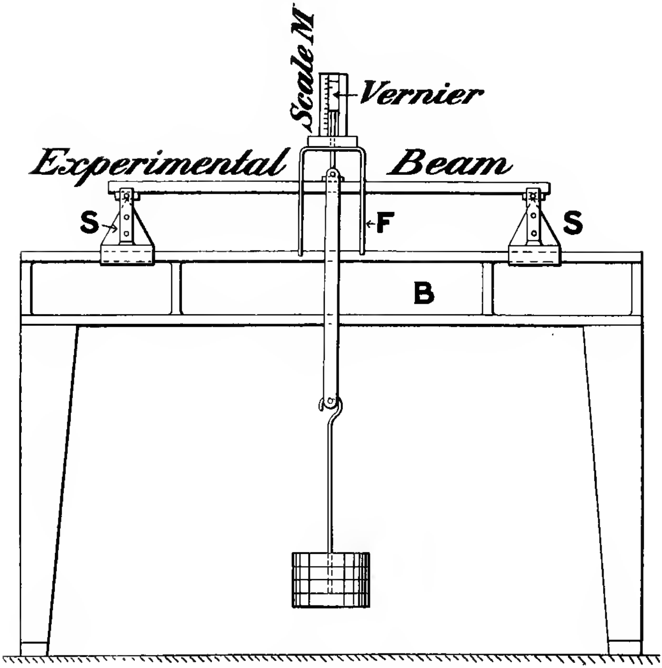

FIG. 238.—Beam-bending apparatus.

above the knife-edge to prevent wood beams from being indented.

A spider frame, F, carries the scale M and the guides between which the vernier slides. The vernier slide is fixed to a vertical rod which bears upon the stirrup, by which the load is put on the beam.

In the first experiment, find the relation between the load

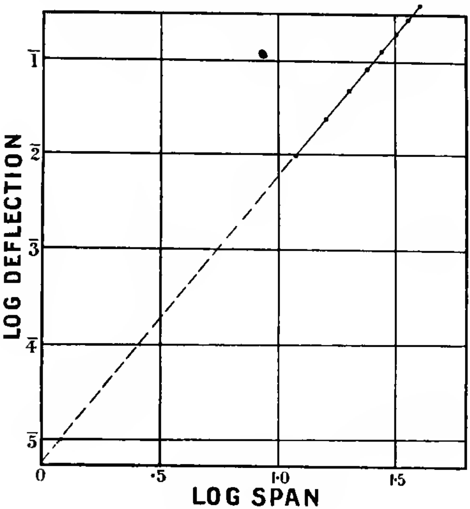

FIG. 239.

and the deflection, using the longest length of beam. It will be found that the deflection is proportional to the load producing it as with the spiral spring.

In the second experiment, vary the length of the beam by shifting the supports S, and use the same load for every length. Be careful to take a zero reading with every length, because beams are seldom quite straight.

Plotting the length and deflection, we get a curve. Find its equation by the method suggested in the Appendix. The results of an experiment are shown in the following table and plotted in Fig. 239.

Span in.	Deflection in.	Log span.	Log deflection.
12	0·01	1·0792	$\bar{2}$·0
16	0·025	1·2041	$\bar{2}$·3979
20	0·05	1·3010	$\bar{2}$·6990
24	0·085	1·3802	$\bar{2}$·9290
28	0·13	1·4472	$\bar{1}$·1139
32	0·2	1·5051	$\bar{1}$·3010
36	0·3	1·5563	$\bar{1}$·4771
40	0·42	1·6021	$\bar{1}$·6232

Equation to line AB, Fig. 239—

$$\text{log deflection} = 3 \text{ log span} + \bar{6}\cdot77$$
$$\text{and deflection} = 0\cdot0000059 \text{ span}^3$$

In other words, the deflection of a beam is directly proportional to the cube of its span.

In the third experiment, vary the depth of the beam, keeping the load and the span constant.

It will be found, in a manner similar to that just described, that—

$$\text{Deflection} = \frac{c}{\text{depth}^3}$$

Lastly, vary the width of the beam, keeping the load and span constant; and then—

$$\text{Deflection} = \frac{k}{\text{width}}$$

Now connect these equations together, as was explained on page 155 in connection with the Attwood's machine, and we get—

$$\text{Deflection} = \text{coefficient} \times \frac{\text{length}^3}{\text{width} \times \text{depth}^3}$$

The coefficient should be found for different materials.

The Work done in bending a Beam can now be quickly found if the deflection due to one load is known. The load-deflection curve in the previous experiment was found to be a straight line through the origin. In Fig. 240 the deflection AP was caused by the load PQ. Join the origin A to Q, then AQM is the deflection curve within the elastic limit.

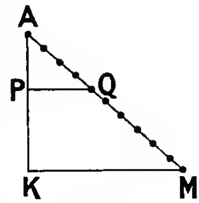

FIG. 240.

The work done in deflecting the beam through AP is given by the area of the triangle APQ, and the work of deflecting it through AK is given by the area AKM. This work is often called the *resilience* of the beam, because it is equal to *the work done by the beam in coming back to the unstrained condition.*

However a beam is bent, within the elastic limit, the work done in bending it is represented by a triangle, in which one side is the amount of deflection, and the side perpendicular to this represents the steady load which would cause this deflection.

A Live Load is Twice as Destructive as a Dead Load.

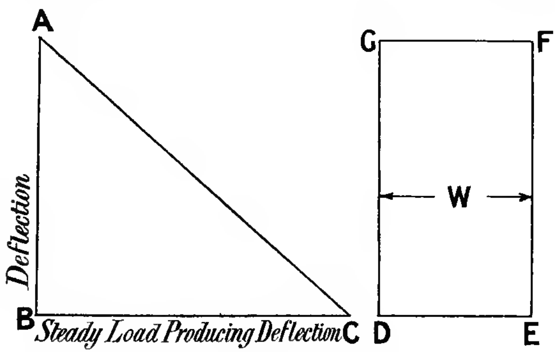

FIG. 241.

—This can be shown in the following manner: Let the load

in question, W, be supported in contact with the beam, but *the beam bears none of the load.* Then let the load's support be removed, and the load will come suddenly upon the beam, and will cause it to deflect.

Let the deflection be GD, Fig. 241. The work done by the weight on the beam in falling = W × GD = area DEFG.

From the "Principle of Work," this must be used in bending the beam.

From above, the work done in bending the beam is given by the area of the triangle ABC (Fig. 241), in which AB = deflection and BC the *steady* load which would cause the deflection. Equating these quantities, we get—

$$\text{area ABC} = \text{area DEFG}$$
$$\text{or } \tfrac{1}{2}\text{ deflection} \times \text{BC} = \text{deflection} \times \text{W}$$

that is—

$$\text{BC} = 2\text{W}$$

or the effect of a live load is the same as that which would be produced by a steady load of twice the amount of the live load.

Bending Moment and Moment of Resistance.—In Fig. 242 a line is drawn across the beam between W_2 and W_3.

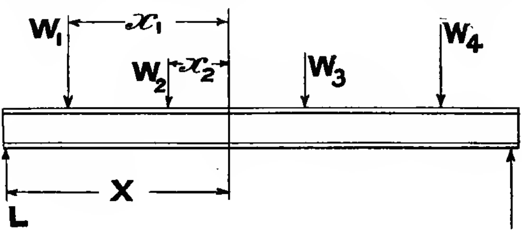

FIG. 242.

Consider the part of the beam to the left of this line. The forces trying to turn it round an axis perpendicular to the paper and passing through the centre of the beam and the line of section are the left supporting force L, W_1, and W_2.

The combined effect of these forces is a tendency to turn or bend the left part of the beam round the axis mentioned above in the direction of the hands of a clock. Taking each force separately and considering clockwise direction as positive, the moment of L is LX, the moment of W_1 is $-W_1x_1$, and the moment of W_2 is $-W_2x_2$. The resultant moment equals the sum of these moments, namely—

$$LX - W_1x_1 - W_2x_2$$

This resultant moment is called the *bending moment* at the section in question.

If the left part of the beam is in equilibrium, as it must be, the above resultant moment must be balanced by an equal and opposite moment, which is called the *moment of resistance.* It is produced by the action of the right portion of the beam upon the left.

In Fig. 243, a section of the beam is shown on the left. It

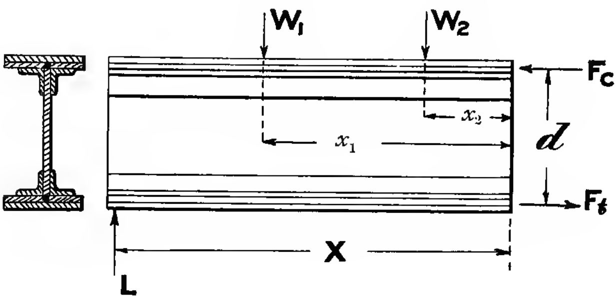

FIG. 243.

consists of some plates and angles forming the top and bottom flanges, which are connected together by a web-plate. The upper flange is in compression and the lower flange in tension. That being so, the action of the right part of the upper flange on the left part is a force F_c, and similarly at the lower flange we get a tensile force F_t. These must be the directions of the forces, because they are the only directions which permit of a

moment of resistance in the opposite direction to the bending moment.

The forces F_c and F_t act approximately at the dots in the section, these dots being at the centre of gravity of the flanges.

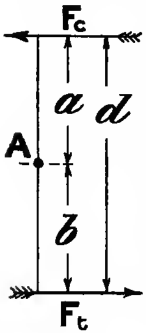

FIG. 243*a*.

These forces, F_t and F_c, must be equal, because the first law of equilibrium says that the sum of the horizontal components is zero, and as there are only two forces, one must be equal and opposite to the other. Further, their moment round any point in the section is the product of one of the forces and their distance apart, or Fd, for the moment of F_c round A is F_ca, and the moment of F_t round A is F_tb. The sum of these is $F(a + b) = Fd$. This is called the moment of resistance, and equals the bending moment. Hence—

$$Fd = LX - W_1x_1 - W_2x_2$$

We can find L by the method given on page 80, and if we know d we can find F. Then, assuming a working or safe stress, say f tons per square inch—

$$F = f \times \text{area A of flange}$$

from which the size of the flange can be calculated.

There is a force not yet taken account of in the above beam problem.

The first law of equilibrium states that if the part of the beam (Fig. 243) is in equilibrium, the sum of the vertical components acting on it must be zero. It will be found that $L - W_1 - W_2$ is not zero, but equal to some force, say S. This force S is the vertical force exerted by the right part of the beam upon the left part at the section, and is called the *shearing force* at that section.

This shearing force is resisted almost entirely by the web-plate of the beam.

The student should bear in mind that the bending moment and also shearing force are the same, whatever the shape

of the beam section. They depend respectively only on the disposition of the loads.

But the moment of resistance depends entirely on the shape of the section. The easiest section has been taken above, and practically the only one that can be dealt with at this stage of the subject.

Summary of Chapter IX.

Work = force exerted × distance moved by body while force is acting.

Force = rate of doing work per unit of distance.

Power = rate of doing work per unit of time.

One horse-power = 33,000 ft.-lbs. per minute.

One horse-power = 746 watts.

Tension on the tight side of a belt is assumed to be twice the tension on the slack side.

Deflection of a beam is proportional to the load.

Effect of a live load is twice that of a dead load at any section of a beam.

Bending moment = moment of resistance.

Bending moment = sum of moments of forces on one side of section.

Shearing force = sum of forces on one side of section.

Moment of a couple = one force × perpendicular distance apart of forces.

Examples on Chapter IX.

1. Define the unit of work. What name is given to this unit? In drawing a load a horse exerts a constant pull of 120 lbs. How much work will be done in 15 minutes, supposing the horse to walk at the rate of 3 miles an hour? *Ans.* 475,200 ft.-lbs.

2. When a body is raised through a given height, how is the work done estimated? A body weighing 8 cwt. is drawn 100 ft. along an incline which rises 2 ft. for every 5 ft. along the incline. Find the work done, neglecting friction. *Ans.* 35,840 ft.-lbs.

3. Define the terms: Force, work, and power. A horse drawing a cart at the rate of 2 miles per hour exerts a force of 156 lbs. Find the work done in 1 minute. *Ans.* 27,456 ft.-lbs.

4. Distinguish between force and power. How is each respectively measured? A traction-engine draws a load of 20 tons along a level road, the tractive force on the load being 150 lbs. per ton. Find the work done upon the load in drawing it through a distance of 500 yards.

Ans. 4,500,000 ft.-lbs.

5. How is the work done by a force measured? The resistance to motion on a level road is 150 lbs. per ton of weight moved. How many foot-pounds of work are expended in drawing 6 tons through a distance of 15 yards? *Ans.* 40,500 ft.-lbs.

6. Find the work done in raising 136 cub. ft. of water to a height of 20 yards. *Ans.* 510,000 ft.-lbs.

7. Explain the method of calculating the work done by a force, and define the unit of work. The surface of the water in a well is at a depth of 20 ft., and when 500 gallons have been pumped out, the surface is lowered to 26 ft. Find the number of units of work done in the operation, the weight of a gallon of water being 10 lbs. *Ans.* 115,000 ft.-lbs.

8. The plunger of a force-pump is $8\frac{3}{4}$ in. in diameter, the length of the stroke is 2 ft. 6 in., and the pressure of the water is 50 lbs. per square inch. Find the work done in one stroke. *Ans.* 7516 ft.-lbs.

9. A weight of 4 tons is raised from a depth of 222 yards in a period of 45 seconds: calculate the amount of work done. *Ans.* 5,967,360 ft.-lbs.

10. If a horse, walking at the rate of $2\frac{1}{2}$ miles per hour, draws 104 lbs. out of a well by means of a cord going over a wheel, how many units of work would he perform in 1 minute? *Ans.* 22,880 ft.-lbs.

11. Four cwt. of material are drawn from a depth of 80 fathoms by a rope weighing 1·15 lbs. per linear foot: how many units of work are expended? What horse-power would be required to raise the material in $4\frac{1}{2}$ minutes? *Ans.* 347,520 ft.-lbs.; 2·34 H.P.

12. A chain, weighing 2 lbs. per foot, passes over a fixed smooth pulley, so that 14 ft. hangs over on one side and 6 ft. on the other: show by a diagram the work which will be done in pulling round the wheel until the upper end of the chain is 1 ft. above the lower end. Hence explain the use of a conical drum as applied to coal-winding machinery.

Ans. 31·5 ft.-lbs.

13. A spiral spring is stretched through 1 in. by a force of 10 lbs. Find the work done in stretching it through an additional length of 2 inches. Draw the diagram of work done, giving dimensions. *Ans.* 40 in.-lbs.

14. What must be the effective horse-power of a locomotive engine which moves at a steady speed of 40 miles per hour on a level rail, the resistance being 15 lbs. per ton, and the weight of the engine and train being 100 tons? If the rails were laid on a gradient of 1 in 100, what additional horse-power would be required? *Ans.* 160 H.P.; 238·9 H.P.

15. A train of 200 tons ascends an incline which has a rise of 5 ft. in

1000, with a uniform speed of 30 miles per hour: what is the effective horse-power of the engine, the friction being 5·5 lbs. to the ton?

Ans. 267·2 H.P.

16. A cistern, 22 ft. long, 14 ft. broad, and 12 ft. deep, has to be filled with water from a well 7 ft. in diameter. The vertical height of the bottom of the cistern above the free surface of the water in the well is 100 ft. when the operation of filling the cistern is commenced. Water flows into the well at the rate of 462 cub. ft. per hour. Find the horse-power of the pump in filling the cistern, supposing 30 minutes are required in the operation.

Ans. 35·1.

17. A belt is required to transmit 4 horse-power from a shaft running at 120 revolutions to one at 160 revolutions per minute. Find the stresses in the belt, the small pulley being 2 ft. in diameter, and the ratio of the tensions on the belt being as 7 is to 4. Find also the width of belt that would be required in the above case, if the stress is taken at 100 lbs. per inch of width. *Ans.* 306·25 lbs.; 175 lbs.; 3·06 in.

18. In the transmission of energy by a rope, the wheel carrying the rope is 14 ft. in diameter and makes 30 revolutions per minute, the tension of the rope being 100 lbs.: find the amount of power transmitted. *Ans.* 4 H.P.

19. A pulley, 3 ft. 6 in. in diameter, and making 150 revolutions per minute, drives by means of a belt a machine which absorbs 7 horse-power. What must be the width of the belt so that its greatest tension shall be 70 lbs. per inch of width, it being assumed that the tension in the driving side is twice that in the slack side? *Ans.* 4 in.

20. Show that a force acting at any point of a body is equivalent to an equal and parallel force acting at any other point and a couple about that point.

21. Show that a couple has no particular point of application, but may be shifted anywhere in the same plane without disturbing the equilibrium of a body to which it is applied. Is this true of a force? Explain the difference, if any.

22. What is a *foot-pound* of work? Is it the same at different parts of the earth? What is a *horse-power?*

A reservoir of water is 11 ft. deep, and is fed by a stream supplying 60,000 gallons per hour. The water runs out at the same rate at the bottom and turns a turbine. If the turbine uses 60 per cent. of the potential energy which the water loses, find the horse-power which it supplies.

Ans. 2 H.P.

23. The average pressure on the piston of a steam-engine is 60 lbs. to the square inch, the area of the piston is 1 sq. ft., and the length of stroke 18 in. The engine registers 8 horse-power. How many strokes does it make per minute? *Ans.* $20\frac{10}{27}$.

24. A snow slope rises a height of 50 ft. in a slope of 200 ft. A sledge weighing 400 lbs. is drawn up it by a rope parallel to the surface of the snow. Find a triangle representing in magnitude the forces acting, and find the pull on the rope when the sledge is going steadily up. Find the

work done in pulling the sledge up the slope. Friction is to be neglected. *Ans.* 20,000 ft.-lbs.

25. In machinery, where one pulley drives another by means of an endless belt, there is a difference of tension in the two parts of the belt. Why is this? The pulley on an engine shaft is 5 ft. in diameter, and it makes 100 revolutions per minute. The motion is transmitted from this pulley to the main shaft by a belt running on a pulley, and the *difference in tension* between the tight and slack sides of the belt is 115 lbs. What is the work done per minute in overcoming the resistance to motion of the main shaft? *Ans.* 180,550 ft.-lbs.

CHAPTER X.

FLUIDS.

THE different states of matter are (1) *Solid*, (2) *Liquid*, and (3) *Gaseous*. It is difficult to draw a hard-and-fast line between these different states, if absolute accuracy is required; but it is convenient in describing a portion of matter to use the above terms; and although every one knows what is generally meant by these terms, it will be useful for future use to define them as nearly as we can.

A Solid body is one which retains its shape and size without lateral support when left to itself; for example, a penholder will maintain its size and shape unless it is wilfully broken or cut to pieces, and it does not require any lateral support to keep its shape.

A Fluid body is one which *does* require lateral support to maintain its shape. Fluids may be subdivided into Liquids and Gases.

A Liquid body is one which requires lateral support to maintain its shape, and its volume is the same whatever may be the shape it occupies. Water is a liquid, and while it may be poured from one vessel to another of a different shape, its volume does not alter. Further, the sides of the vessel are necessary to provide the lateral support to prevent it from altering its shape.

A Gaseous body is one which requires lateral support to maintain *both* its shape and volume. If permitted to do so, it will alter its shape and volume indefinitely.

Pressure at a Point in a Liquid.—By this we mean the pressure or force exerted by the liquid on a square inch or

square foot, as the case may be, when the centre of that square inch or square foot is situated at the point in question.

Consider the prism immersed in the tank of water (Fig. 244), and let the prism weigh exactly the same as would the quantity of water which would just fill the space occupied by the prism. If this condition is satisfied, the prism will remain anywhere within the water in which it may be placed. Also let δ = weight of one cubic foot of water. Then the weight of the prism must equal the number of cubic feet it contains multiplied by the weight of one cubic foot. Let V be the number of cubic feet in the prism (= volume), then its weight is $V\delta$ lbs.

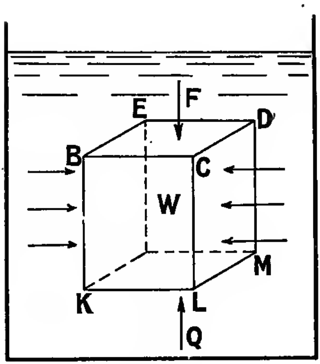

FIG. 244.

Let h be the height of the prism in feet, and A the area of each end in square feet.

Then $A \times h$ = volume of prism

and $V \times \delta = A \times h \times \delta$ = weight of prism

Now consider the equilibrium of the prism. It is maintained in equilibrium by—

(1) The force or total pressure, F, of the water over the upper end BCDE.

(2) The total pressure Q over the under side of the prism.

(3) The weight of the prism $= A \,.\, h \,.\, \delta$ lbs.

(4) The horizontal pressures H on the right and left vertical faces, and

(5) The horizontal pressures on the back and front vertical faces.

We have learnt from page 51 that the mutual pressures between two surfaces in contact must be perpendicular to the surfaces at the points of contact. The water touches the vertical faces of the prism, and therefore the pressure of

the water on the vertical faces must be perpendicular to those faces; that is, they are horizontal in direction. Further, the pressure on one face must be equal and opposite to the pressure on the opposite face; for if these forces are not equal, they must have a resultant which will make the prism move in the direction of the resultant. In the same way, the pressures on the back and front faces must balance one another.

There are now left for consideration the forces mentioned in paragraphs 1, 2, and 3. These being all vertical, we can add them together with their proper signs, and by the first law of equilibrium the result must be zero. That is—

$$\text{F} + \text{weight of prism} = \text{Q}$$
$$\text{or F} + \text{A} \,.\, h\delta = \text{Q}$$

Now let the prism grow longer until the top surface is coincident with the free surface of the water (Fig. 245). F is now the pressure of the atmosphere only on the prism, and equal to $14 \cdot 7 \times 144 \times \text{A}$ lbs.

Then $\text{Q} = 14 \cdot 7 \times 144 \times \text{A} + \text{A} \,.\, h \,.\, \delta$

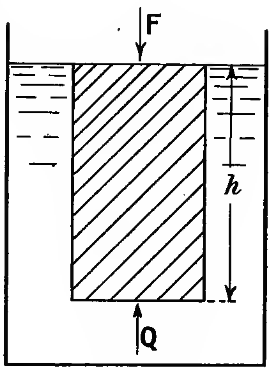

FIG. 245.

But as Q is the total or resultant pressure on the lower horizontal surface of the prism, we can interpret the above equation as—

Resultant pressure of a liquid on a horizontal surface immersed in it = pressure of the atmosphere on an equal surface + area of surface in square feet × depth of surface below free surface of liquid × density of liquid.

If the atmosphere were replaced by a gas under pressure or by steam, as in a boiler, the above equation holds good if the word "atmosphere" be replaced by "gas" or "steam," as the case may be.

If, as is generally the case, we require to find the pressure

due to the liquid only, without any atmosphere or gas pressing on the free surface of the liquid, put the atmospheric or gas-pressure equal to zero, and we get resultant pressure on horizontal surface *due to the liquid only* $= A \cdot h \cdot \delta$.

The student should, if possible, now carry out an experiment to verify the above statement. This can easily be done by connecting a delicate pressure-gauge with a column of water or mercury, as suggested in Fig. 246. The height of the water-column K is measured on the scale S. The supply-valve is shown at A, and the drain at D. The valve E is for the purpose of shutting off the gauge if necessary.[1]

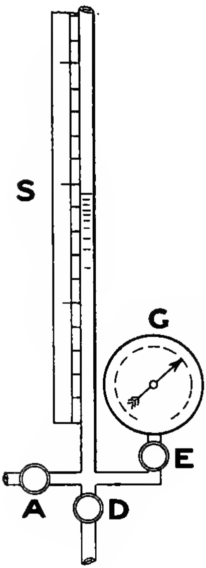

Fig. 246.

The gauge G measures the pressure of the water in pounds per square inch, and if the above formula is true, and we put the area $A = \frac{1}{144}$ square foot and $\delta = 62{\cdot}5$, we get—

Pressure per square inch due to a column of water h ft. high $= \frac{1}{144} \times h \times 62{\cdot}5$
$= 0{\cdot}43h$

Take observations of the pressure-gauge indications and the height of the water-column, and after entering in a table, plot them with the height, h, of water-column along the base and the gauge-pressures vertically upwards. Deduce the equation to the resulting curve, and compare it with that found above, namely—

$$\text{Pressure in pounds per square inch} = 0{\cdot}43h \text{ ft.}$$

[1] In fitting up this gauge, it should be carefully filled with water and the water retained in it by keeping the valve E closed, except when being used. The gauge may be filled with water by attaching to a steam main and allowing the steam to condense in it. With a Schaffer gauge this is not necessary.

Example.—A tank, which is full of water, is 7 ft. deep, 2 ft. wide, and 3 ft. long. What is the resultant pressure on its base?

$$\begin{aligned}\text{Resultant or total pressure on base} &= \text{pressure of atmosphere only} \\ &\quad + \text{pressure of water only} \\ &= 14{\cdot}7 \times 144 \times A + A \,.\, h \,.\, \delta \\ &= 2117 \times 6 + 6 \times 7 \times 62{\cdot}5 \\ &= 12{,}702 + 2625 \\ &= 15{,}327 \text{ lbs.}\end{aligned}$$

If the atmosphere is exhausted from the tank, then the pressure on the base is due to the water alone, and equals 2625 lbs.

If the tank had been full of oil, each cubic foot of which weighed 42 lbs., the pressure on the base would be found by substituting 42 for δ in the above equation.

We have so far only considered the pressure on a *horizontal* surface. Consider the portion of the liquid in Fig. 247, shown

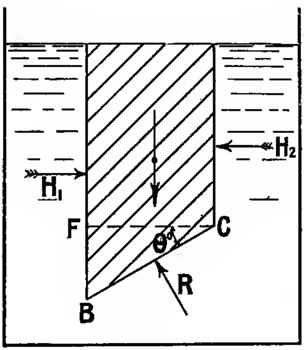

Fig. 247.

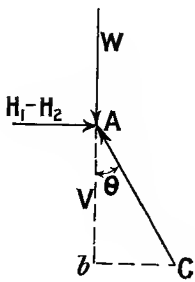

Fig. 248.

shaded. The pressures on the different surfaces must be perpendicular to their respective surfaces.

The shaded part is in equilibrium under the action of its weight W, the resultant side pressures H_1 and H_2, the resultant back and front pressures (which balance each other), and the resultant pressure R on the inclined base. (See also Fig. 248.)

Resolving these forces in the vertical direction, we have V, the vertical component of R, balancing the weight W; then—

$$W = V$$

But W = volume of shaded portion × weight per cubic foot

= area of horizontal section × average height × δ

and area of horizontal section = area of base resolved horizontally

$$\text{or } \frac{\text{area of horizontal section}}{\text{area of base}} = \frac{FC}{BC}$$

$$\left.\begin{array}{r}\text{and therefore area of}\\ \text{horizontal section}\end{array}\right\} = \frac{FC}{BC} \times \text{area of base}$$

Also, average height of shaded portion = depth of centre of gravity of BC below free surface ED of liquid = h say;

$\therefore$ weight of shaded portion = W

$$= \text{area of horizontal section} \times h \times \delta$$

$$= \left(\frac{FC}{BC} \times \text{area of base}\right) \times h \times \delta$$

Again, the triangle cab is similar to the triangle CFB, because ca is perpendicular to BC, and ba perpendicular to FC, and the angle between two lines equals the angle between the two perpendiculars, that is, the angle bac = the angle FCB. Further, the angles at F and b are right angles, therefore the remaining angles at B and C must be equal. (See Introduction.) The triangles being similar, their corresponding sides are proportional; that is—

$$\frac{FC}{BC} = \frac{ba}{ac} = \frac{V}{R} = \frac{W}{R}$$

$$\text{and } W = R \times \frac{FC}{BC}$$

But from above we have—

$$W = \frac{FC}{BC} \times \text{area of base} \times h \times \delta$$

$$\therefore R \times \frac{FC}{BC} = \frac{FC}{BC} \times \text{area of base} \times h \times \delta$$

that is, R = area of base × h × δ

But R = resultant pressure on base

therefore—

Resultant pressure on inclined surface (base) = area of the wetted surface in square feet × depth (h ft.) of the centre of gravity of wetted surface below the free surface of the liquid × weight of 1 cub. ft. of the liquid.

The reason for calling the immersed surface "the wetted surface" is because some of it may under certain conditions be out of the liquid, and we are only concerned with that part in contact with the liquid, and which of necessity must be wetted.

Example.—A rectangular water-tank is 6 ft. deep, 10 ft. wide, and 15 ft. long: calculate the pressure on the base, and on each side separately.

Substituting these numbers in the equation in heavy type above, we have—

$$\text{Total or resultant pressure on base} = (15 \times 10) \times 6 \times 62{\cdot}5$$
$$= 56{,}250 \text{ lbs.}$$

The area of one end is $6 \times 10 = 60$ sq. ft.

The centre of gravity of the end will be halfway down it, or 3 ft. below the free surface; hence—

$$\text{Total pressure on end} = 60 \times 3 \times 62{\cdot}5$$
$$= 11{,}250 \text{ lbs.}$$
$$\text{total pressure on one side} = (6 \times 15) \times 3 \times 62{\cdot}5$$
$$= 16{,}875 \text{ lbs.}$$

Example.—A triangular vessel, shown in Fig. 249, has a pipe 1 ft. diameter fixed in its upper side. The vessel is 2 ft. deep perpendicular to the paper. Calculate the pressure on each side of the vessel.

The centre of gravity, *a*, of the left side is at *a* (50 + 2) ft. below the free surface; hence—

$$\text{The total pressure on it} = \text{area in square feet} \times 52 \times 62{\cdot}5$$
$$= 4 \times 2 \times 52 \times 62{\cdot}5 = 26{,}000 \text{ lbs.}$$

$$\left.\begin{array}{l}\text{the total pressure on the}\\ \quad\text{sloping side}\end{array}\right\} = (5 \times 2) \times 52 \times 62{\cdot}5 = 32{,}500 \text{ lbs.}$$

$$\left.\begin{array}{l}\text{the total pressure on the}\\ \quad\text{top side}\end{array}\right\} = \text{area} \times 50 \times 62{\cdot}5$$

$$= \left(6 - \frac{\pi}{4} \times 1^2\right) 50 \times 62{\cdot}5 = 16{,}297 \text{ lbs.}$$

The centre of gravity of the triangular face is $\frac{4}{3}$ ft. down from the top side, or $50 + 1\frac{1}{3}$ below the free surface. The area of $ABC = \frac{1}{2} \times 4 \times 3 = 6$ sq. ft.

Total pressure on face $ABC = 6 \times 51\frac{1}{3} \times 62{\cdot}5 = 19{,}250$ lbs.

The student will realize from working through this problem

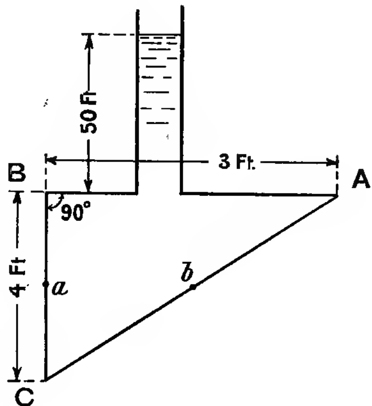

Fig. 249.

the use of a vertical stand-pipe, as it is called, for producing pressure. One will be found in connection with the pumping-stations of most waterworks.

Example.—A sluice-gate, 2 ft. wide and 3 ft. deep, has its upper edge 26 ft. below the free surface of the water. It is raised and lowered by means of a screw, the collar which supports the weight being made as the nut to the screw, which latter does not turn. The collar nut has 25 teeth on its periphery, into which gears a worm-wheel, which is turned by means of a handle 10 in. long.

What force must be applied to the handle to lift the sluice-gate, if the efficiency of the whole is $0{\cdot}12$, the pitch of the screw being $\frac{1}{2}$ in., and the coefficient of friction of sluice $= 0{\cdot}3$? Weight of gate and other parts lifted $= 300$ lbs.

$$\left.\begin{array}{l}\text{Resultant pressure of water on}\\ \quad\text{gate}\end{array}\right\} = \text{area of gate} \times \text{depth of centre of gravity} \times 62\cdot5$$

$$= 3 \times 2 \times 27\cdot5 \times 62\cdot5$$

$$= 375 \times 27\cdot5 \text{ lbs.}$$

$$\text{resistance of friction} = 0\cdot3 \times 375 \times 27\cdot5 \text{ lbs.}$$

$$= 3100 \text{ lbs.}$$

$$\text{total resistance to motion} = \text{friction} + \text{weight of gate}$$

$$= 3100 + 300 = 3400 \text{ lbs.}$$

$$\left.\begin{array}{l}\text{work done in lifting this during}\\ \quad\text{one turn of worm-wheel}\end{array}\right\} = 3400 \times 0\cdot5 \text{ in.-lbs.}$$

During same time the worm and handle have made 25 turns.

Let F = average force on end of handle; then, as the efficiency is 0·12, the portion of F doing useful work in lifting = 0·12F lbs. The useful work done by this during 25 turns of the handle $= 2\pi \times 10 \times 25 \times 0\cdot12F$ in.-lbs., and this must equal the work done in lifting sluice, or—

$$2\pi \times 10 \times 25 \times 0\cdot12F = 3400 \times 0\cdot5$$

from which—

$$F = 9 \text{ lbs. approximately}$$

The expression obtained on page 289 for the pressure on *any*

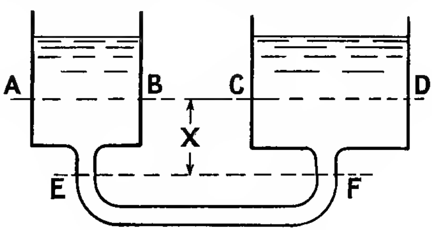

Fig. 250.

surface immersed in a liquid did not contain the inclination of the surface to the horizon; hence the pressure must be the same, whatever the inclination; that is, it is the same in all directions at a point.

We have also seen, in Fig. 245, that the pressure is the same

all over any horizontal plane [1]; hence the pressure can be transmitted without loss from one point in a horizontal plane to another point in the same plane. In Fig. 250, if the two cylinders are connected as shown, the above statement shows that the pressure per square inch over the section AB must be the same as that over the section CD. Similarly, the pressure at E is the same as that at F. The pressure per square inch at E = the pressure per square inch at B + the increase of pressure due to the head X ft. of water.

As in ordinary machinery X is generally small, say only a few feet, and the pressure per square inch from 500 to 3000 lbs. per square inch, we do not trouble to include the small extra pressure due to X ft. of water, and hence the statement that **the pressure per square inch is the same throughout the fluid connections in hydraulic machinery.**

Example.—In Fig. 251 are shown two cylinders, B and C, fitted with pistons, which are assumed to be watertight, and at the same

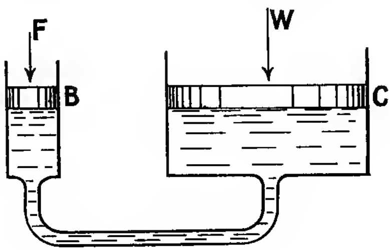

FIG. 251.

time free from friction. If B is 2 in. in diameter, and C 10 in. in diameter, find W, if F = 100 lbs.

From the above statement in heavy type, we know that the pressure per square inch throughout the whole of the connections between B and C is constant (if we neglect the small effect of the weight of the water). Call this pressure p lbs. per square inch.

[1] This is so, because the pressure at a point is proportional to its depth below the free surface, which depth is constant in the case of a horizontal plane, and consequently the pressure is constant over that plane.

For the equilibrium of the small piston (neglecting its own weight)—

F − total pressure of water on under side = 0

$$\text{or } F = p \times \frac{\pi}{4} \times 2^2$$

$$\text{then } p = \frac{F}{\pi} \text{ lbs. per square inch}$$

Again, for the equilibrium of the large piston—

W − upward pressure of water = 0

$$\text{or } W = p \times \frac{\pi}{4} \times 10^2$$

$$\text{from which } p = \frac{4W}{100\pi}$$

Equating the two equivalents of p, we get—

$$\frac{F}{\pi} = \frac{4W}{100\pi}$$

$$\text{or } W = \frac{100F}{4} = 25F = 2500 \text{ lbs.}$$

If in the above problem 45 per cent. of the total work was done against friction, what would then have been the value of W?

The part of F which was useful was—

$$\left(\frac{100 - 45}{100}\right)F = 0{\cdot}55 \times 100 = 55 \text{ lbs.}$$

Substituting this value for F above, we get—

$$W = 25F = 25 \times 55 = 1375 \text{ lbs.}$$

The Hydraulic Press in its simplest form is shown in Fig. 252. The press proper is shown in section on the left, but the pump on the right of the figure is required to work it, and should be included with any description of the press.

The plunger P of the pump is forced up and down by means of the hand-lever H. During the upward stroke of the plunger P a partial vacuum is formed in the chamber A, and the pressure of the atmosphere on the surface of the water in the tank forces it up the suction pipe K, past the suction-valve S, into the

chamber A. During the down stroke of the plunger, pressure is put on the water by it, which pressure is conveyed throughout the water and lifts the delivery-valve D and passes along the connecting-pipe M to the cylinder of the press. Here the

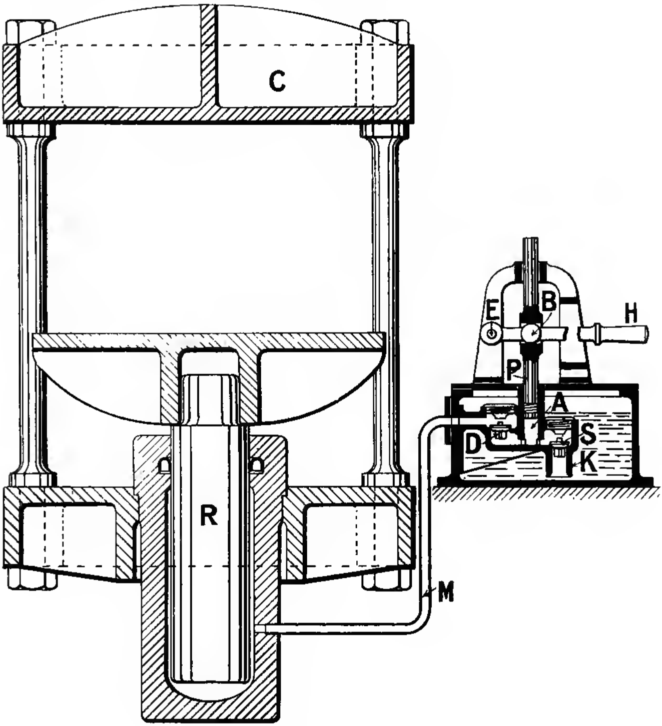

FIG. 252.—Hydraulic press and hand-pump.

pressure in the water produced by the pump acts on the base of the ram R and forces it upward.

The pump-plunger corresponds to the small piston in Fig. 251, while the press-ram corresponds to the larger piston.

Moderate size pump-plungers, or small rams, may be made watertight by means of leather or rubber packing, C (Fig. 253),

in the form of a cup, on which the water presses, forcing it against the pump-barrel and preventing the escape of any water. The greater the pressure, the harder is the packing pressed against the barrel. P is a plate for the purpose of keeping the packing in proper working position. In Fig. 252, the packing consists of a gasket of leather or other soft packing, wound in the groove turned in the plunger. The press-ram is made watertight by the ∩ leather ring U in Fig. 254; the water-pressure on the sides of the ∩ forcing them against the neighbouring metal, which prevents the escape of water. Sometimes a metal ∩ is placed inside the ∩ leather for the purpose of keeping the wearing surfaces in their working position. There is also a bye-pass from the pipe M back to the suction-tank, this being fitted with a valve to allow the ram R to return to its lowest position after being lifted.

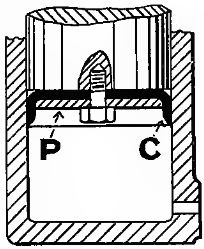

FIG. 253.—Cup-leather packing.

Example.—In Fig. 252 assume that HB is 25 in., and BE

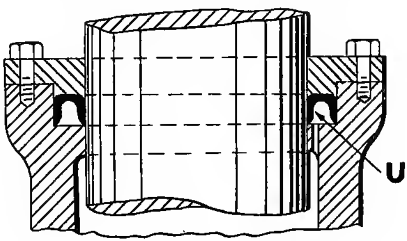

FIG. 254.—∩-leather packing.

$2\frac{1}{2}$ in. Also that the diameter of the plunger P is 0·8 in., while that of the ram R is 10 in. If the average pressure exerted on the hand-lever at H is 30 lbs., calculate the pressure exerted by the ram R on the material above it, (1) assuming no friction, and (2) assuming the efficiency of the machine is 35 per cent.

Consider the equilibrium of the hand-lever (Fig. 255). The sum of the moments round E must be zero, or—

$$(30 \times 27{\cdot}5) - (x \times 2{\cdot}5) = 0$$
$$\text{or } x = 330 \text{ lbs.}$$

where x is the resistance of the plunger.

Now consider the equilibrium of the plunger.

The pressure of the lever upon it (= 330 lbs.) is balanced by the upward pressure of the water on it $= \frac{\pi}{4} \times 0{\cdot}8^2 \times p$ lbs., where p = pressure per square inch of the water in the pump barrel.

Solving for p, we get—

$$p = \frac{330}{\frac{\pi}{4} \times 0{\cdot}64} \text{ lbs. per square inch}$$

This pressure is conveyed through the liquid connections to the

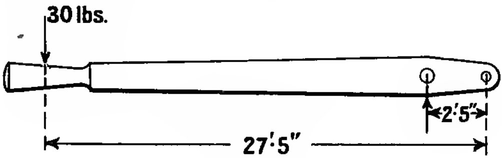

FIG. 255.

press cylinder. For the equilibrium of the ram R, we must have the upward resultant pressure of the water on it *minus* the resistance offered to the ram by the material to be pressed = zero;

$$\text{or } p \times \frac{\pi}{4} \times 10^2 = \text{resistance offered to ram by material}$$
$$= \text{force exerted by ram on material}$$
$$= \text{W say}$$

Substituting for p from the equation above, we get—

$$W = \frac{330}{\frac{\pi}{4} \times 0{\cdot}64} \times \frac{\pi}{4} \times 100$$
$$= 51{,}750 \text{ lbs.}$$

Had the efficiency been 35 per cent., the value of W would have een 0·35 of 51,750 = 18,200 lbs. nearly.

The Hydraulic Jack is an extremely useful machine by which heavy weights may be lifted while it is being worked by

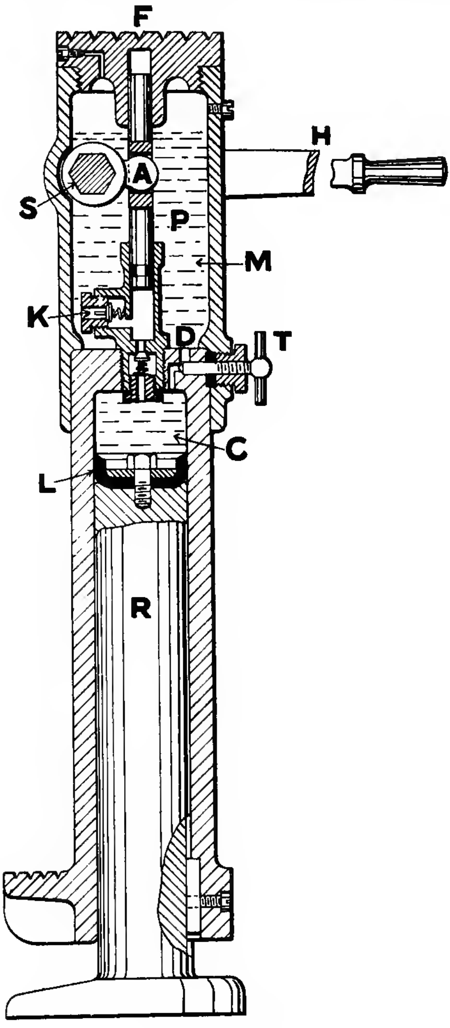

FIG. 256.—Hydraulic jack.

one man. It is in reality the pump and press of Fig. 252, very neatly and compactly arranged in a portable form. It is shown in section in Fig. 256. The plunger P is made to reciprocate by means of the hand-lever H fixed to the hexagonal spindle S, on which is threaded the small arm A, which fits into a slot in the plunger. The suction valve is shown at K, and the delivery

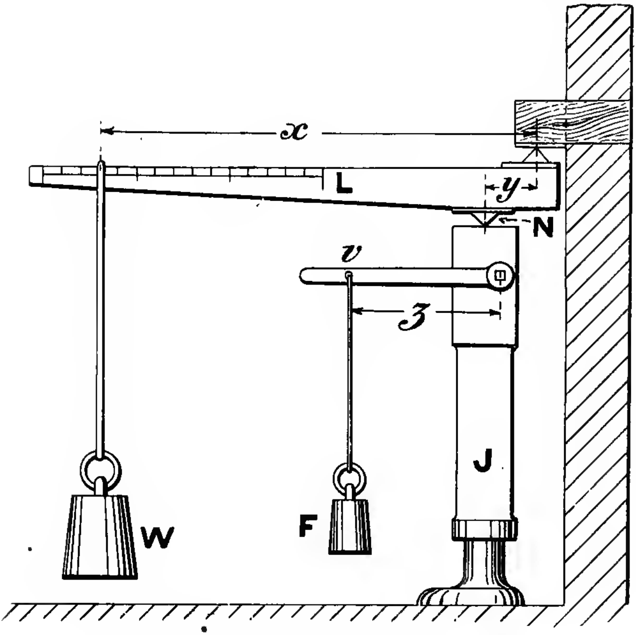

FIG. 257.—Arrangement of hydraulic jack for experiment.

valve at D. The ram R, over which slides the cylinder C, is made watertight by means of the cup leather L. As the hand-lever is worked up and down, the water in the cistern M is pumped into the cylinder C, thus lifting the whole of the machine (except the ram) together with the load on F. When it is desired to lower the load, the little thumbscrew T is withdrawn a small amount, and the pressure of the load F forces the water from the cylinder C back into the cistern M.

An experiment may be carried out with this jack in the following manner. The maximum load which one of these jacks will lift is over a couple of tons, even if it be a small one, and consequently it will be necessary to put on the load in some artificial way. A lever, L (Fig. 257), fitted with a couple of knife-edges, is used for this purpose, and a strong block of wood is cemented into a thick wall with plenty of weight above it. The knife-edge N bears upon the top of the jack while a hole is drilled in the hand-lever at v, through which a small cord may be passed to support the weight F, which here plays the part of driving effort. Another weight W is suspended from the lever L by a single cord, its position being noted on the scale of inches upon the lever. The actual load upon the top of the jack is—

$$\left(W \times \frac{x}{y}\right) + \text{the weight of the lever} \times \frac{\text{distance of its centre of gravity from right knife-edge}}{y}$$

Attach a weight F lbs. to the cord at v, and adjust another weight W along the lever L until the weight F will only just slowly move the hand-lever downwards. Insert the necessary numbers in the following table, and repeat with other weights for F :—

EXPERIMENT WITH A HYDRAULIC JACK.

Date, , . Observer, , .

Distance y = in.

z = in.

Distance of centre of gravity of lever from right knife-edge =

Weight of lever, lbs. =

Corresponding pressure on top of jack =

Driving effort, F lbs.	Weight W, lbs.	Distance x in.	Total load on top of jack. lbs.	Mechanical advantage.	Mechanical efficiency.

The mechanical advantage will equal $\frac{\text{total load on top of jack}}{F}$

The velocity ratio can be found by measuring the distance of the outer end of the lever L from the floor, and then working the hand-lever up and down *through its full travel* a number of times (say 50), and then measuring the height of the lever end from the floor again.

Also measure the movement of the point *v* in the hand-lever during one downward motion, and multiply by the number of downward movements.

This latter gives the distance moved by the driving end, while the following end (top of jack) will move in the same time through a distance equal to—

$$\text{Movement of end of lever} \times \frac{Z}{\text{length of lever}}$$

Plot the driving effort, mechanical efficiency, and mechanical advantage on a total-load base, and deduce the equations connecting them with the load, including friction.

The Hydraulic Accumulator.—When a small pump is working a press, whether large or small, the press has to be stopped repeatedly, the period of stoppage varying from a few seconds to some minutes.

It is undesirable and inconvenient to start and stop the engine or other motor for driving the pump every time the press is started and stopped, and, further, it is generally very desirable to work the press at a speed much greater than would be possible if it were connected direct to the pump. These conditions necessitate a sort of storage reservoir for the water under pressure, somewhere between the pump and the press, and it takes the form of a ram working in a cylinder, the top of the ram being loaded with a dead weight.

A section of a small accumulator is shown in Fig. 258. A number of annular cast-iron weights W are slung to the cross-head C by long rods, R. Each circle or ring of weights is divided into four parts for convenience in handling, but the bottom ring is in one piece. When the press or other hydraulic

machine is stopped, the pump simply forces water into the accumulator cylinder, lifting the ram with its load. When it is desired to operate the press, a valve is opened and the water

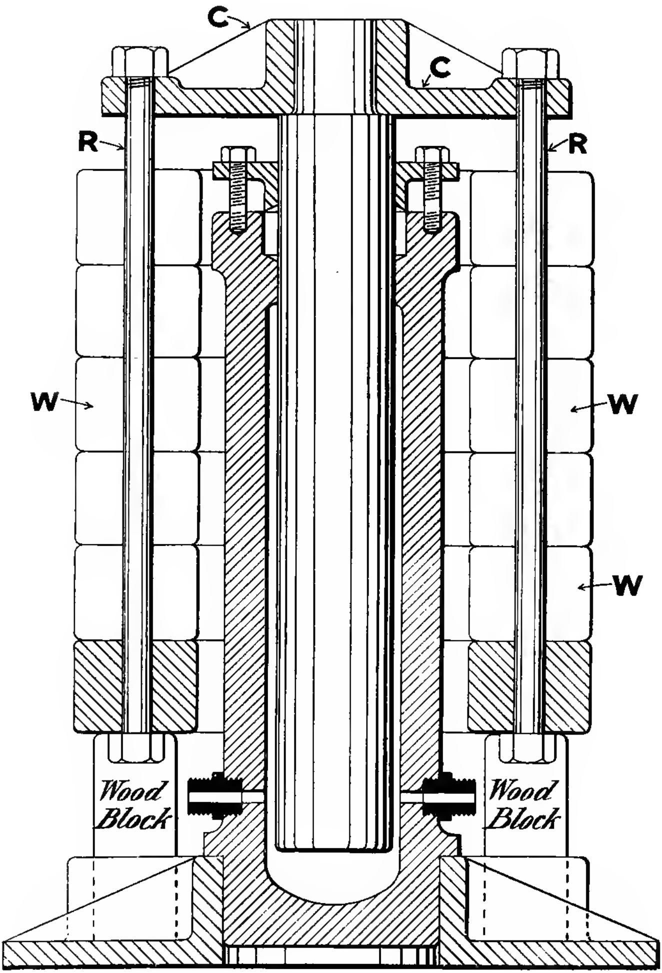

FIG. 258.—Hydraulic accumulator.

under pressure is admitted to the press. If the pump does not work fast enough to supply the press with water, the accumulator ram descends and makes up the deficit, while if the pump works faster than is necessary to supply the press, the accumulator ram is lifted. To prevent any damage to the accumulator, it is so arranged that as it approaches the top of its stroke it strikes a lever which opens a loaded safety-valve connected to the accumulator pipes, and thus permits some water to flow out of the ram cylinder. In large installations the above lever first closes the steam-valve on the pumping-engine, so as to waste as little water as possible.

Example.—A hydraulic accumulator ram is 9 in. diameter and 12 ft. long. It is supplied with water by a single-acting pump, x in. in diameter, and $4x$ in. stroke, and it makes 40 double strokes per minute.

If the pump will supply enough water in 10 minutes to lift the accumulator ram through its length, what is x?

Also, how many foot-pounds of work are stored in the accumulator (neglecting the weight of the water), and what must be the load on the top of the ram if the water-pressure is 706 lbs. per square inch?

$$\left.\begin{array}{l}\text{Volume of water entering ac-}\\ \quad\text{cumulator cylinder during}\\ \quad\text{one stroke of the ram}\end{array}\right\} = \text{length} \times \text{sectional area of ram}$$

$$= 144\frac{\pi}{4} \times 81 \text{ cub. in.}$$

$$\left.\begin{array}{l}\text{volume of water delivered}\\ \quad\text{by pump in one stroke}\end{array}\right\} = \frac{\pi}{4} \times x^2 \times 4x \text{ cub. in.}$$

$$\text{volume per minute} = \pi x^3 \times \text{number of useful strokes per minute}$$

As one stroke only out of every two actually delivers water to the accumulator, there will be 40 useful strokes per minute; hence—

$$\text{Volume of water delivered in 10 minutes} = 10 \times 40 \times \pi x^3$$

and as this equals the volume entering the accumulator during one stroke of the ram—

$$400\pi x^3 = 144 \times \frac{\pi}{4} \times 81$$

$$\therefore x^3 = 7{\cdot}3$$

$$\text{and } x = 1{\cdot}94 \text{ in.}$$

$$\left.\begin{array}{l}\text{Work done in lifting accumulator}\\ \quad\text{ram through 12 ft.}\end{array}\right\} = 700 \times \frac{\pi}{4} \times 81 \times 12$$

$$= 622{,}600 \text{ foot-lbs.}$$

$$\text{Load on ram (neglecting friction)} = \text{total pressure of water on lower end of ram}$$

$$= 700 \times \frac{\pi}{4} \times 81 = 44{,}550 \text{ lbs.}$$

$$= 19{\cdot}9 \text{ tons}$$

If the water were used by some hydraulic machine so quickly that the ram descended the 12 ft. in 30 secs., the rate at which the machine was doing work was—

$$622{,}600 \times 2 \text{ ft.-lbs. per minute}$$

$$\text{or } \frac{622{,}600 \times 2}{33{,}000} = 37{\cdot}7 \text{ horse-power}$$

The horse-power required to drive the pump (neglecting friction)

$$= \frac{\text{total press. on piston} \times \text{length of stroke} \times \text{no. useful strokes per min.}}{33{,}000}$$

$$= 700 \times \frac{\pi}{4} \times 1{\cdot}94^2 \times \frac{4 \times 1{\cdot}94}{12} \times \frac{40}{33{,}000}$$

$$= 1{\cdot}89$$

This calculation shows how the accumulator is capable of doing work very rapidly for a short time, while the driving pump is supplying the energy much more slowly, in this case at one-twentieth of the rate at which the accumulator enables the hydraulic machine to work. It must not be forgotten that the accumulator requires a period twenty times as great as that occupied by the ram in descending, in which to store the next charge of energy, so that no energy is created by the accumulator. It only allows the energy to be stored slowly, and given out rapidly if necessary; and in this way permits of the use of a *small* pump and motor to drive it, where otherwise a large one would be necessary.

Experiment to measure the Efficiency of a Pump.—The pump-shaft must be driven in such a way that the rate at

which energy is transmitted to it can be measured in horse-power. This can be done by an electric motor, or a transmission dynamometer, such as that described on page 261. We will call the rate of transmission of energy to the pump-shaft the *pump horse-power*, and the rate of transmission to the *water* in the pump, the *water horse-power.*

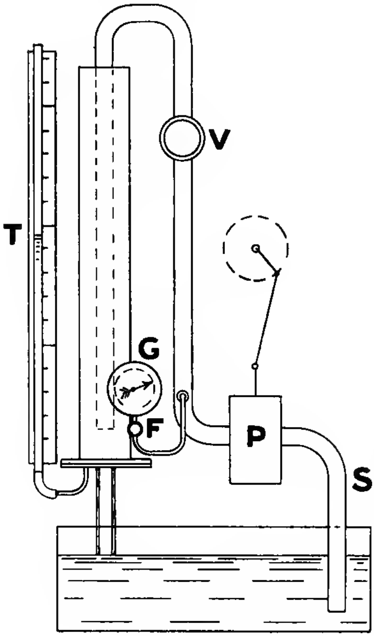

FIG. 259.

The efficiency of the pump must then be—

$$\frac{\text{water horse-power}}{\text{pump horse-power}}$$

This is the quantity we want to find for different lifts.

The pump is fitted up (Fig. 259) so that the weight of water pumped per minute can be easily measured on the scale at T, and the pressure against which the water is delivered is indicated by the gauge G. This pressure can be converted into feet of water by dividing the gauge pressure by 0·43. Call this h_d, the delivery head, and measure the distance from the centre of the gauge [1] to the water-level in the suction-tank. Call this h_s, the suction head. Then the total height through which the water is lifted is $h_s + h_d$; and the work done per minute = weight of water delivered per minute × total lift in feet = $W(h_s + h_d)$ foot-pounds.

[1] The gauge is here supposed to be at the same level as the centre of the pump cylinder.

$$\text{Water horse-power} = \frac{W(h_s + h_d)}{33{,}000}$$

The pressure measured by the gauge G is produced by partly closing the valve V. *It should never be quite closed.* If the gauge pointer oscillates, it should be checked by partly closing the small valve F. **Continued oscillation of the pointer will spoil the gauge.**

RECORD OF EXPERIMENT ON A PUMP.

Date, . Observer, .

Object of Experiment.—To determine the efficiency of the pump at different lifts and speeds.

Kind and dimensions of pump, .
Suction head, ft. .

TABLE OF WATER QUANTITIES.

Time.	Delivery head.		Weight of water delivered per minute. W lbs.	Water horse-power.
	Gauge-pressure, lbs. per sq. in.	Feet of water.		

Results—Mean total lift in feet =
Mean weight of water pumped per min., lbs. =
Mean water horse-power =
Mechanical efficiency =

The Density of any material is the mass of 1 cub. ft. in English measure, and is expressed as so many pounds per cubic foot. In the metric system, it is the mass of 1 cub. centimetre of the material, and is expressed as so many grammes per cubic centimetre.

The density of a substance may be obtained by weighing a known volume of the substance; for instance, assume a piece

of wood 1 ft. long, 3 in. wide, and 6 in. deep is weighed, and its weight is 4·2 lbs., its volume is $1 \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ cub. ft., and therefore 1 cub. ft. will weigh $4·2 \times 8 = 33·6$ lbs. Its density is then 33·6 lbs. per cubic foot.

Example.—A piece of iron 5 in. long and 2 in. diameter weighs 4·25 lbs.: what is its density?

$$\text{Volume of piece of iron} = 5 \times 2^2 \times \frac{\pi}{4} \text{ cub. in.}$$

$$= \frac{5\pi}{1728} \text{ cub. ft.}$$

Then, number of cubic feet × weight of 1 cub. ft. = 4·25 lbs.

$$\text{or } \frac{5\pi}{1728} \times \text{density} = 4·25$$

$$\text{and density} = 4·25 \times \frac{1728}{5\pi}$$

$$= 465 \text{ lbs. per cubic foot}$$

It is seldom convenient to weigh a piece of material of a regular shape, such that its volume can be readily calculated. A more general method is that of **weighing in air and water.**

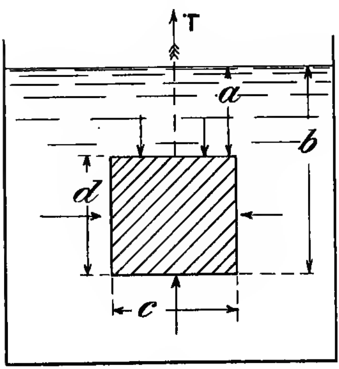

FIG. 260.

Consider the rectangular solid immersed in a liquid, Fig. 260, and supported in it by a fine cord whose tension is T. The length, width, and thickness of the solid are respectively *l*, *c*, and *d* ft. The pressure on the left face is balanced by that on the right face. Similarly, the pressure on the front face is balanced by that on the back face. These *balanced* pressures can be left out of any calculation respecting the resultant pressure or the equilibrium of the solid. If the solid

is in equilibrium, the sum of the vertical component of the forces acting on it must be zero; or—

Upward pressure on base of solid − weight of solid W lbs. + tension of cord T lbs. − pressure of liquid on top of solid = 0

The pressure on base of solid = $(l \times c) \times b \times \delta$

where δ = density of liquid in pounds per cubic foot; also—

$$\text{Pressure on top of solid} = (l \times c) \times a \times \delta$$

Inserting in the above equation, we get—

$$l \times c \times b \times \delta - W + T - l \times c \times a \times \delta = 0$$

$$\text{or } lc\delta(b - a) = W - T$$

$$\text{But } b - a = d$$

$$\text{and } l \times c \times d = \text{volume of solid}$$

hence—

$$\text{Volume of solid} \times \text{density of liquid} = W - T$$

The left side of this equation is equal to the mass of liquid which would exactly fill the same space as the solid, or, as we say, it is the mass of the liquid displaced by the solid.

The right side of the equation is the apparent loss of weight of the solid while immersed in the liquid; that is, it is the portion of the weight of the solid which is supported by the liquid. Hence we may state—

The vertical supporting force offered by a liquid to a solid immersed in it equals the weight of a quantity of liquid equal in volume to that of the solid.[1]

[1] This may be shown more easily as follows:—
The side pressures balance each other.

The resultant pressure of the liquid on the solid } = pressure on base − pressure on top

$$= l \times c \times b \times \delta - l \times c \times a \times \delta$$
$$= lc\delta(b - a) = lc\delta d$$

Dividing the above equation by the density of the liquid, we get—

$$\text{Volume of solid} = \frac{W - T}{\text{density of liquid}}$$

Also by definition—

$$\text{Weight of solid} = \text{volume of solid} \times \text{density of solid}$$

$$\text{or } W = \frac{W - T}{\text{density of liquid}} \times \text{density of solid}$$

that is—

$$\mathbf{\frac{W}{W - T} \times density\ of\ liquid = density\ of\ solid}$$

If water (density 62·5 lbs. per cubic foot) is the liquid used,

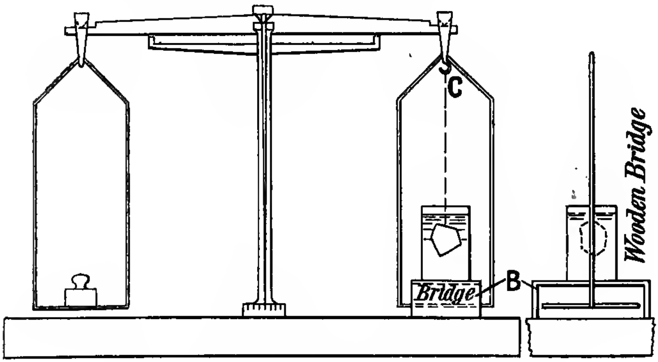

FIG. 261.—Chemical balance arranged for determining density.

then the density of a solid may be found by weighing the solid first in air and then in water, and inserting the weights

---

= volume of solid × density of liquid
= mass of a quantity of liquid whose volume is equal to that of the solid

The resultant pressure is upward, because the pressure on the base is greater than on the top. Hence the support offered by a liquid to a solid immersed in it equals the weight of an equal volume of liquid.

in the above equation. This is the commonest method of finding the density of a solid heavier than water.

The density of the solid having been obtained, the density of another liquid may be obtained by weighing the solid when it is immersed in this other liquid; and inserting the numbers so obtained in the above equation.

The weighing in air and water can best be done with a chemical balance, but approximate results can be obtained by means of a good spring-balance when a large piece of solid is used. Fig. 261 shows the method of using the ordinary chemical balance, where B is a light wooden bridge to support a beaker for weighing in water. The substance is first weighed by attaching it by a fine thread to the hook C, after which the beaker of water is placed upon the bridge, and the body weighed in water as shown in the figure.

Great care should be exercised in removing with a camel-hair brush any air-bubbles which may adhere to the solid, and it is better to use water which has been previously boiled, so that it does not contain any dissolved air.

A quicker, though more crude, method is to use a much larger piece of material and a spring-balance. Here 20, 30, or 40 lbs. of material may well be used, whereas only a small fraction of a pound would be used with the chemical balance.

Nicholson's Hydrometer may be used for finding density, and the method of using it depends upon the fact expressed on page 307, that *the support offered by a liquid to a solid equals the weight of an equal volume of liquid.* The hydrometer consists of a hollow water-tight vessel, H, Fig. 262, fixed upon the wire stem, S, which carries two pans, P and Q, the whole floating in a vessel of water. There is a notch, M, cut in the wire S.

The hydrometer, without anything in the pans, floats as shown in Fig. 262. Now add standard weights, W, to the upper pan P until the mark M has sunk to the surface of the water (Fig. 263). Then, because the support offered by the water = weight of a volume of water equal to that of the instrument up to M, and as the instrument is sunk every time to M, the weight of instrument + total weight added to pans =

support of water, which is constant because volume of instrument immersed is constant. Also the weight of the instrument is constant; hence we can write the above equation as—

$$\begin{aligned}\text{Total weight added to pans} &= \text{support of water} - \text{weight of instrument}\\ &= \text{constant}\\ &= W\end{aligned}$$

Remove W and place the body in the upper pan P, adding weights, W_u, until the instrument sinks to M again.

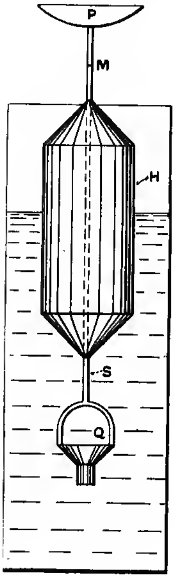

FIG. 262.

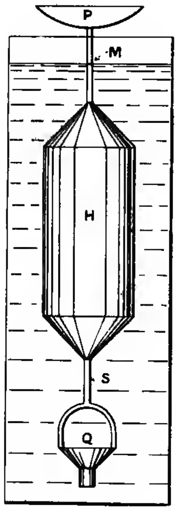

FIG. 263.

Nicholson's hydrometer.

As the total weight added to pans = W, we have—

$$\text{Weight of body in air} + W_u' = W$$
$$\text{and weight of body in air} = W - W_u$$

Now remove W_u, and place body in lower pan, Q. Add weights, W_l, to the upper pan to bring the instrument down to M again. Then—

$$\text{Weight of body in water} + W_l = W$$
$$\text{and weight of body in water} = W - W_l$$

But from page **308** we get—

$$\frac{\text{weight of body in air}}{\text{weight of body in air} - \text{weight of body in water}} \times \text{density of water} = \text{density of body}$$

$$= \frac{W - W_u}{W - W_u - (W - W_l)} \times \text{density of water}$$

$$= \frac{W - W_u}{W_l - W_u} \times \text{density of water}$$

Example.—It was found that 15 grms. were required to sink a Nicholson's hydrometer to the mark M in water. It required 5 grms. to sink it with the body in the upper pan, and 8 grms. to sink it with the body in the lower pan. What was the density of the body?

$$\text{Density of body} = \text{density of water} \times \frac{15 - 5}{8 - 5}$$
$$= 1 \times \tfrac{10}{3}$$
$$= 3{\cdot}33 \text{ grms. per cubic centimetre}$$

The Common Hydrometer consists of a glass bulb, B (Fig. 264), having a smaller bulb at one end weighted to cause stability while floating, and a hollow stem protruding from the other side, the stem being uniform in cross-section throughout its length.

The support offered by a liquid to a solid equals the weight of liquid which would occupy the same volume as that part of the solid below the surface of the liquid.

In Fig. 264, the weight of the hydrometer must equal the weight of liquid which would occupy the volume of that part of the instrument below the surface AA.

Let V = volume of instrument below AA;
and δ = density of liquid in which it floats.

$$\text{Then } V \times \delta = \text{weight of instrument} = \text{a constant}$$

Let suffix $_1$ refer to one liquid, say water, and suffix $_2$ refer to another liquid. Then, as $V \times \delta$ is always constant for all liquids—

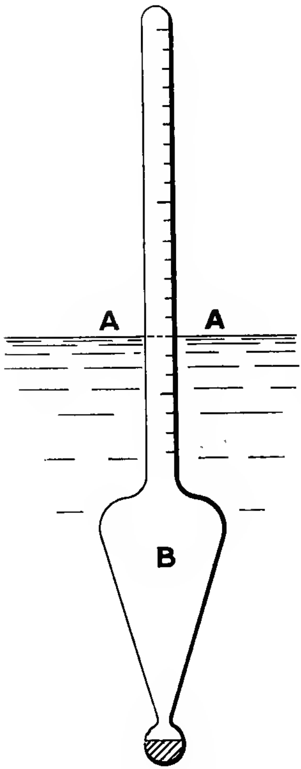

FIG. 264.—The common hydrometer.

$$V_1 \times \delta_1 = V_2 \times \delta_2$$

$$\text{and } \delta_2 = \delta_1 \times \frac{V_1}{V_2}$$

From this equation we see that the density δ_2 of the liquid is inversely proportional to the volume of the instrument under the liquid. Hence the lighter the liquid the deeper will the hydrometer sink, and thus a scale of densities can be inscribed on the stem of the instrument.

Let a = sectional area of the stem;
and L = length of unimmersed part of stem;
also V = volume of complete hydrometer.

$$\text{Then } V_1 = V - aL_1$$

$$\text{also } V_2 = V - aL_2$$

$$\text{and } \delta_2 = \delta_1 \times \frac{V - aL_1}{V - aL_2}$$

The instrument can be floated in water to get L_1, the sectional area a can be obtained from the diameter of the stem, and V can be found by immersing in a graduated burette or by other more accurate methods. L_2 can be measured in the same way that L was measured, and δ_2 can then be

calculated. As a rule these hydrometers have their stems graduated to read $\frac{V_1}{V_2}$ direct. In many instruments the decimal places are omitted, and numbers such as 700, 950, 1244 really mean 0·7, 0·95 and 1·244 respectively. The density in the English system is obtained by multiplying these numbers by 62·5. In the metric system they represent density, because 1 cub. cm. of water weighs 1 grm.

Specific Gravity.—We can write the previous equation—

$$\delta_2 = \delta_1 \times \frac{V_1}{V_2}$$

$$\text{as } \frac{\delta_2}{\delta_1} = \frac{V_1}{V_2}$$

The left side is the ratio of the density of a substance to that of water, and is called the specific gravity of the substance. Hence the numbers on the hydrometer stem are specific gravities × 1000.

We may then define specific gravity as—

$$\frac{\text{weight of a body}}{\text{weight of an equal volume of water}}$$

In the metric system the density is the same as the specific gravity.

The Density Bottle is useful in many cases for the determination of density. It is shown in Fig. 265, and consists of a small glass bottle containing a glass stopper ground in position and having a fine hole through it.

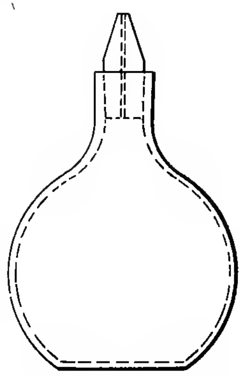

Fig. 265.—The density bottle.

The bottle can be filled with a liquid and the stopper inserted, when the superfluous liquid will be expelled through the hole in the stopper. If the bottle were not covered in, it would be uncertain when it was *exactly* full, as a liquid will rise slightly above the top edge of a vessel

without overflowing. The stopper also permits finely divided substances being inserted.

The density of a liquid can be found by means of the bottle. Clean and dry the bottle and weigh it. Then fill it with water and weigh it again. Dry it and fill it with the liquid whose density is required, and weigh it again.

Let weight of bottle $= w$, and weight of bottle + water $= W_w$.

Then weight of water $= W_w - w$

But weight of water = volume × density of water

$$\therefore \text{volume} = \frac{\text{weight of water}}{\text{density of water}}$$

$$= \frac{W_w - w}{\text{density of water}}$$

Let weight of bottle + liquid $= W_l$.

Then weight of liquid $= W_l - w$

and weight = volume × density

$$\text{or } W_l - w = \frac{W_w - W}{\text{density of water}} \times \text{density of liquid}$$

From which we get—

$$\text{density of liquid} = \text{density of water} \times \frac{W_l - w}{W_w - w}$$

The density of a solid insoluble in water is found in the following manner by means of the bottle:—

$$\text{weight of bottle} = w$$
$$\text{weight of solid} = W_s$$
$$\text{weight of bottle full of water} = W_w$$
$$\text{then weight of water in bottle} = W_w - w$$

Put the solid in the bottle, and add water until the bottle is quite full, and weigh again. Let this weight be W. Then—

$$\left.\begin{array}{l}\text{Weight of water required to occupy volume}\\ \quad\text{of bottle less the volume of solid}\end{array}\right\} = W - W_s - w$$

As the weight of water (full) $= W_w - w$, and weight of water which will occupy same volume as solid $= (W_w - w) - (W - W_s - w) = W_w - W + W_s$.

Again, as weight = volume × density—

$$\text{Volume of solid} = \frac{W_w - W + W_s}{\text{density of water}}$$

$$\text{also weight of solid} = \text{volume of solid} \times \text{density}$$

$$\text{or density of solid} = \frac{\text{weight of solid}}{\text{volume of solid}}$$

$$= \frac{W_s}{W_w - W + W_s} \times \text{density of water}$$

Example.—Find the density of a solid from the following observations:—

weight of solid = 70 grms.
weight of bottle empty = 5 grms.
weight of bottle quite full of water = 55 grms.
weight of bottle + solid + water make up = 112 grms.

Substituting from these observations in the above equation, we have—

$$\text{Density of solid} = \text{density of water} \times \frac{70}{55 - 112 + 70}$$

$$= 62{\cdot}5 \times 5{\cdot}35$$

$$= 334 \text{ lbs. per cubic foot}$$

In the metric system the density would be 5·35 grms. per cubic centimetre, which is the specific gravity also.

Density of a Solid which will float in Water.—This may be found most easily by a chemical balance, as in Fig. 261, but with the addition of a *sinker* for the purposes of completely immersing the solid. Let us take an example—

Weight of solid = 12 grms.
" sinker = 40 grms.
" sinker in water = 35 grms.
" sinker + solid in water = 33·5 grms.

The weight of water displaced by sinker $= 40 - 35 = 5$ grms.

$$\left.\begin{array}{r}\text{Also weight of water dis-}\\ \text{placed by sinker and solid}\end{array}\right\} = \begin{array}{l}\text{weight of (solid + sinker)} - \text{weight} \\ \quad \text{of (solid + sinker) in water}\end{array}$$

$$= 12 + 40 - 33{\cdot}5 = 18{\cdot}5 \text{ grms.}$$

$$\left.\begin{array}{l}\text{therefore weight of water}\\ \text{displaced by solid}\end{array}\right\} = 18{\cdot}5 - 5 = 13{\cdot}5 \text{ grms.}$$

From page 308 we have—

$$\left.\begin{array}{l}\text{Density of}\\ \text{solid}\end{array}\right\} = \text{density of water} \times \frac{\text{weight of solid}}{\text{wt. of water displaced by solid}}$$

$$= 1 \times \frac{12}{13{\cdot}5}$$

$$= 0{\cdot}888 \text{ grm. per cubic centimetre}$$

$$\text{or } = 55{\cdot}3 \text{ lbs. per cubic foot}$$

Centre of Pressure.—The point of application of the resultant fluid pressure on a surface is called the centre of

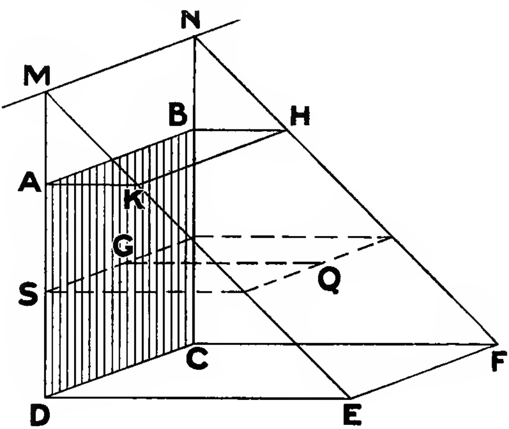

FIG. 266.

pressure. Let the rectangle ABCD (Fig. 266) be the surface in question, in which the edge AB is below the free surface of the liquid. Assume for the sake of simplicity that the edge AB is parallel to the free surface MN of the liquid.

We know from page 285 that the pressure at a point in a liquid is proportional to its depth below the free surface; hence the pressure at the point D is proportional to MD. Set out DE perpendicular to the surface at D to represent the pressure at D to some scale. For simplicity, set out DE = DM. Do the same at the points A, B, and C, and we get the lines AK, BH, and CF, representing the pressures at those points. This may be repeated at a number of other points, and it will be found that the ends of the perpendiculars will all lie in the plane EMNE, because AK was made proportional to AM, and DE proportional to DM; hence we have two triangles, MAK and MDE, in which the sides MA and AK are proportional to the two sides MD and DE; and, further, the angles at A and D are right angles, therefore the two triangles are similar to their corresponding angles equal (see Introduction)—that is, the angle at M in one triangle equals the angle at M in the other, and therefore the side MK is coincident with the side ME, and consequently the line MKE is straight, and therefore the surface KHFE is plane.

It is also convenient to notice that—

$$\left.\begin{array}{l}\text{The total pressure}\\ \text{on ABCD}\end{array}\right\} = \text{area of ABCD} \times \text{depth of the centre of gravity G of ABCD below MN} \times \text{density } \delta \text{ of the fluid}$$

$$= \text{shaded area} \times \text{MS} \times \delta$$

$$= \text{shaded area} \times \text{GQ} \times \delta$$

Now turn the figure 266 round through 90° in anti-clockwise direction, as shown in Fig. 267. Only that portion standing out from the shaded surface has been reproduced. The line GQ is the average height of the solid shown, and consequently the right-hand side of the above equation equals the weight of the solid in Fig. 267, resting on the shaded surface. The point about which this solid will balance is a point through which the resultant pressure line of action must pass (see chapter dealing with centre of gravity and parallel forces). The resultant pressure must act perpendicular to the surface, hence find the centre of gravity g of the solid in Fig.

267, and draw through g a line perpendicular to the shaded surface. The point X, where this line meets the surface, is the centre of pressure.

Rotating the figure back again, we get X in its true position. This is the point of application of the resultant fluid pressure on the shaded area.

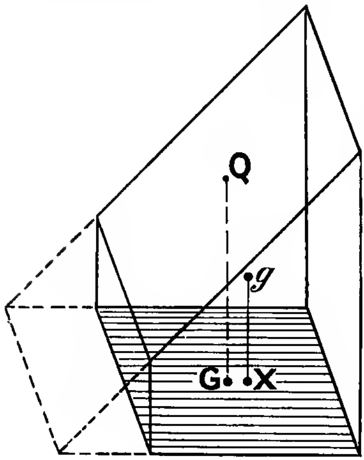

FIG. 267.

We may apply the above reasoning to a simple rectangle with one edge in the free surface, but the more difficult cases are outside the scope of this work.

In Fig. 268 the rectangle ABCD has the edge AB in the free surface of the liquid. The centre of pressure, X, will be the point in which a line perpendicular to ABCD drawn through the centre of gravity of the triangular prism meets the surface ABCD. Take a side view of the prism, Fig. 268. The centre of gravity of the triangle is at G, one-third of the length of the dotted line up from the base. Through this point draw a line perpendicular to the shaded surface, meeting it in X. This is the centre of pressure, EX being two-thirds of AD.

In general, the centre of pressure on a plane area is found in the following manner :—

Draw perpendiculars to the surface (at the outline of the surface) whose lengths represent the pressures of the liquid at their feet. Find the centre of gravity of the solid enclosed between the perpendiculars, and draw through this centre of gravity a perpendicular to the surface in question. The point in which it meets the surface is the centre of pressure.

The Flow of Liquids.—The first case we shall consider is the flow of a liquid (water) through a hole in the side or

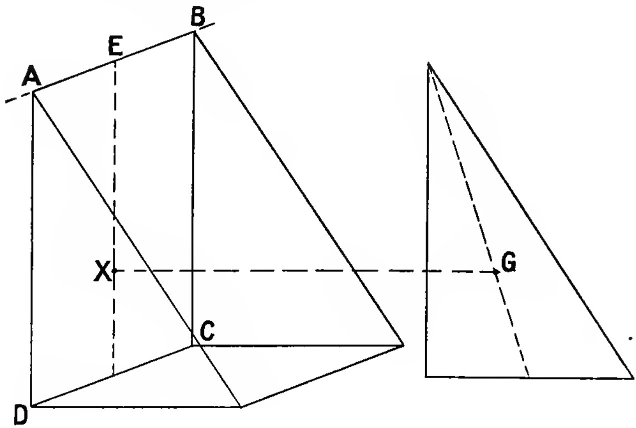

Fig. 268.

base of a tank. It will be noticed in Fig. 269 that the diameter of the stream is not so great as the diameter of the hole or orifice—that is, the stream has contracted in flowing through the orifice. The amount of contraction can be roughly measured with a pair of calipers, when it will be found that the *sectional area* of the stream at a little distance from the orifice is about 0·65 times that of the orifice. This number is called the *coefficient of contraction.*

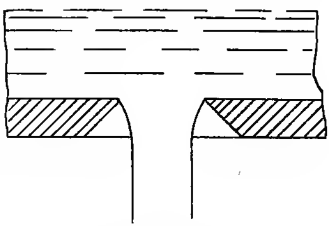

Fig. 269.—Showing the shape of a jet of liquid issuing from a sharp-edged orifice.

If the amount of water flowing from the orifice is caught in a measuring tank or weighed, the volume flowing out per second can be calculated. This will vary according to the *head* or height of water above the orifice. It is necessary to

find by experiment the relation between the *head* and the discharge per second. This is done by catching the water discharged during a number of minutes with a definite and constant head and calculating the discharge per second. This is repeated for a number of different heads, and a table filled up something like that below.

RECORD OF EXPERIMENT ON THE DISCHARGE FROM A SIMPLE SHARP-EDGED ORIFICE.

Date, Observer,

Object—To determine the relation between the head and the discharge per second.

Duration of flow, minutes.	Head, ft.	Amount of water caught.	Volume discharged per second, cubic feet.

Plot the volume discharged per second on a *head* base. The result is a flat curve. Adopt the method given in the Appendix to find the equation to this curve, that is, plot the logarithms of the heads and volumes. The result ought to be a straight line. Deduce its equation. It will be found to be approximately—

$$\text{Cubic feet discharged per second} = 4{\cdot}95 Ah^{\frac{1}{2}}$$

where A = area of orifice in square feet;
and h = head in feet.

The number 4·95 is very nearly $0{\cdot}62\sqrt{2g}$, and hence the discharge per second is very nearly $0{\cdot}62A\sqrt{2gh}$.

The number 0·62 is called the *coefficient of discharge.* It is not constant, but varies with the head and the size and shape of the orifice. The above is the average value.

But the volume flowing out per second = sectional area of stream × velocity = $0{\cdot}65A \times v$.

Equating the two expressions for the discharge per second, we get—

$$0{\cdot}65Av = 0{\cdot}62A\sqrt{2gh}$$

$$\text{or } v = 0{\cdot}96\sqrt{2gh}$$

That is, the velocity of outflow is as nearly as possible the same as if each particle of water fell freely through a height equal to the head.

The number 0·96 is called the *coefficient of velocity.*

The Calibration of a Tank is necessary if there is no weighing machine on which the tank can be placed for weighing

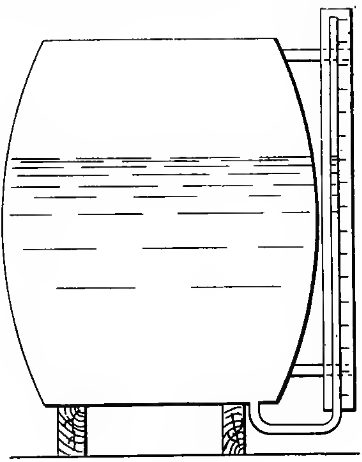

FIG. 270.

the discharge. Any irregularly shaped tank can be used for the purpose. The tank should be fitted with a gauge-glass, as in Fig. 270, for indicating the height of the water in the tank.

Known weights of water are poured into the tank, and the height of water in the gauge-glass noted on a scale of inches or centimetres placed there temporarily. Plot the weight of water in the tank vertically, as in Fig. 271, and the height of water in the gauge-glass horizontally, and then project up the

tens to a slip of paper, AB, which is going to be the final scale. The smaller divisions can be afterwards projected, but sometimes the graduations are nearly equal, when the larger

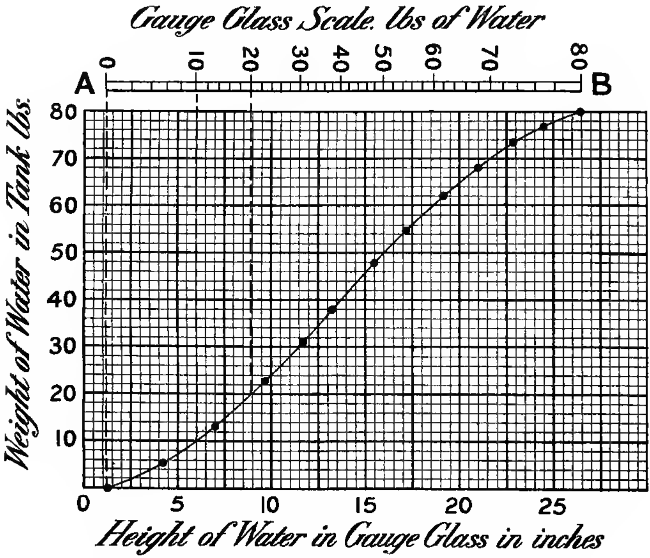

FIG. 271.—Calibration curve for tank in Fig. 270.

ones can be divided with small compasses. In Fig. 271 the small divisions have been projected.

Bernouilli's Theorem states that if a liquid flow in a closed channel, the channel always being full, and if there is no friction of the fluid against the solid, and, further, if the density of the fluid remain constant, then—

$$\frac{P}{\delta} + \frac{v^2}{2g} + z = \text{constant}$$

where P = pressure in pounds per square foot at *any* point;
δ = density of fluid in pounds per cubic foot;
v = velocity of fluid at the point in feet per second;
and z = the height of the point in feet above some selected datum line.

If the sum of the above three quantities is the same for any and every point in the channel, we can write it in a slightly

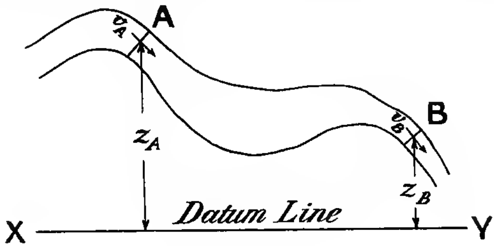

FIG. 272.

different form. Take any two sections of the stream A and B (Fig. 272); then—

$$\frac{P_A}{\delta} + \frac{v_A^2}{2g} + Z_A = \frac{P_B}{\delta} + \frac{v_B^2}{2g} + Z_B$$

must be true, the suffixes A and B referring to the sections A and B respectively.

Consider for the moment the fluid between two normal sections, such as AB (Fig. 273), these sections being close

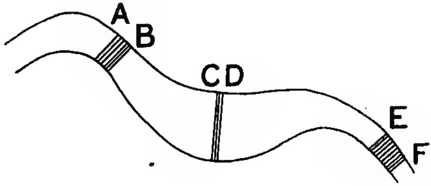

FIG. 273.

together. It will assist in following the argument if the fluid between A and B be assumed to be coloured. With the motion we are considering, it is assumed that the coloured fluid always remains between the two sections, as if these sections were bounding plates which could expand or contract automatically to fill the channel. As the channel became larger

in diameter, the plates must come closer together because the volume of coloured fluid is constant.

This is called *motion in plane layers*, and although the friction of the fluid against the sides of the channel prevents this from actually taking place in most cases in practice, yet it enables us to formulate an expression for the flow of a fluid which coincides very nearly with actual facts.

In Fig. 274 consider the motion of the part AB of the

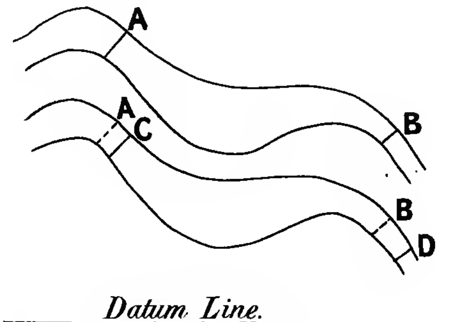

FIG. 274.

stream during a very short interval of time. We can apply the principle of work to the part AB, thus—

The kinetic energy possessed by the fluid AB in the upper position + the work done upon or received by the fluid AB during motion to position CD = the kinetic energy possessed by fluid AB in the position CD + the work done by or given up by AB during motion.

We will consider each of these items separately.

The work done *by* the neighbouring fluid *on* the part AB (Fig. 274) in direction of motion
= total pressure × distance moved through by point of application
= P_A × area of plate or section at A × AC
= P_A × volume of channel between A and C

Similarly, the work done *by* the fluid AB on the neighbouring fluid at section B in the direction of motion = $P_B \times$ volume of channel between B and D.

But it was shown on the previous page that the volume of AC was the same as that of BD, which we may call V; therefore the work done on, or received by, the fluid AB = $P_A V$ ft.-lbs., and the work done *by* the fluid AB on other fluid at section B = $P_B V$.

We have assumed the velocity at any section to remain constant at that section; for example, the velocity at section C will be v_C during the time that motion continues. For this

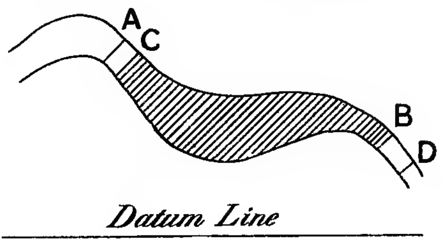

FIG. 275.

reason, the velocity at any section of the shaded part (Fig. 275) will remain constant during the motion, and consequently the kinetic energy of the shaded part remains constant. . If the sections A and C are close together, the velocity between A and C will be as nearly as possible v_A. Similarly, between B and D the velocity will be v_B.

Call the kinetic energy of the shaded part S.

Then the kinetic energy of the part AB in the initial position } = KE of AC + KE of shaded part

$$= \text{mass of AC} \times \frac{v_A^2}{2g} + S$$

Also the kinetic energy of the part AB in the final position CD } = KE of shaded part + KE of BD

$$= S + \text{mass of BD} \times \frac{v_B^2}{2g}$$

The centre of gravity of the shaded part remains at the same height above the datum line, and consequently no work can be done by it.

Before motion began, we had a part of the fluid AB between A and C, and none between B and D. At the end of the motion we have none between A and C, and some between B and D. This is in effect the same as allowing the fluid between A and C to flow or fall direct to the position between B and D. The work done by the earth on it = its weight × vertical distance fallen through = weight $(Z_A - Z_B)$ ft.-lbs. Now insert all these quantities in the equation on page 324, and we get—

$$\text{Mass of AC} \times \frac{v_A^2}{2g} + S + P_A \times \text{volume of AC} + \text{weight of AC}(Z_A - Z_B) = \text{mass of BD} \times \frac{v_B^2}{2g} + S + P_B \times \text{volume of BD}$$

As the mass of AC is the same as the mass of BD, which equals the volume of AC × its density = δV, and S cancels, the above equation becomes—

$$\delta V \times \frac{v_A^2}{2g} + P_A V + \delta V(Z_A - Z_B) = \delta V \frac{v_B^2}{2g} + P_B V$$

Divide both sides by δV, and we have—

$$\frac{v_A^2}{2g} + \frac{P_A}{\delta} + Z_A - Z_B = \frac{v_B^2}{2g} + \frac{P_B}{\delta}$$

And after rearranging the terms—

$$\frac{\mathbf{P}_A}{\delta} + \frac{\boldsymbol{v}_A^2}{\mathbf{2g}} + \mathbf{Z}_A = \frac{\mathbf{P}_B}{\delta} + \frac{\boldsymbol{v}_B^2}{\mathbf{2g}} + \mathbf{Z}_B$$

This is called Bernouilli's equation.

Example.—Find the velocity of the water as it emerges from the end B of the pipe in Fig. 276. The pipe is parallel, and is

supplied from a tank overhead, the sectional area of which is many times that of the pipe. Water is kept flowing into the tank to maintain the level at A constant. Also plot a curve on the dotted line AB as base, showing the pressure of the water from A to B.

In problems of this sort, the quantities relating to the two extreme sections A and B of the channel must be inserted in Bernouilli's equation. Of these quantities P_A is the pressure of the atmosphere per square foot, because the liquid at section A is in contact with it. The same holds with section B, and hence $P_A = P_B$, and these can at once be cancelled in the above equation.

The velocity v_A of the surface A is zero, because the surface is maintained at a constant level. Even were no water allowed to come in, the velocity of the surface would be so small as to be negligible.

The height $Z_A = 20$ ft. if we take the level of B as our datum line. At the same time $Z_B = 0$.

Inserting these quantities in Bernouilli's equation, we get—

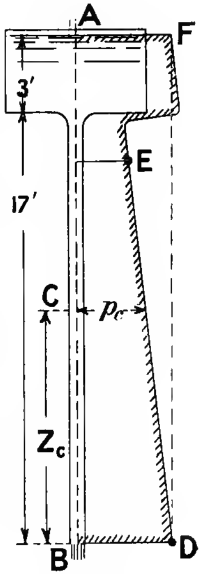

FIG. 276.

$$20 = \frac{v_B^2}{2g}$$

and $v_B = 35{\cdot}8$ ft. per second

Now take two sections, B and C, the latter being at Z_C above B, and find the pressure in the liquid at the section C. Bernouilli's equation for these sections is—

$$\frac{P_B}{\delta} + \frac{v_B^2}{2g} + Z_B = \frac{P_C}{\delta} + \frac{v_C^2}{2g} + Z_C$$

If we assume 15 lbs. per square inch as the atmospheric pressure, $P^B = 15 \times 144$ lbs. per square foot, and $\frac{P_B}{\delta} = \frac{15 \times 144}{62{\cdot}5} = 34{\cdot}7$ ft.

This is the head of water equivalent to 15 lbs. per square inch. Also $v_B = v_C$, because the pipe is parallel in bore. Inserting these quantities in Bernouilli's equation, we get—

$$34{\cdot}7 = \frac{P_C}{\delta} + Z_C$$

$$\text{or } (34{\cdot}7 - Z_C)\delta = P_C \text{ lbs. per square foot}$$

$$\text{or } (34{\cdot}7 - Z_C)\frac{62{\cdot}5}{144} = p_c \text{ lbs. per square inch}$$

This reduces to—

$$p_c = 15 - 0{\cdot}432Z_C$$

and is the equation to a straight line. Putting $Z_C = 0$, we get—

$$p_c = 15 \text{ lbs. per square inch}$$

also when $Z_C = 10$,

$$p_c = 10{\cdot}68 \text{ lbs. per square inch}$$

Hence, plot these two points, D and E, in Fig. 276, and join them with a straight line. This is the pressure diagram, and is shown hatched.

In the upper part of the figure the line AF represents the atmospheric pressure (15 lbs. per square inch). If we take section A and another section further down the tank at a height Z from the datum line, then the velocity there is the same as at A; that is zero, or nearly zero, because the tank has parallel sides.

Substituting in Bernouilli's equation, we get—

$$\frac{P_A}{\delta} + 0 + 20 = \frac{P}{\delta} + 0 + Z$$

$$\text{or } P = \delta(20 - Z) + P_A$$

$$\text{and } p = \frac{62{\cdot}5}{144}(20 - Z) + p_A$$

$$= 15 + 0{\cdot}43 \times \text{depth below A}$$

Plotting this line, we get the part of the curve immediately below F.

From this we see that if a hole were made in the *tank*, water would come out through it, because the pressure is everywhere greater than that of the atmosphere; but if a hole were made in the *pipe* no water would come out, but air would pass into the pipe, because the pressure of the atmosphere is greater than that of the water inside the pipe.

Example.—A siphon is arranged (Fig. 277) with its longest leg extending h feet below the free surface in the tank, and with the

top of the siphon H ft. above the free surface. If h is maintained constant at 6 ft., and the sectional area of the pipe at B is twice that at A, what is the greatest height H of the top of the siphon above the free surface so that it will work?

As the same volume of water passes A and B per second, we must have—

$$\text{Volume per second} = v_A A_A = v_B A_B$$

where A = sectional area of pipe. This is called *the equation of continuity*, and simply expresses the fact that the same volume passes both sections during the same time. From it we find—

$$v_A = v_B \times \frac{A_B}{A_A} = 2v_B$$

To find v_B, insert all the known quantities in Bernouilli's equation, using the free surface of the water as the datum level, and also for one section (no suffix), the other being at B. $P = P_B$ = pressure of the atmosphere, and $v = 0$, while $Z_B = -6$ ft. Then, after cancelling P and P_B, we have—

$$0 + 0 = \frac{v_B^2}{2g} - 6$$

$$\text{or } v_B = 19{\cdot}6 \text{ ft. per second}$$

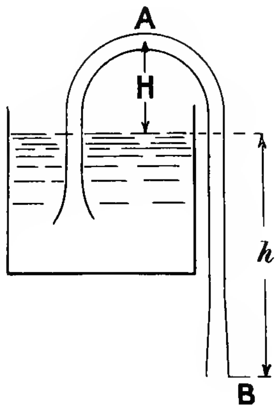

FIG. 277.

If the stream through the pipe is to be perfect, the pressure must not go below zero, because a liquid cannot sustain a tension. Hence the maximum value of H corresponds to $P_A = 0$. Inserting this in Bernouilli's equation for sections A and B, we get—

$$0 + \frac{v_A^2}{2g} + H = \frac{15 \times 144}{62{\cdot}5} + \frac{v_B^2}{2g} - 6$$

and as $v_A = 2v_B = 39{\cdot}2$ ft. per second, we have—

$$H = 34{\cdot}7 + 6 - 6 - 24$$
$$= 10{\cdot}7 \text{ ft.}$$

Hence, if the top of the siphon is raised more than 10·7 ft.

above the free surface of the water in the tank the stream will break, and the siphon stops working.

Fluid Friction, unlike solid friction, does not depend upon the pressure of the fluid; but it does depend upon the surface of contact between the pipe and the fluid, and also upon the velocity of the fluid over that surface.

The loss of head between two sections of a horizontal pipe, measured in feet of water, is given approximately by the expression—

$$\frac{cl}{d} \cdot \frac{v^2}{2g}$$

where l is the length of the pipe in feet, d is the diameter of the pipe in feet, and c is a coefficient.

This expression can be verified in the following manner:—

Arrange a long length of horizontal pipe (preferably of small diameter) with a series of tees. Attach a pressure gauge to the first of these, and another gauge to each of the others in turn. The difference in the readings of the two gauges will be found to be proportional to their distance apart along the pipe.

Next carry out an experiment at different rates of flow, and calculate the velocity in each case. Deduce the relation between difference of gauge-readings and velocity.

Different-size pipes must be used to show that the loss of head is inversely proportional to the diameter of the pipe.

The General Equation for a Gas in Equilibrium. By the term gas we generally infer that a gas far removed from condensation is meant. A gas near condensation is called a *vapour*, and behaves differently to a gas far removed from condensation.

Let P = pressure of gas in pounds per square foot;

p = " " " " inch;

V = volume of 1 lb. gas in cubic feet (called the specific volume of the gas);

Let T = absolute temperature of the gas
= 273 + *t* Centigrade,
or = 468 + *t* Fahrenheit, where *t* = temperature on the ordinary scale of the thermometer;
R = a constant, which is different for different gases.

Then—

$$PV = RT$$

is an equation which holds good for all gases with a near approach to accuracy.

If some other system of units be used instead of the foot-pound and second system, the value of R will be different.

In the above equation let T be kept constant, then—

$$PV = \text{a constant}$$

which generally goes by the name of Boyle's Law. If suffixes $_1$, $_2$, $_3$, etc., denote different stages in some operation at constant temperature, then—

$$P_1V_1 = P_2V_2 = P_3V_3 = \text{a constant}$$

Returning to the original equation, and making P constant, we get—

$$V = \frac{R}{P}T = T \times \text{a constant}$$

which indicates that the volume of a given mass of gas increases directly as the absolute temperature, under constant pressure.

This relation enables us to construct a thermometer in which a gas is the element which expands and contracts with change of temperature instead of mercury.

Let suffix $_0$ indicate the zero of an ordinary thermometer; then, if A is the absolute temperature of the ordinary zero—

$$V = \frac{R}{P}T = \frac{R}{P}(A + t)$$

$$\text{and } V_0 = \frac{R}{P}(A + 0)$$

Dividing one equation by the other, we get—

$$V = V_0\left(\frac{A + t}{A}\right)$$

$$= V_0\left(1 + \frac{1}{A}t\right)$$

Let $\frac{1}{A}$ be represented by α; then—

$$V = V_0(1 + \alpha t)$$

or the volume at any temperature $t°$ on the ordinary scale equals the volume at ordinary zero (freezing-point) multiplied by (1 + coefficient of expansion × temperature). The quantity α in this case is called the *coefficient of expansion at constant pressure*, and it is independent of the kind of gas; that is, it is the same for all gases. This is approximately the case, as will be seen from the table below:—

Hydrogen	0·003661
Air	0·003670
Carbonic oxide	0·003669

If we put $\alpha = 0·00366$ in the equation—

$$V = V_0(1 + \alpha t)$$

and remember that according to a previous equation the absolute temperature was zero when the volume was zero, we find, when putting $V = 0$, that $(1 + \alpha t) = 0$, and—

$$t = -\frac{1}{0·00366} = -273°\ \text{C.}$$

The absolute zero of temperature is then somewhere about 273° C. below freezing-point. Hence the absolute temperature of a body is 273 + the temperature on the scale of an ordinary thermometer.

Returning again to the equation—

$$PV = RT$$

make V constant; then—

$$P = \frac{R}{V}T$$

$$\text{also } P_0 = \frac{R}{V}T_0$$

Divide one equation by the other;

$$\text{then } P = P_0\frac{T}{T_0} = P_0\left(\frac{A+t}{A}\right)$$

$$= P_0(1 + at)$$

Hence the coefficient of expansion of a gas at constant volume is the same as at constant pressure.

In the above equations V has represented the volume of 1 lb. of gas. If ρ be the density of the gas, that is, the mass in pounds of 1 cub. ft., or the mass in grammes of 1 c.c., the mass of V cub. ft. must be $V\rho$ lbs. But the mass of V cub. ft. is 1 lb. by definition; hence—

$$1 = V\rho$$

$$\text{and } \rho = \frac{1}{V}, \text{ or } V = \frac{1}{\rho}$$

Therefore in any of the preceding equations we may write $\frac{1}{\rho}$ for V, and express the different relations in terms of density rather than specific volume.

Example.—A quantity of gas occupies a volume of 3 cub. ft. at a pressure of 25 lbs. per square inch. It is compressed at constant temperature (37° C.) till its volume is 1·2 cub. ft.: find its pressure. It is now raised in temperature at constant volume to 127° C.: find its pressure. Its temperature is maintained constant, and the gas is expanded to a pressure of 10 lbs. per square inch: what is its volume? If it is now cooled to 10° C. at constant pressure, what is its volume?

At constant temperature—

$$P_1V_1 = P_2V_2$$

$$\text{or } 25 \times 144 \times 3 = 144 \times p_2 \times 1{\cdot}2$$

$$\text{and } p_2 = 62{\cdot}5 \text{ lbs. per square inch}$$

Again—

$$P_2V_2 = RT_2$$
$$\text{and } P_3V_3 = RT_3$$

Dividing one equation by the other, and remembering that $V_2 = V_3$, we get—

$$\frac{P_2}{P_3} = \frac{T_2}{T_3} = \frac{273 + 37}{273 + 127} = \frac{31}{40}$$

$$\text{hence } P_3 = P_2 \times \tfrac{40}{31} = 62{\cdot}5 \times 144 \times \tfrac{40}{31}$$

$$\text{and } p_3 = 62{\cdot}5 \times \tfrac{40}{31} = 81 \text{ lbs. per square inch nearly}$$

At constant temperature—

$$P_3V_3 = P_4V_4$$

and as $V_3 = V_2$, the volume being constant during that change—

$$144 \times 81 \times 1{\cdot}2 = 144 \times 10 \times V_4$$
$$\text{and } V_4 = 9{\cdot}72 \text{ cub. ft.}$$

Again—

$$\frac{P_4V_4}{P_5V_5} = \frac{RT_4}{RT_5}$$

$$\text{or } V_5 = V_4 \times \frac{T_5}{T_4} \text{ when } P_4 = P_5$$

$$\text{that is, } V_5 = 9{\cdot}72 \times \frac{273 + 10}{273 + 127}$$
$$= 6{\cdot}9 \text{ cub. ft.}$$

Example.—If a mass of gas occupied a volume of 9 cub. ft. at 20° C., what will it occupy at 120° C. if its coefficient of expansion is 0·00366 and the pressure is constant?

The simplest way of solving this problem is as follows:—

$$P_{20}V_{20} = RT_{20}$$
$$\text{and } P_{120}V_{120} = RT_{120}$$
$$\text{as } P_{20} = P_{120}$$
$$V_{120} = V_{20} \times \frac{T_{120}}{T_{20}}$$
$$= 9 \times \frac{273 + 120}{273 + 20}$$
$$= 12{\cdot}07 \text{ cub. ft.}$$

It can be solved as follows :—

$$V_{20} = V_0(1 + \alpha \times 20)$$

$$\text{hence } V_0 = \frac{9}{1{\cdot}0732} = 8{\cdot}386 \text{ cub. ft.}$$

Again—

$$\begin{aligned} V_{120} &= V_0(1 + a \times 120) \\ &= 8{\cdot}386 \times 1{\cdot}4392 \\ &= 12{\cdot}07 \text{ cub. ft. nearly} \end{aligned}$$

Summary of Chapter X.

1. The pressure of a fluid on a surface immersed in it = area of surface × depth of centre of gravity of surface × density of fluid.

2. In hydraulic machinery, the pressure per square inch is the same throughout all the connections.

3. The support offered by a liquid to a solid immersed in it = weight of water displaced by solid.

4. Density is the mass of unit volume.

5. Specific gravity $= \dfrac{\text{density of substance}}{\text{density of water}}$

$$= \frac{\text{weight of a body}}{\text{weight of an equal volume of water}}$$

6. Volume of liquid discharged from an orifice per second = $c \, . \, A \, . \, \sqrt{2gh}$.

7. When a liquid flows (full bore) through a closed channel (Bernouilli's equation)—

$$\frac{P}{\delta} + \frac{v^2}{2g} + Z = \text{constant}$$

and (equation of continuity)—

$$A \, . \, v \, . \, \delta = \text{constant}$$

8. Loss of head due to friction $= c\dfrac{L}{d} \cdot \dfrac{v^2}{2g}$

Examples on Chapter X.

1. In a hydraulic press the pump-plunger is a cylinder 1 cm. in diameter, and makes a stroke 7 cms. long. The plunger of the press is 20 cms. in diameter. Calculate (*a*) the pressure in the press when a weight of 100 lbs. is applied to the pump-plunger (ignoring friction); (*b*) the force acting on the press-plunger; (*c*) the number of strokes which the pump must make in order to raise the press-plunger 10 cms.

2. Describe the hydraulic press. How many tons to the square inch would you expect at a depth of 5 miles in the ocean, if 1 cub. ft. of sea-water weighs 66 lbs.?

3. Describe experiments to prove that the upward force which a fluid exerts on an incompressible solid immersed in it depends only on the bulk of the body, the density of the fluid, and the intensity of gravity, and is independent of the depth of immersion and of the shape of the body. What difference would it make if the body were readily compressible?

4. Define density or specific gravity, and calculate the mass of 1 c.c. of a certain solid from the following data: a mass of 720 grms. hanging from one pan of a balance is totally immersed in water, and found to be counterpoised by a weight of 645 grms. in the other pan.

5. A spherical leaden bullet is cast with a hollow in its interior. How could you ascertain the size and position of the hollow without spoiling the bullet?

6. A solid body weighs 117 grms. in air, 98 in water, and 101 in another liquid. Calculate the specific gravity of the solid and of the liquid.

7. Define *density*. A specific gravity bottle, completely full of water, weighs 38·4 grms.; and when 22·3 grms. of a certain solid have been introduced, it weighs 49·8 grms. Calculate the density of the solid.

8. The upper ends of two long vertical glass tubes are connected together and to the receiver of an air-pump. The lower end of one dips into a beaker of water, that of the other into a beaker of copper sulphate. On working the air-pump, the liquids rise in the two tubes, but to different heights. Explain the cause of this.

9. A flask when empty weighs 120 grms., when full of air it weighs 121·3 grms., and when full of water, 1120 grms. . calculate the density of the air.

Explain whether it is or is not necessary to take account of the weight of air displaced.

10. Explain the construction of a barometer, what it measures, and how it measures it. Translate pressure measured in terms of the height of a barometer mercury column (say either 27 in. or 60 cms.) into absolute units of pressure.

11. Prove that the surface of a heavy liquid at rest in a vessel is a horizontal plane.

12. A cylinder, loaded so as to float vertically, and weighing 2 grms. altogether, just sinks overhead in water when half a gramme extra is put on its top; otherwise it protrudes 7 cms. above the surface. What length will protrude above the surface of a liquid whose density is five-sixths that of water, if the cylinder be set floating in it without the extra load?

13. An artificial lake, $\frac{1}{4}$ mile long and 100 yards broad, with a gradually shelving bottom varying from nothing at one end to 88 ft. at the other, is dammed at the deep end by a masonry wall across its entire breadth. Find the total pressure on the embankment when the lake is full of water weighing three-quarters of a ton to the cubic yard. Find also the total weight of water in the lake.

14. A block of wood floats with 10 cub. in. above the surface in fresh water, and with 40 cub. in. above the surface in salt water of specific gravity 1·025. What is its total volume? *Ans.* 1240 cub. in.

15. A cylinder, weighing 1 lb., floating in water with its axis vertical and each of its ends horizontal, requires a weight of 4 ozs. to be placed on its upper surface to depress it to the level of the water. Find the specific gravity of the cylinder. *Ans.* 0·8.

16. Describe an experiment which shows that when a body is weighed in water the loss of weight is equal to the weight of water displaced.

17. A lock-gate in a canal is 8 ft. wide, the water on one side is 10 ft. deep, and on the other side is 6 ft. deep. Find the pressure on each side if 1 cub. ft. of water weighs 62·4 lbs. *Ans.* 24,960 lbs., and 8085·6 lbs.

18. A solid floats in water with nine-tenths of its volume immersed. What proportion of its volume will be immersed when it floats in mercury? [Specific gravity of mercury, 13·6.] *Ans.* $\frac{9}{136}$.

19. A glass bulb will just stand an excess of inside over outside pressure of 200 grms. per square centimetre. It is sealed up at a place where the barometer stands at 75 cms., and then taken uphill till it bursts. What is the height of the barometer at the place where this occurs? *Ans.* 60·3 cms.

20. A wide-mouthed bottle full of air is closed with a well-ground glass stopper, 5 cms. in diameter, when the barometer stands at 772 mm. What weight must the stopper have in order that it may just be blown out if the barometer goes down to 730, the temperature remaining the same?

21. What do you know about the density of gases in relation to temperature and pressure? Describe experiments which show that the density of a gas at constant temperature is proportional to its pressure. A uniform tube closed at top, open at bottom, is plunged into mercury, so that it contains 25 c.c. of gas at atmospheric pressure of 76 cms.; the tube is now raised until the gas occupies 50 c.c.: how much has it been raised?

22. A balloon is filled with a gas whose specific gravity is one-tenth of that of air at the pressure of 760 mm. of mercury at 0° C. Compare the lifting power of the balloon in air when the height of the barometer is 750 mm. with its lifting power when the barometer stands at 760 mm. The temperature in both cases is 0° C., and the volume of the balloon is supposed to remain unaltered.

23. How would you test Boyle's Law for air at pressures less than the atmospheric pressure?

Gas is sold by the cubic foot, but the illuminating power is proportional to the mass burnt per hour. A 15-candle-power jet costs 1*d.* per 6 hours when the barometer stands at 30 in. What will it cost for the same candle-power in the same time when the barometer is at 28 in.? (Neglect the difference of pressure between the gas and the atmosphere.)

Ans. The mass of gas which is sold for 1*d.* at the higher pressure will cost $\frac{30}{28}d.$ or $1\frac{1}{14}d.$ at the lower pressure, and this is the required price of the jet for 6 hours.

24. State Boyle's Law carefully, and describe how to verify it for any permanent gas.

25. How can the atmospheric pressure be measured and expressed in grammes per square centimetre?

26. A barometer reads 30 in. at the base of a tower, and 29·8 in. at the top, 180 ft. above. Find the average mass of a cubic foot of air in the tower, taking the specific gravity of mercury as 13·5, and the mass of a cubic foot of water as 62·4 lbs. *Ans.* 0·078 lb.

27. A diving-bell, having a capacity of 125 cub. ft., is sunk in salt water to a depth of 100 ft. If the specific gravity of salt water be 1·02, and the height of the water-barometer be 34 ft., find the total quantity of air at atmospheric pressure required to fill the bell.

28. Two hydraulic cylinders, $1\frac{1}{2}$ in. and 7 in. radius respectively, are connected by a pipe. If the smaller piston is depressed through 4 ft., how much will the larger piston be raised, and why?

29. In a hydraulic press the pump ram is $1\frac{1}{2}$ in. in diameter, the press ram 15 in. in diameter. Through how many inches must the pump ram move if the press ram is raised 16 in.? What pressure must be exerted on the pump ram to secure a pressure of 36 tons by the press? Assume the efficiency is 70 per cent. *Ans.* 1600 in., and 1152 lbs.

30. Make a sectional sketch of a hydrostatic press suitable for giving a pressure of 100 tons, showing the valves and pump, and by what contrivance the leakage of water is prevented. The pump for such a press has a cylindrical plunger 1 in. in diameter, with a leverage of 10 to 1. What should be the least diameter of the ram which should give 100 tons pressure when a force of 56 lbs. was applied at the end of the pump lever? Friction is neglected. What form is most suitable for the base of the ram cylinder? and for what reason is a special form adopted?

31. Describe and sketch in section a hydraulic accumulator, showing how the ram is kept tight in the cylinder. A hydraulic press, having a ram 16 in. in diameter, is connected with an accumulator which has a ram 8 in. in diameter and is loaded with 50 tons of ballast. What is the total pressure on the ram of the press?

32. A hydraulic accumulator has a loaded plunger of 12 in. diameter, and the lift of the same is 8 ft. Sketch the arrangement. What load, in tons, will give a pressure of 700 lbs. per square inch in the water passing

from the accumulator to the hydraulic cranes? What amount of energy may be stored up in such an accumulator? How is a continuous supply of air under pressure provided for in a forge bellows?

33. The accumulator in a hydraulic plant has a ram 8 in. in diameter, with a stroke of 20 ft. The pressure of water adopted in the system is 1200 lbs. per square inch. What must be the load placed on the ram, in tons? and what horse-power can be supplied by this plant, assuming the machines worked off it use 50 cub. ft. per minute? The average efficiency of the machines is 58 per cent., and the pumps supplying the accumulator are sufficient to keep it steady with this consumption.

Ans. 26·9 tons, and 152 H.P.

34. Draw a section of a hydraulic press, and explain the principle of its action.

When the ram is exerting a very great pressure, why may a comparatively weak pipe be strong enough to contain the water which transmits the force from the driving points?

35. A gauge in a water-pipe indicates a pressure of water equal to 40 lbs. on a square inch. What is the depth of the point below the free surface? Sketch and explain the action of some form of gauge suitable for the above purpose.

36. Describe the construction and action of an ordinary suction pump for raising water from a well. If 200 gallons of water are raised per hour from a depth of 20 ft., and if the efficiency of the pump is 60 per cent., what horse-power is being given to the pump?

37. Water at 750 lbs. per square inch pressure acts on a piston 1 sq. ft. in area, through a stroke of 1 ft. What is the work that such water does per cubic foot, and per gallon? If a hydraulic company charges 18*d.* for 1000 gallons of such water, how much work is given for each penny?

38. Explain the use of an intensifying accumulator in hydraulic machinery, and sketch its construction. What are the advantages of using such an arrangement?

How many horse-power are available in a stream of water in which 50 cub. ft. per second are passing, if the head of the water is 92 ft.?

39. Water is flowing uniformly in a nearly rectangular channel through a meadow; it has fallen from a height of 30 ft. State how you would proceed to find, with a very rough approximation to accuracy, the water-power of the fall. What measurements would you find it necessary to make?

40. Calculate how many horse-power can be transmitted by a 6-in. pipe conveying water under a pressure of 1120 lbs. per square inch. Select your velocity of flow. Sketch a suitable joint for connecting two lengths of pipes for such a pressure.

41. In a hydraulic press worked by hand, the hand-lever was 5 ft. long and the plunger was connected to the lever at a distance of 4 in. from the end. The diameter of the press ram was 1 ft., and the pressure it exerted was 15 tons. If the diameter of the pump-plunger was 1 in., what force

must a man exert at the end of the hand-lever to produce this pressure on the ram? Work out the problem from first principles.

42. State how the pressure at any point of a fluid is measured. P is a point 10 ft. below the surface of a fluid, 1 cub. in. of which weighs $\frac{1}{30}$ lb. Two men, A and B, are talking about the pressure at P; A says it is 4 lbs., B says it is 576 lbs.; the difference is simply due to neither of them saying exactly what he means. Show this by completing their statement.

43. Explain what is meant by the pressure of a fluid at a point, and by the resultant fluid pressure on a surface. If a sphere is held half immersed in water, find the magnitude of the resultant pressure of the water and the line along which it acts.

44. State the method of finding the magnitude of the resultant pressure on a plane area immersed in a liquid. A thin lamina is in shape a regular hexagon, and its sides are 3 ft. long; it is immersed in water with one side in the surface, and its plane inclined at an angle of 45° to the vertical: find the magnitude of the resultant pressure on one of its faces, and the average pressure-intensity over that face.

45. State the equation for finding the magnitude of the resultant pressure of a liquid on a plane area immersed in it. A square whose side is 8 ft. long has its plane vertical and its upper edge of the surface of the water in which it is immersed: find the magnitude of the resultant pressure on one face of it. If the square were fixed and the surface of the water were raised a foot, what would now be the magnitude of the resultant pressure?

46. When a body is partly immersed in a liquid, along what line does the resultant of the fluid pressures act?

ABCD is a rectangular lamina; it is held in a vertical plane with the diagonal AC on the surface of water: show in a diagram the line along which the resultant of the fluid pressure acts.

If AB, BC are 10 ft. and 4 ft. long respectively, and if the thickness of the lamina is 1 in., find the magnitude of the resultant pressure.

CHAPTER XI.

MECHANISM.

A machine was defined, on p. 209, as a combination of material parts, each of which can only move in its own particular path.

When considering the geometrical properties of, or the relations between, the movements of different parts of a machine, we often speak of it as a mechanism.

A machine is made up of parts or pieces, each of which is called a *link*, and the mechanism is often spoken of as a *kinematic chain.*

A kinematic chain does not become useful until one of its links is fixed. It is then, and only then, a machine. A link is sometimes called an *element.*

Two consecutive links forming part of a machine must be connected in some way or other. They then form a *pair* of elements or links.

Pairs are of two kinds: (1) lower, and (2) higher.

Lower Pairs consist of *rigid* elements or links, between which *contact* takes place over a *surface.* There are three kinds, namely—

(1) A turning pair, in which a portion of one element turns inside a portion of the other element without movement in the axial direction, as a shaft turns in its bearing. (Note that if two links are connected by a pin, the pin is assumed to form part of one of the links, as if it were solid with it.)

(2) A sliding pair, in which one element slides in or over another element without turning, as a piston in a cylinder.

(3) A screw pair, in which one element has a motion

relative to the other, which is a combination of turning and sliding, as a nut moves over a bolt.

Higher Pairs are pairs of elements which are not lower pairs. These may contain non-rigid elements such as a belt (Fig. 278) or a rope (Fig. 199); or a liquid element, such as in Figs. 251, etc.; or contact between two links or elements may take place along a *line* (instead of a surface), such as in toothed wheels and cams.

The above gives an idea of how mechanisms may be analyzed, but it is a class of work more suited to the senior than the elementary student, and hence is out of place in a work of this character. Still, some knowledge of how different motions are produced by the aid of mechanism must be of great value to the elementary student, especially if he be engaged in any technical work, and in consequence a few illustrations will now be given of how certain typical results are produced.

For the purpose of our elementary discussion of the subject, simple mechanisms may be classified as follows:—

(1) Transmission of circular motion.
(2) Transmission of linear motion.
(3) Conversion of circular to linear motion or *vice versâ*.
(4) Conversion of continuous to intermittent motion.

A belt and a pair of pulleys are the simplest contrivance for transmitting circular motion from one shaft to another. This has already been dealt with arithmetically on p. 266. It is sometimes necessary to drive a shaft at a number of different speeds by a belt from another shaft rotating at a constant speed. This is accomplished by an arrangement of *speed cones* (Fig. 278), in which the sum of the diameters of any pair of pulleys on which the belt can run simultaneously is constant. For example—

$$\begin{aligned}\text{Diameter of C} + \text{diameter of D} &= \text{diameter of E} + \text{diameter of F}\\ &= \text{diameter of H} + \text{diameter of K}\\ &= \text{a constant}\end{aligned}$$

Of course the length of the belt must be the same for every pair of pulleys, otherwise the same belt would not work on every pair. This is exactly the case if the belt is crossed, but

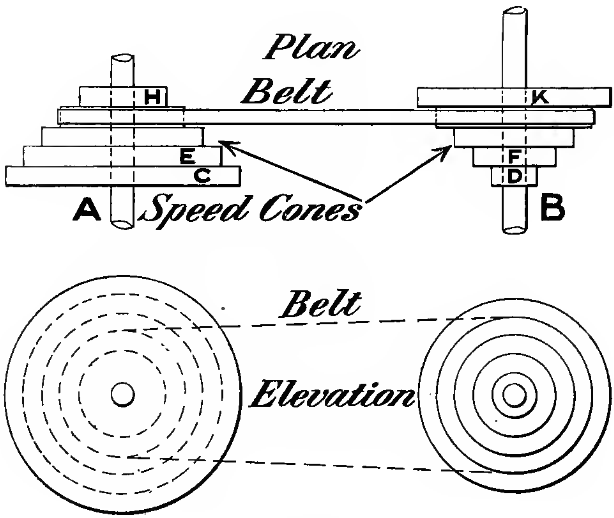

FIG. 278.—Speed cones.

when the belt is used " open," as in the figure, there is a *slight* difference in the length for different pairs of pulleys, but this is easily taken up by the elasticity of the belt. These sets of pulleys are called **speed cones**, because their edges lie on a cone, the slope of the two cones which will work together being the same.

It is sometimes desirable to be able to transmit circular motion in either direction at will. One method of doing this is shown in Fig. 279. The driving shaft is D, and the driven or following shaft which is required to be rotated in either direction at the will of the attendant is F. Open and crossed belts are used together with one fast pulley C, and two pulleys B and D, which are quite free to rotate upon the shaft F and are called loose pulleys. A fork, or pair of fingers, whose ends

are shown in Fig. 279 at *aa*, and between which the belt runs, can be made to push the belt on to the pulley B by urging the fork towards the left. The belt then rotates the loose pulley only, and the shaft F remains at rest. The fork *aa* is rigidly connected to the fork *cc* so that as the open belt is moved from

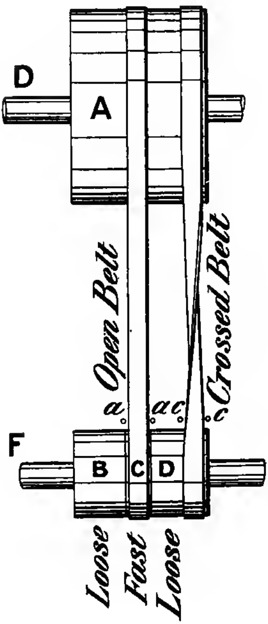

FIG. 279.

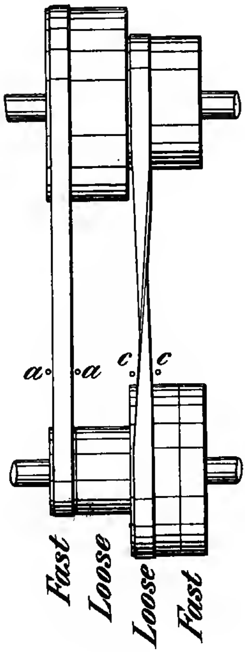

FIG. 280.

the fast to the loose pulley B, the crossed belt is simultaneously moved *towards* the fast pulley C. It is necessary that the open belt should completely leave the pulley C before the crossed belt comes into contact with it, hence the distance between the two belts must be at least equal to the width of the pulley C. This necessitates the loose pulleys B and D being of *double width*. When the forks are urged right across, so that the

crossed belt comes on to the fast pulley, the motion of the shaft is reversed.

In some machines it is desirable that the time occupied in one direction should be less than that in the other direction; or, in other words, the backward and forward speeds must be different. Fig. 280 shows how this may be accomplished. The two loose pulleys may be together, as in Fig. 280, or the two fast pulleys may be together and the two loose pulleys outside.

Another mechanism for producing a result something similar to Fig. 279 is shown in Fig. 281. It is used for the purpose of

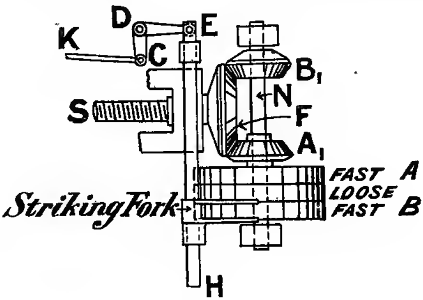

FIG. 281.—Automatic reversing arrangement for planing-machine.

giving to the table of a planing-machine its backward-and-forward motion, and it is automatic in its action. The screw S works in a nut fixed to the movable bed of the planing-machine, and as it is rotated will make the bed move in the direction of the axis of the screw. To the screw is keyed the bevel wheel F (whose teeth are not shown), and gearing with F are two other bevel wheels, namely, B fixed upon the shaft N, and A fixed upon a sleeve, or hollow shaft, which can rotate freely round the shaft N. To this sleeve is fixed the belt pulley A, so that A and A_1 rotate together. The pulley B is fixed upon the shaft N, so that B and B_1 rotate together. A single belt passes between the fork shown, and always runs in the same direction. As the moving table of the planing-machine comes near the end of its travel, it strikes a collar on the rod K, moving it from left to right, thereby carrying the fork and belt over from the pulley B

to A. The pulley A, while turning in the same direction as B, will rotate the bevel wheel F in the opposite direction, and thereby cause the table to move from right to left until it strikes another collar on the rod K, and carries the fork and belt over to the position shown, thus reversing the direction of the motion again.

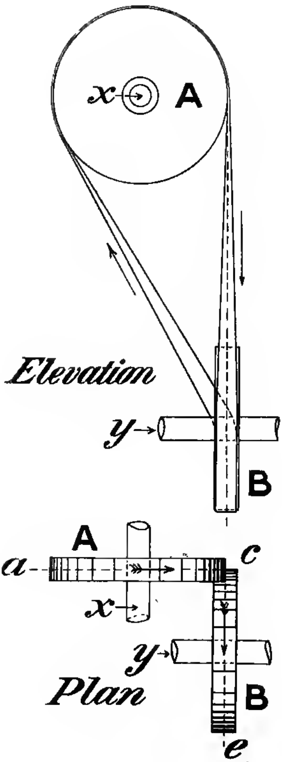

FIG. 282.—Half-crossed belt drive.

The loose pulley is necessary for the same purpose as the extra width of the pulleys B and D in Fig. 279, to enable the belt to get quite clear of one fast pulley before moving on to the other.

An automatic attachment may be added to Figs. 279 and 280 to produce a result similar to that of Fig. 281.

It is sometimes necessary to drive a shaft from another not parallel to it. A particular case is that shown in Fig. 282, in which it is required to drive the shaft y from the shaft x.

The condition which must be satisfied in all belt arrangements is that the belt, as it leaves one pulley, must be in the central plane of the other pulley to which it is advancing.

We see in Fig. 282, both in elevation and plan, that when the belt leaves the pulley A, it is then in the central plane ce of the pulley B; and further, when the belt leaves the pulley B, it is then in the central plane ac of the pulley B. If the direction of motion is reversed, the above condition is not satisfied, and the belt immediately comes off the pulleys.

It will be noticed in any workshop that the surface of a pulley on which a belt runs is generally slightly convex, as shown at P in Fig. 283. This causes the belt to keep to the centre of the pulley without any guides or shrouds. In the figure the belt is shown running on a cone. The tension of the

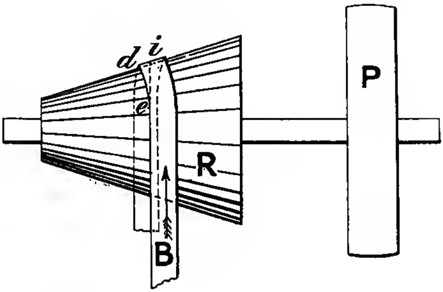

Fig. 283.

belt causes it to try and fit tightly upon the surface of the cone. To do this, the part in contact with the cone takes the shape shown in the figure with a point in the edge at *e* further up the cone than the point *d* in the same edge. Every point in the surface of the cone moves in a circle whose plane is perpendicular to the shaft, and consequently the point *e* will be carried round with the cone surface to *i*, assuming the belt does not slip over the cone. Now, the point *e* is higher up the cone than the point *d*, and in this way the belt will continue to *creep* up the cone *towards the larger diameter*. By making the surface of a pulley convex, as at P, the belt running upon it will always tend to creep to the largest diameter, which is in the central plane.

A Claw Clutch (Fig. 284) is used to enable a wheel or shaft (in the same line as the driving shaft) to be rotated or stopped when required, while the driving shaft continues to rotate. The spur-wheel, which is quite free upon the shaft, has a number of projections or claws, C_2, formed on its boss, which will fit into corresponding recesses in the clutch C_1, a perspective view of which is shown on the right at C. A fork fits into the

groove A in the clutch, and by its means the clutch may be moved forwards on the left into gear with the spur-wheel or

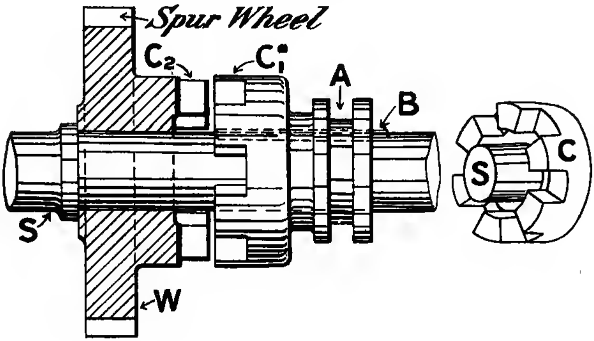

FIG. 284.—Claw clutch.

back again. The clutch is compelled to rotate with the shaft continually by the feather-key B. These clutches can only be

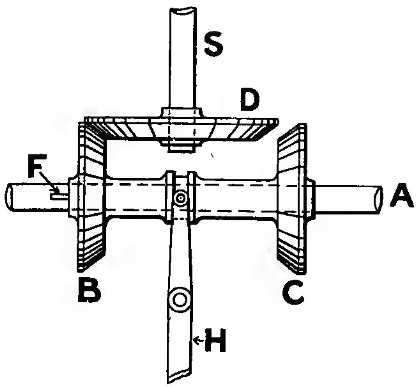

FIG. 285.

used where they can be put into and out of gear while the shaft is at rest, or when turning slowly, as the shock which the teeth

of the spur-wheel would receive by being thrown into gear when the shaft was turning rapidly would probably break the teeth of the spur-wheel.

Another arrangement of a similar nature, where the two shafts are at right angles instead of being coaxial, is shown in Fig. 285. The cones B, C, and D are the pitch surfaces of three bevel wheels, the teeth not being shown. The wheels B and C are formed in one casting, and their position is controlled by the lever H. In addition to being able to start and stop the rotation of the wheel D by bringing the teeth of the wheel B into gear with those of wheel D, the direction of rotation can be reversed by bringing the wheel C into gear with the wheel D. In this case also the wheels cannot be put into gear while the driving shaft is rotating rapidly, as some of the teeth may get broken in so doing.

The Friction Clutch (Fig. 286) gets over this difficulty. The shaft A requires to be started and stopped at will. To it is secured the socket C of the clutch. The spigot K is made to turn with the shaft B by the feather-key F, and it is also capable of sliding axially along the shaft B, which is continually in motion, the position of the spigot K being controlled by the lever L through the screw S and the hand-wheel W.

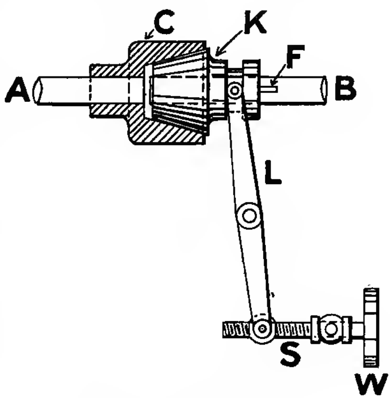

Fig. 286.—Cone friction clutch.

The conical surfaces of contact of the spigot and socket are made smooth, and the one slips over the other while the spigot is being pressed into the socket, and until the speed of the shaft A has increased to that of shaft B. As a rule, the cone surfaces are much larger in diameter than those shown in the figure, and the wedging action enables great pressure to be produced

between the surfaces in contact, and a correspondingly large amount of friction.

Bevel Wheels.—The nature of bevel wheels can be more easily understood from Fig. 287. Here no teeth are shown. It will be seen that the wheel on the shaft B is part of the cone EOC, and similarly the wheel on the shaft A is part of the cone DOC. These two cones have their vertices at the same point, O, and they will roll together if there is no slipping of one past the other; that is, if a circle on one cone moves over the same distance in the same time as the corresponding circle on the other cone which is in contact with it. Thus for perfect rolling we must have the circle DC moving at the same rate as the circle EC, and the circle *ab* must move at the same rate as the circle *cb*. Consider first the rolling circles DC and EC. Let N be the revolutions per minute of shaft A, and *n* the revolutions of shaft B. Then from page 219 we have—

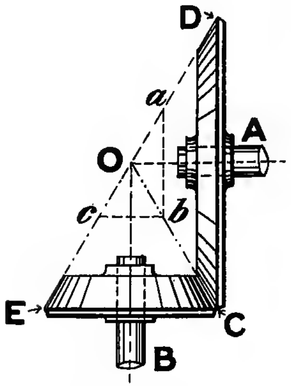

FIG. 287.—Bevel wheels.

$$N \times DC = n \times EC, \text{ or } \frac{N}{n} = \frac{EC}{DC}$$

$$\text{similarly, } N \times ab = n \times cb, \text{ or } \frac{N}{n} = \frac{cb}{ab}$$

$$\text{hence } \frac{EC}{DC} = \frac{cb}{ab}$$

Again, if the two cones have their vertices at the same point O, and *cb* is parallel to EC and *ab* parallel to CD, then from similar triangles—

$$\frac{Ob}{Oc} = \frac{cb}{EC}, \text{ also } \frac{Ob}{Oc} = \frac{ab}{CD}$$

$$\text{hence } \frac{cb}{\text{EC}} = \frac{ab}{\text{CD}}$$

$$\text{or } \frac{cb}{ab} = \frac{\text{EC}}{\text{CD}}$$

which is the same equation as that previously found above, on the condition that any two circles of contact should roll together; hence two cones will roll together if their vertices are coincident.

Variable-feed Gear.—In some machines it is necessary to vary the rate of feed, either between wide limits or con-

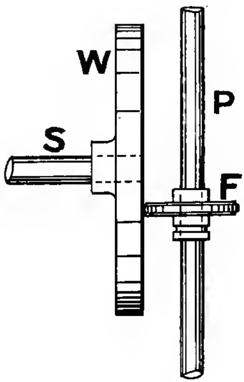

FIG. 288.

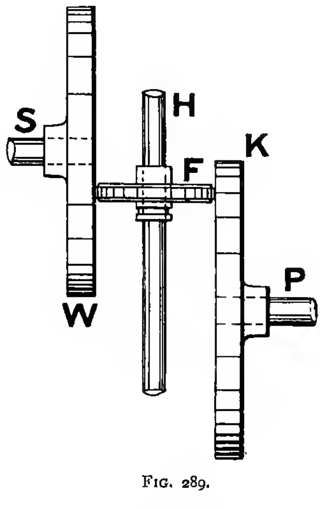

FIG. 289.

tinuously from a quick to a slow rate, or *vice versâ*. In Fig. 288 a mechanism is shown in which S is the main shaft rotating at a constant speed. On the end of S is fixed a disc, W, in contact with which is a rolling disc, F, which turns with the shaft P, but is capable of sliding along it, its position being determined by a fork, lever, and screw, as in Fig. 286. The shaft P is connected to the feed gear. The linear speed of a point near the centre of the disc W (Fig. 288) is less than that

of a point near the circumference; and hence, if the wheel F is in contact with the disc near the centre of W, the shaft P will turn more slowly than when the wheel F is further from the centre. The direction of the feed will be reversed if the wheel F is moved to the other side of the centre of the disc W. As there is only frictional contact, the effort transmitted cannot be great.

A much greater range of speed may be produced by the arrangement in Fig. 289, but no reversal can take place. In the position of the wheel F in the figure, the disc W is driving with a small radius, and the disc K is following or being driven with a large radius. The corresponding speed of the shaft P is much smaller than would be the case if the wheel F were near the centre of the disc K.

Epicyclic Trains of Wheels.—Another but more com-

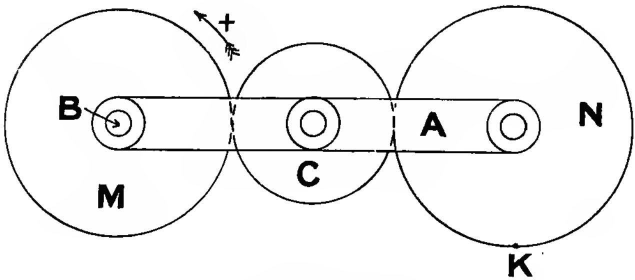

FIG. 290.—Epicyclic train of wheels.

plicated mechanism for the transmission of circular motion is an epicyclic train of toothed wheels, one form of which is shown in Fig. 290. A is a rod or arm which can turn loosely round the shaft B. It carries the shafts on which are the wheels C and N. If by some means we can fix the arm A and rotate the wheel M at a given speed, we can calculate the speeds of C and N by the method given on page 219. Hence, to be able to make calculations or measurements, we must first reduce the arm A to rest. We can do this by giving to the

whole mechanism the same number of revolutions as the arm, but in the *opposite* direction to that of the arm.[1]

If the arm previously made a revolutions per minute, while the wheel M made m revolutions per minute and the wheel N made n revolutions per minute, then, after giving to every part of the mechanism a number of revolutions (a) in the opposite direction to the arm's motion, the arm will make $a - a = 0$ revolutions, the wheel M will make $m - a$ revolutions per minute, and the wheel N will make $(n - a)$ revolutions per minute.

But with the arm fixed, we have the relation [2]—

M's revolutions × M's teeth = N's revolution × N's teeth

the wheel C being an idle wheel. This may be written as—

$$^{3}\ \frac{\text{M's teeth}}{\text{N's teeth}} = \frac{\text{N's revolutions}}{\text{M's revolutions}} = \frac{n - a}{m - a}$$

If a clock train is used, then the left-hand side of this equation becomes [4]—

$$\frac{\text{number of teeth of each driver multiplied together}}{\text{number of teeth of each follower multiplied together}}$$

In making calculations relating to epicyclic trains, great care should be taken to prefix the proper signs before n, m, and a, so far as they may be known; clockwise direction being considered negative.

Example.—Let the wheels M and N (Fig. 290) have fifty teeth each. Prevent M from moving, and make the arm rotate twenty

[1] As an illustration, a boat is allowed to drift down a stream at say 3 miles per hour. This corresponds to the motion of the arm. If now the boat is rowed *against* the stream at the rate of 3 miles per hour, the boat will be stationary, because the rowing just neutralizes the effect of the stream.

[2] See page 219.

[3] If M and N turn in opposite directions when the arm is fixed, a minus sign should be placed before the fraction $\frac{\text{M's teeth}}{\text{N's teeth}}$.

[4] See page 220.

times a minute in clockwise direction : how many revolutions does N make per minute ?

Here $m = 0$, and $a = -20$. Then—

$$\frac{50}{50} = \frac{\text{M's teeth}}{\text{N's teeth}} = \frac{n - a}{m - a} = \frac{n - (-a)}{0 - (-a)}$$

$$\text{or } a = n + a$$

and consequently $n = 0$; that is, the wheel N does not turn upon its axis, which is the same as saying that its lowest point K remains the lowest point throughout the motion.

Example.—Take away the wheel N (Fig. 290), and prevent the wheel C from rotating upon its axis. If the arm makes ten revolutions per minute, how many will M make, if M and C have each fifty teeth ?

Here $a = 10$, and $n = 0 =$ revolutions of C.

As there are two wheels only, they will turn in opposite directions, and therefore—

$$-\frac{50}{50} = \frac{\text{M's teeth}}{\text{C's teeth}} = \frac{n - a}{m - a}$$

$$\text{or } -1 = \frac{0 - 10}{m - 10}$$

then $m = 20$, or the wheel M makes twice as many turns as the arm in the same direction.

Example.—In Fig. 291, let the arm make twenty turns per

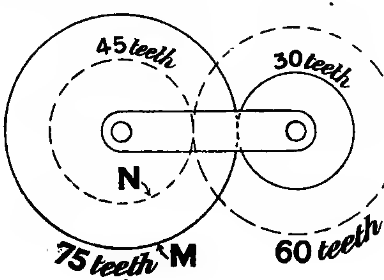

FIG. 291.

minute in the positive direction round the same shaft as the wheels N and M, while the wheel M makes thirteen turns per minute

in the same direction: how many turns does the wheel N make per minute?

The wheels M and N turn in the same direction when the arm is fixed, and hence—

$$\frac{\text{product of teeth in drivers}}{\text{product of teeth in followers}} = +\frac{75 \times 60}{30 \times 45} = \frac{10}{3}$$

$$\text{and } \frac{10}{3} = \frac{n - a}{m - a} = \frac{n - 20}{13 - 20}$$

which gives $n = -3\frac{1}{3}$ revolutions per minute.

If M had made fourteen turns per minute, N would have come out zero; while if M had made twenty revolutions per minute, the equation—

$$\frac{10}{3} = \frac{n - 20}{m - 20}$$

would become—

$$\frac{10}{3} = \frac{n - 20}{0}$$

which is only possible when $n = 20$. Then all the wheels and the arm would rotate together as if they formed one solid piece.

The Transmission of Linear Motion is not so common

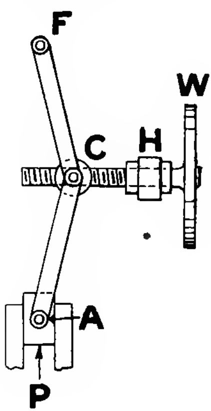

FIG. 292.—The toggle joint.

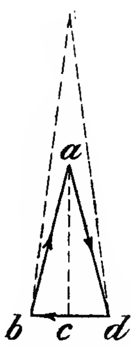

FIG 293.

as the transmission of circular motion, though it is somewhat difficult to separate them. An example may be found in the *toggle joint* in Fig. 292.

A link, FC, is pivoted at F and connected to another link, AC, at C, the end A being compelled to move in a vertical path by the guides shown. If the joint C be urged in a horizontal direction, tending to straighten the links FCA, then a small force applied at C will require a large force, P, to resist the motion of A. This is shown in Fig. 293, which is a triangle of forces for the pin C. The horizontal force on C is represented by *db*, and the forces in the links by *ba* and *ad*. For the equilibrium of the pin A we have the forces *ab*, *bc*, and *ca*, the last being P. The straighter the links become, the further does *a* recede from *c*, and therefore the larger does P become. The force *db* is put on by means of hand-wheel W and screw. This joint is used in some friction clutches and presses.

Some other instances of linear motion may be found in weighing-machines, though here again we also have motion round a point, as in circular motion.

The Roman Steelyard is shown diagrammatically in Fig. 294. Taking moments round the suspension hook at A, we have—

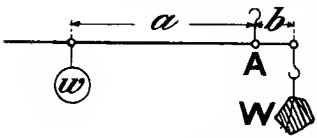

FIG. 294.—The Roman steelyard.

$$w \times a - Wb = 0$$

$$\text{or } W = w \times \frac{a}{b}$$

If b and w are kept constant, then W is proportional to a, and that arm may be graduated in pounds or any other measure.

It is sometimes convenient to use a number of levers instead of one long one. Such a system is shown in Fig. 295, where a large weight, W, is balanced by a small weight, w. Considering the uppermost lever, we have—

$$\text{force at A} \times b - w \times a = 0$$

And for the second lever—

$$\text{force at B} \times d - \text{force at D} \times c = 0$$

As the force at A = force at B, the two equations above together give—

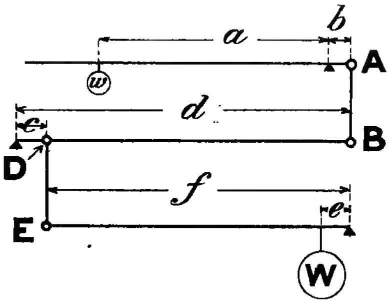

FIG. 295.—A system of levers.

$$\text{force at D} = w \times \frac{a}{b} \times \frac{d}{c}$$

In the same manner—

$$W = w \times \frac{a}{b} \times \frac{d}{c} \times \frac{f}{e}$$

If it is desired to bring the two hooks in Fig. 294 very close together, it will be found very difficult and perhaps impossible.

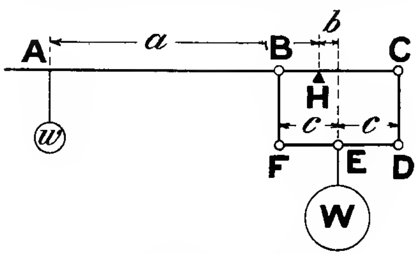

FIG. 296.—The differential weighing lever.

The arrangement shown in Fig. 296 may then be adopted. Here the weight W is equally divided between the points F and D, and transmitted to the points B and C.

$$BH = c - b, \text{ and } CH = c + b$$

Taking moments of forces acting on the lever round the knife-edge at H, we have—

$$w \times a + \frac{W}{2} \times BH - \frac{W}{2} \times CH = 0$$

$$\text{or } wa = \frac{W}{2}(CH - BH)$$

$$= \frac{W}{2}[c + b - (c - b)]$$

$$= \frac{W}{2} \times 2b$$

$$\text{and } w \times \frac{a}{b} = W$$

It is most inconvenient in an ordinary weighing-machine to have to sling everything up to a hook.

A platform is much more convenient, on which the article to be weighed is placed. Now, the machine must be arranged so that the true weight is indicated, whatever be the position of the article upon the platform. The platform is shown in outline in Fig. 297 by the dotted rectangle, and it possesses four knife-edges, which rest upon the levers at the points of the arrows.

A bifurcated lever, HFDG, turns about knife-edges at D and G. Two other levers, CA and QM, rest respectively upon fixed knife-edges at A and M, and upon the bifurcated lever at C and Q; the lengths AB and DE being equal, and DC and AC being equal.

The bifurcated lever is connected to the weighing-lever PNL by a link, LH.

Let the weight W of the article be distributed in *any* manner between the four knife-edges, such as W_1, W_2, W_3, and W_4. The pressure of the lever AC on the bifurcated lever at C is obtained by taking moments round A, when it will be found to be $W_1 \times \frac{AB}{AC}$. Similarly, the pressure of MQ on the lever at Q is $W_4 \times \frac{AB}{AC}$.

Now, consider the equilibrium of the bifurcated lever. Take moments round the line joining D to G. Then, if the pressure of the link LH on the lever be represented by f—

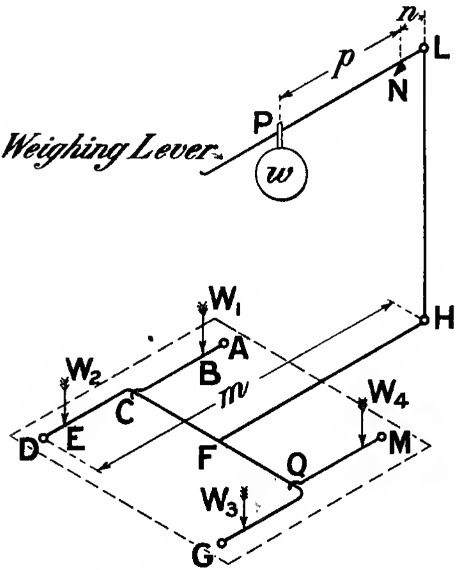

FIG. 297.—Diagrammatic representation of the ordinary weighing-machine.

$$f \times m - W_2 \times ED - W_3 \times ED - W_1 \frac{AB}{AC} \times CD - W_4 \frac{AB}{AC} \times CD = 0$$

Now, ED = AB, and AC = CO, in the ordinary weighing-machine (Fig. 297); then the above equation becomes—

$$\begin{aligned} fm &= W_2 \times ED + W_3 \times ED + W_1 \times \frac{ED}{CD} \times CD + W_4 \times \frac{ED}{CD} \times CD \\ &= (W_2 + W_3 + W_1 + W_4)ED \\ &= W \times ED \end{aligned}$$

Again, considering the equilibrium of the weighing-lever, we have—

$$w \times p - fn = 0$$

$$\text{or } wp = W \times \frac{ED}{m} \times n$$

$$\text{that is, } w \times \frac{p \times m}{n \times ED} = W$$

In the commercial form of the weighing-machine, the jockey weight w can only weigh between certain narrow limits, such as between 0 and 14 lbs., and heavier masses than

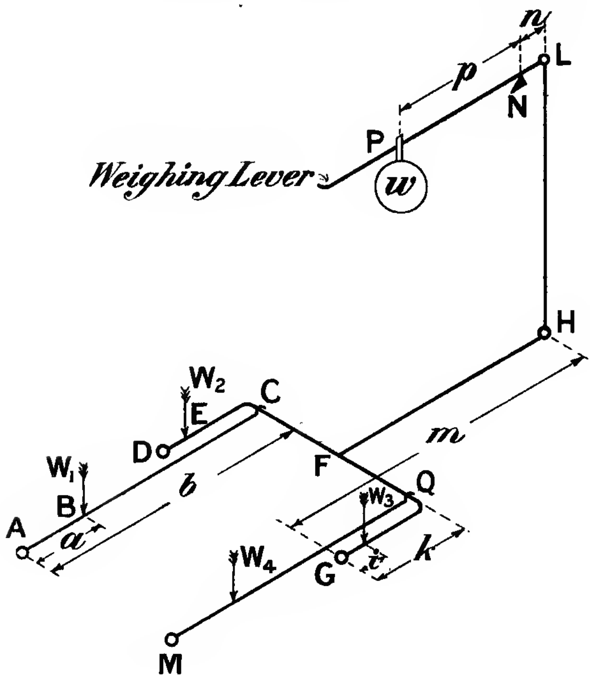

Fig. 298.

14 lbs. are weighed by putting on a load pillar situated at the outer end of the weighing-lever, weights corresponding to 14 lbs., 28 lbs., 42 lbs., etc., the intermediate quantities being

obtained by sliding the jockey weight w along the arm until the latter is balanced. If the jockey weight stands at 9 lbs., and there are weights on the end of the lever labelled 56 lbs. and 14 lbs., the weight registered is $56 + 14 + 9 = 79$ lbs.

A similar arrangement is used in Messrs. Riéhlé Brothers' testing-machine (Fig. 298). The load W on the specimen is divided in any manner into W_1, W_2, W_3, W_4. The levers are so arranged that $\frac{a}{b} = \frac{i}{k}$.

Inserting these equal ratios in the above equation for the equilibrium of the bifurcated lever in Fig. 297, we get the same result as that just found, namely—

$$W = \frac{pm}{ni} \times w$$

A rope-lifting tackle (Fig. 199) is another example of the transmission of linear motion.

An example of the *Conversion of linear to circular motion* is found in the crank and connecting-rod (Fig. 117). When a fluid presses upon the piston and compels the crank to turn, we get the engine mechanism, but when the crank compels the piston to reciprocate, we have a species of pump.

Cams are used for the purpose of converting circular to linear motion. One is shown in Fig. 299 at C, fixed upon the shaft S.

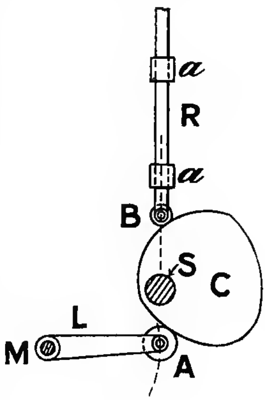

Fig. 299.

During rotation the cam urges the roller B further away from the shaft, and the guides a, a compel the rod R to move vertically.

In some cases it is convenient to replace the rod R by an arm, L, upon a shaft, M, the former carrying a roller, A, to minimize friction. The cam can be so shaped as to give

almost any kind of reciprocating motion. The weight of the rod R maintains it against the cam, while in the case of the arm L the roller A must be pressed against the cam by a spring or other similar means.

The steam-engine governor (Fig. 300) may be cited as an example of the conversion of the circular motion of the weights

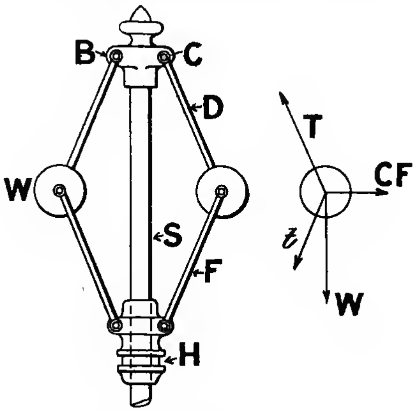

FIG. 300.—Engine governor.

W round B and C into a linear motion of the sleeve H along the spindle S.

At any given speed the right-hand weight is in equilibrium under the action of its weight W, the centrifugal force CF, the tension T of the link D, and the tension *t* of the link E. An increase of speed causes an increase of centrifugal force, which requires a new position of the weights further from the spindle S, to permit of equilibrium.

This outward movement of the weights causes the sleeve H to rise, and the governor-valve connected to H will be partially closed, and the amount of steam regulated to suit the work to be done.

Intermittent Motion.—This kind of motion is often used for feed purposes in machine tools. A simple example is given in Fig. 301. A crank, FE, turns about F at a constant

speed. An arm, DA, turning about a spindle, A, is connected to E by a rod, ED. The rotation of the crank causes the arm DA to reciprocate between the limiting positions BA and CA. During the motion from C to B the pawl slides over the teeth of the ratchet wheel, but during the return movement from B to C, the pawl catches against one of the teeth and the ratchet wheel is turned through the angle BAC.

It is necessary that when the resistance of a tooth acts

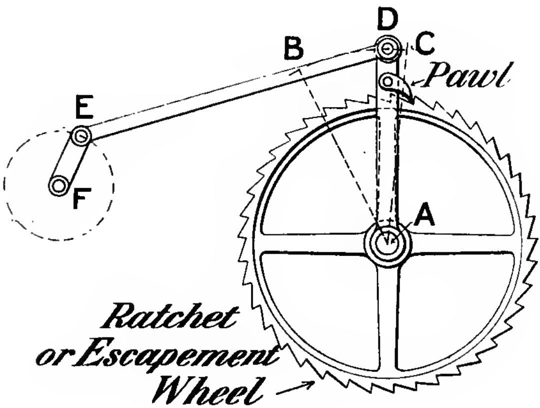

FIG. 301.

on the point of the pawl, the latter shall not slip out of its position. This is ensured by arranging the pawl and the teeth of the ratchet-wheel so that the pressure, R (Fig. 302), of the teeth on the point of the pawl shall pass between the pawl-pin and the wheel, thus producing a moment on the pawl equal to $R \times a$ in a direction tending to keep it in contact with the wheel.

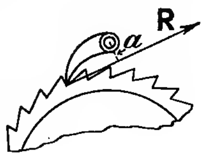

FIG. 302.

It is sometimes necessary to use a pair of pawls, such as P and Q in Fig. 303, when it is desirable to minimize the backlash or lost motion between the

teeth and the pawl. It will be noticed in the figure that the point of the pawl P is half a tooth further in advance than the point of the pawl Q. Using the two pawls is equivalent to doubling the number of teeth in the ratchet-wheel.

A friction feed is shown in Fig. 304.

A ring, S, fits loosely over the plain wheel W. A bent

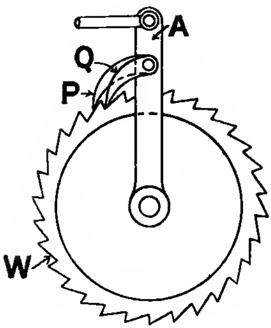

FIG. 303.

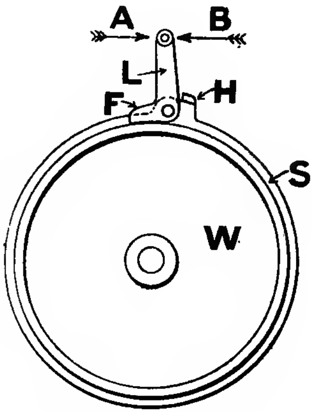

FIG. 304.—A friction ratchet-gear.

lever, L, is attached to the ring S with a pin. When the upper end of the lever is moved in the direction of the arrow A, the foot F is lifted from the wheel, and after the lever comes against the lug H, the ring and lever are carried round together as one body. On the return stroke, when the lever is moved in the direction of the arrow B, the foot F is pressed against the periphery of the wheel W, and the friction so produced is sufficient to carry lever, ring, and wheel forward together. It will be seen that this mechanism is equivalent to that in Fig. 301, and if there were no lost motion of the lever L, it would be equivalent to a ratchet-wheel with an infinite number of teeth.

Fig. 305 is a ratchet gear in which the feed stops after the speed of the mechanism has reached a certain limit. It is in

reality a gas-engine governor. Every time the spindle H is pushed towards the left a charge of gas enters the engine, and useful work is done by the exploding gas upon the piston.

This is normally brought about by the arm BA being moved to the left, and causing the finger D to push in the spindle H. A spring, DC, maintains the arm DAF normally in the position shown; but if the speed exceeds the normal,

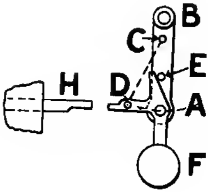

Fig. 305.

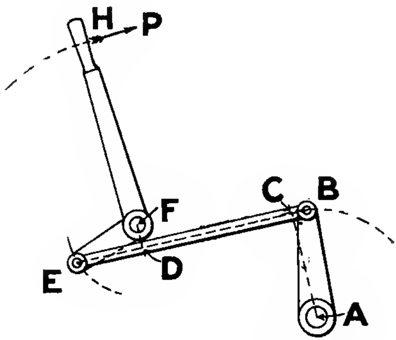

Fig. 306.—Stanhope levers.

the bob F tends to lag behind and stretches the spring DC, so that the finger D misses the spindle H, and no gas enters the engine until the speed comes back to the normal again.

A mechanism which can hardly be classed under intermittent motion is the **Stanhope Levers** in Fig. 306. HFE is a bent lever, and BA is a plain crank fixed upon the spindle A, which, in a form of printing machine, is screwed. For the equilibrium of the lever HFE we must have—

$$\text{pressure on pin E} \times \text{FD} = \text{P} \times \text{HF}$$

$$\text{that is, pressure on pin E} = \text{P} \times \frac{\text{HF}}{\text{FD}}$$

Again—

$$\left.\begin{array}{c}\text{the turning moment}\\ \text{on the spindle A}\end{array}\right\} = \text{tension in EB} \times \text{CA}$$

$$= \text{P} \times \frac{\text{HF}}{\text{FD}} \times \text{CA}$$

The further the lever HFE is moved the smaller does FD become, and consequently the greater does the turning moment become. In this way great pressure can be exerted by the screw in an axial direction.

APPENDIX

The Theory and Use of the Vernier.—The vernier is a small supplementary scale used in conjunction with an ordinary scale for the purpose of measuring to a greater degree of accuracy than is possible with the ordinary scale alone.

One is shown in Figs. 8, 9, 11.

A part of an ordinary scale, with vernier slide attached, is shown in Fig. 307.

The vernier is generally made to read in tenths against a scale

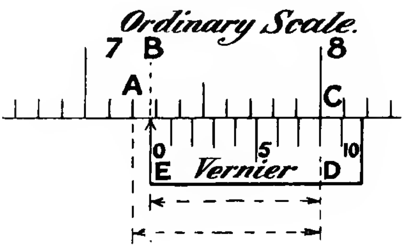

Fig. 307.

also in tenths. For example, if a foot rule contains inches and tenths of an inch, a vernier will enable the rule to be read to hundredths of an inch. Fig. 307 is an enlarged diagram of this arrangement.

The ordinary scale reads 7·2 plus a fraction of one-tenth of an inch, whose length is AB. It is this last piece that the vernier measures.

The vernier is constructed by taking nine ordinary scale divisions and dividing their equivalent length into ten equal parts, as shown in Fig. 308.. Consequently, each vernier division

is shorter than a scale division by one-tenth (or 0·1) of a scale division.

Returning to Fig. 307, it will be noticed that the eighth vernier mark, D, coincides with a scale mark C, therefore the

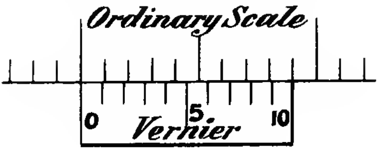

FIG. 308.

length DE (= 8 vernier divisions) is shorter than CA (8 scale divisions) by 8 times 0·1 of a scale division; that is 0·8 of a scale division, which equals 0·08 in. Hence the rule: Look along the vernier scale, and see which mark coincides with a main-scale mark. The number of the vernier mark is the decimal part of the scale division it is desired to measure.

The vernier and scale in Fig. 307 now reads—

$$= 7{\cdot}2 + 0{\cdot}8 \text{ of a scale division}$$
$$= 7{\cdot}2 + 0{\cdot}8 \text{ of } \tfrac{1}{10} \text{ in.}$$
$$= 7{\cdot}2 + 0{\cdot}08 \text{ in.}$$
$$= 7{\cdot}28 \text{ in.}$$

The vernier must be read in the same direction as the ordinary scale.

The vernier need not be made to read in tenths, although it is almost always most convenient to do so.

Let the length of 5 (or $n - 1$) scale divisions be divided into 6 (or n) equal parts to form a vernier, then each vernier division is $\frac{5}{6}$ $\left(\text{or } \frac{n-1}{n}\right)$ of a scale division, and is shorter than a scale division by $\frac{1}{6}$ $\left(\text{or } \frac{1}{n}\right)$ of a scale division. In Fig. 309 the fourth (or r^{th}) vernier mark coincides with a scale mark, hence BC is shorter than AC by $\frac{4}{6}$ $\left(\text{or } \frac{r}{n}\right)$ of a scale division. That is,

AB = $\frac{1}{6}$ $\left(\text{or } \frac{r}{n}\right)$ of a scale division. But each scale division is $\frac{1}{8}$ in., hence AB is $\frac{1}{6}$ $\left(\text{or } \frac{r}{n}\right)$ of $\frac{1}{8}$ in.

We can now state the general rule thus—

Let the principal or unit graduation on the main scale be divided into p equal parts, and let $(n - 1)$ of these be

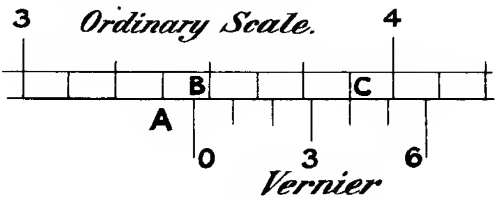

FIG. 309.

divided into n equal parts to form the vernier; then, if the r^{th} mark on the vernier coincide with one on the scale, the small fraction indicated by the vernier is—

$$\frac{r}{pn} \text{ of the unit graduation}$$

The *Screw* Gauge, or Micrometer Caliper.—The instrument

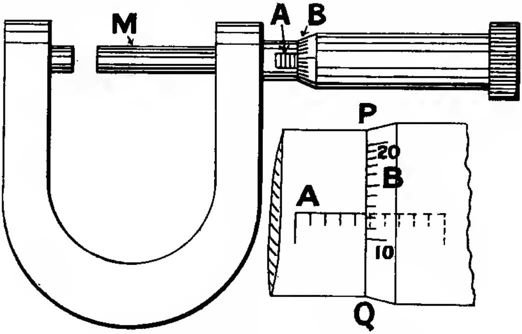

FIG. 310.—The micrometer gauge or caliper.

shown in Fig. 310 is used to measure more accurately than the scale and vernier. It is graduated in millimetres or inches, but

not in both. That in Fig. 310 is graduated in millimetres, the whole number of millimetres being shown on the scale at A. The fraction is read off round the bevelled edge at B in hundredths of a millimetre.

In some makes of instrument this edge is divided into a hundred equal parts; then the number on the bevel coincident with the longitudinal line (touching the top ends of the graduations A) is the fraction of the millimetre required. One turn of the bevelled edge advances the end M just 1 mm. The majority of these gauges are less conveniently graduated than the one just described. The scale A is the same, but that on the bevel does not consist of a hundred equal parts, but of fifty; there being two turns of the bevel to a millimetre of longitudinal movement of M. The bevel is graduated from 0 to 50, so that fractions of the first half of a millimetre are read off directly, but during the second half fifty hundredths have to be added to the number read off on the bevel. In Fig. 310 the graduated parts are enlarged in the lower right-hand corner, and the reading is 4·623 mm., the edge PQ cutting the millimetre in its second half. The bevel reading is not 12·3 hundredths, but 50 + 12·3 hundredths.

Those instruments which read in inches have the scale A graduated in fortieths of an inch; and the bevel B divided into twenty-five equal parts, one turn of the bevel advancing the spindle M through $\frac{1}{40}$ in.

The Equation to a Straight Line.—This is something which is so often used in experiments in mechanics, that it will be necessary to explain here how it is obtained.

Vertical heights from the base ON (Fig. 311) to the line in question, KP, are called ordinates; while horizontal distances from o are called abscissæ; hence a point, P, on the line KP has an ordinate PN = 17·6, and an abscissa oN = 27.

We desire to obtain an equation which will give us the relation between the abscissa and ordinate of *any* point on the line KP.

We notice, to begin with, that the line cuts the vertical axis at the point K seven units above o. The line then rises two units for every five units measured horizontally, that is, a rise of 0·4 for every 1 horizontally. Hence the height of any point, Q, above the base is oK + 0·4 × the horizontal distance of Q from the vertical axis oY. But the height of Q above the base is called the ordinate of Q, and the distance of Q from the vertical axis is called the abscissa of Q; hence the above equation can be written—

$$\text{ordinate} = \text{o}K + 0{\cdot}4 \times \text{abscissa}$$

The number 0·4 was the rise of the line KP, which took place over a horizontal distance of 1, and this must equal the rise over say n units measured horizontally divided by n. If we take any

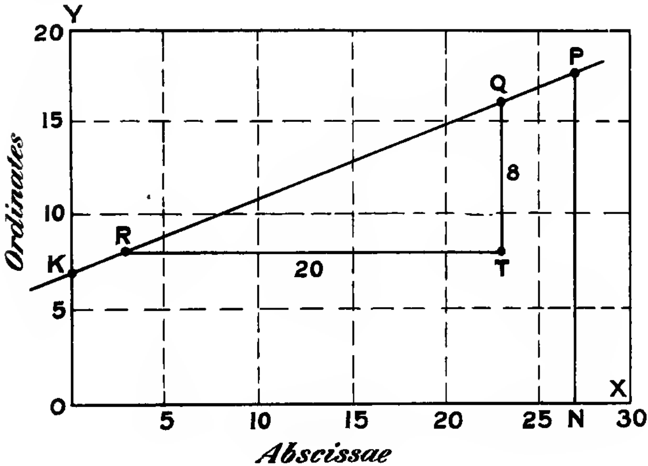

FIG. 311.

two points, R and Q, on the line, and draw a horizontal through R and a vertical through Q, these meeting in T, then QT is the rise over the horizontal distance RT, and the rise over 1 is $\frac{\text{QT}}{\text{RT}}$.

This fraction $\frac{\text{QT}}{\text{RT}}$ is generally called the *slope* of the line, while the length oK may be called the *intercept*, cut off by the line on the vertical axis.

The above equation can now be written as—

$$\text{ordinate} = \text{intercept} + \text{slope} \times \text{abscissa}$$

This will hold for all straight lines, every line having its own intercept and slope as constant quantities. A few examples will now be given.

Find the equation to the line KQ in Fig. 312.

Here the intercept oK is 4. Selecting any two points, R and Q, on the line, and drawing RT and QT, we find RT = 25 and QT = 15.

$$\text{slope} = \frac{QT}{RT} = \frac{15}{25} = 0{\cdot}6$$

Hence ordinate $= 4 + 0{\cdot}6$ abscissa

is the equation to that line.

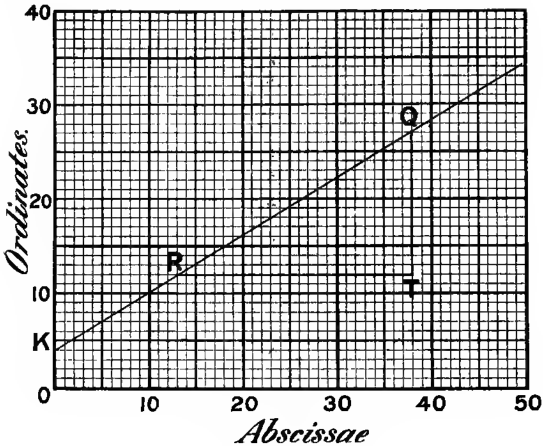

FIG. 312.

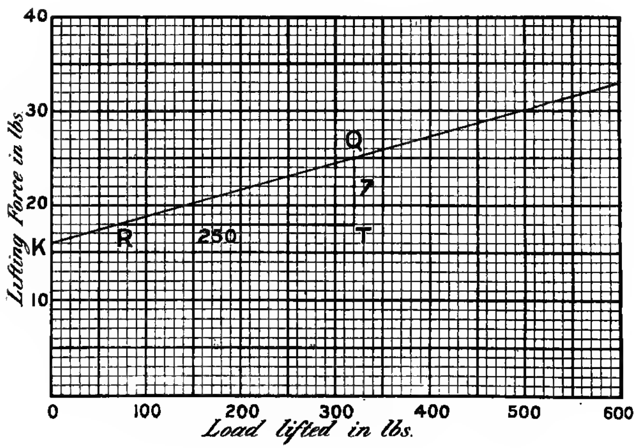

FIG. 313.

Again, find the equation to the line KQ in Fig. 313.

The intercept is 16, the slope is $\frac{7}{250} = 0{\cdot}028$; the ordinates represent lifting forces, and the abscissæ represent loads lifted by the machine, to which this line refers. Thus—

$$\text{lifting force} = 16 + 0{\cdot}028 \text{ load lifted}$$

Find the equation to the line AB (Fig. 314).

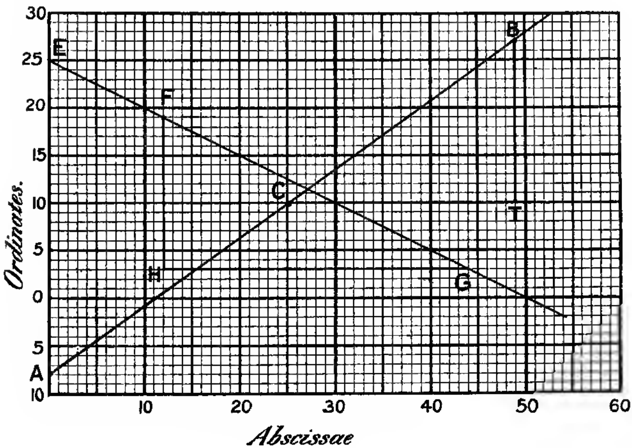

FIG. 314.

Here the intercept is −8, because the line cuts the vertical axis at a point −8 below the origin.

$$\text{The slope is } \frac{BT}{TC} = \frac{17}{24} = 0{\cdot}71$$

$$\text{hence ordinate} = -8 + 0{\cdot}71 \text{ abscissa}$$

The line EG has an intercept of 25 and a slope $= \frac{FH}{GH} = -\frac{16}{32}$ $= -0{\cdot}5$, the minus sign being used to signify that the line slopes upwards from right to left instead of from left to right.

The equation to the line EG is—

$$\text{ordinate} = 25 - 0{\cdot}5 \text{ abscissa}$$

Deducing the Equation to a Curve is not so easy a matter. The few cases that occur in elementary experimental mechanics are dealt with below.

Let y represent the ordinate of a point on the curve, and x

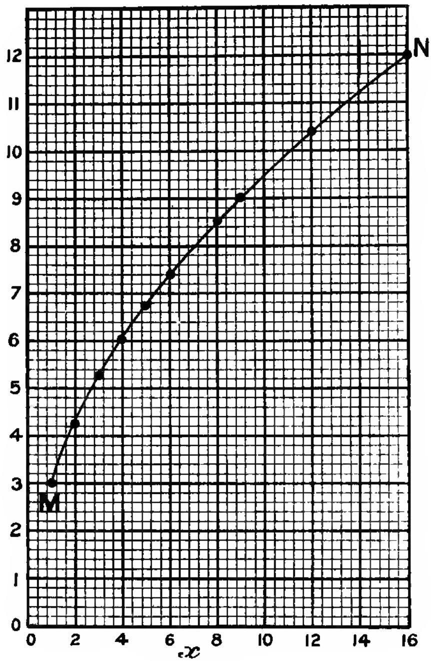

FIG. 315.

the abscissa. Then there are a number of curves that can be represented algebraically by the equation—

$$y = cx^n$$

To find whether a given curve such as MN (Fig. 315) is represented by this equation, proceed as follows :—

x.	y.	Log x.	Log y.
1	3·0	0·0	0·477
2	4·24	0·301	0·627
3	5·21	0·477	0·717
4	6·0	0·602	0·778
5	6·75	0·699	0·829
6	7·4	0·778	0·869
8	8·55	0·903	0·932
9	9·0	0·954	0·954
12	10·4	1·079	1·017
16	12·0	1·204	1·079

Construct a table of corresponding values of x and y. This table is generally obtained from experiment before the curve is drawn. Now obtain the logarithms of the several values of x and y, and plot these as in Fig. 316. The points happen in this case to lie on a straight line. Its equation is—

$$\text{ordinate} = \text{intercept} + \text{slope} \times \text{abscissa}$$
$$\text{or } \log y = 0·477 + 0·5 \times \log x$$

Now take logarithms of both sides of the equation—

$$y = cx^n$$

then—

$$\log y = \log c + n \log x$$

Comparing this with the equation above, we see it is of exactly the same *character*, and that $n = 0·5$, and $\log c = 0·477$. Consequently $c = 3$. The equation to the curve then is—

$$y = 3x^{\frac{1}{2}}$$
$$\text{or } y = 3\sqrt{x}$$

If the plotted logarithms should give a curve instead of a straight line similar to AB (Fig. 316), then the above equation is not the kind of one which represents the curve. Some other form of equation must then be tried. One of these which is used in an experiment is—

$$y = e^{cx}$$

e being the base of the hyperbolic logarithms = 2·718.

Taking hyperbolic logarithms of both sides, we get—

$$(\text{hyp.}) \log y = cx$$

If a table of hyperbolic logarithms is not available, the hyperbolic logarithm may be found by multiplying the common logarithm by 2·3.

Plot the hyperbolic logarithms of y vertically, and the values

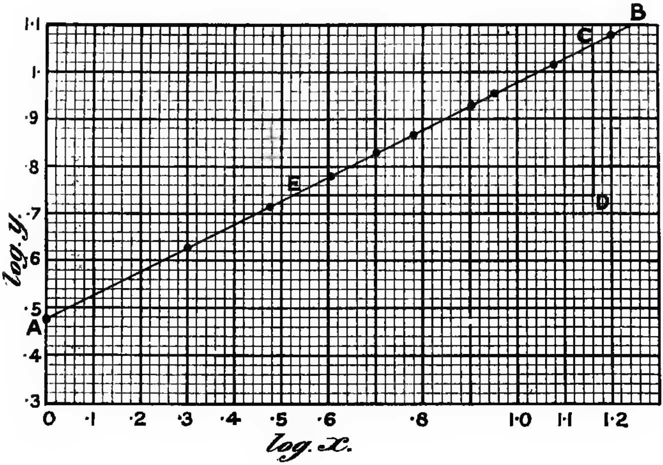

FIG. 316.

of x horizontally. Then, if the curve can be represented by the equation—

$$y = e^{cx}$$

the plotted points will lie on a straight line through the origin, its slope being C. For an equation of this kind, see p. 270.

Average Height and Area of an Irregular Figure.—One of the simplest methods of doing this is to divide the area into a number of parallel strips of equal width. Each strip will be a four-sided figure, with a pair of sides parallel, and hence the average height of a strip will be its middle height, while its area is the product of its width and its average height. The mean height of the whole figure is the average of the middle heights, while the area of the figure is the sum of the areas of the strips.

Referring to Fig. 317, draw a pair of parallel lines touching the ends of the diagram. These are shown dotted, and they pass through A and B respectively.

Set a scale, such as AB in Fig. 317, so that the zero of the scale is somewhere on the left vertical line, and the tenth division on the other vertical line at B. Then mark the middle points, C, D, E, etc., of all the intervals, and through these points draw verticals. These are the positions of the middle heights of the strips.

Then the average height of the figure is the average of the lengths of the dotted middle heights inside the figure. These are

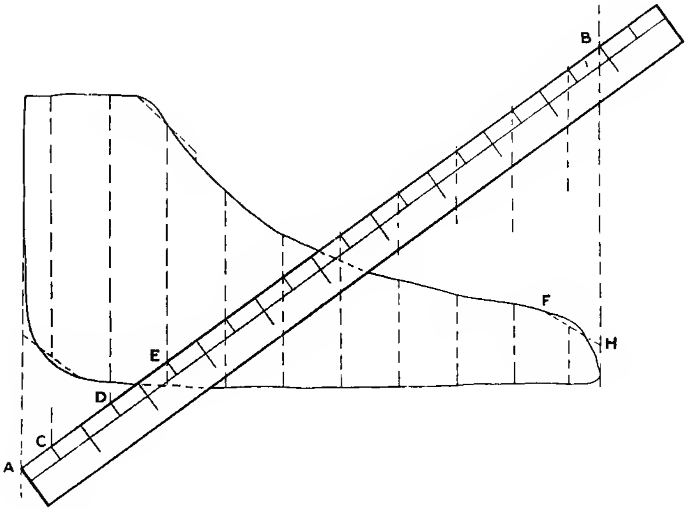

FIG. 317.

given in the first column of the table on page 87. Their sum is 27·62, and consequently the average height of the figure is $\frac{27{\cdot}62}{10} = 2{\cdot}76$ in.

The length of the figure is 9·8 in., hence its area must be $2{\cdot}76 \times 9{\cdot}8 = 27$ sq. in.

The above method is based upon the assumption that the boundaries of the strips are *straight*. This is not approximately true in some cases, notably in the first, third, and last strips in Fig. 317. Here new boundaries are drawn (dotted), which approximately cut away from the figure as much area as they take into it.

The Centre of Gravity of a Triangular Lamina.—In Fig.

318, ABC is a triangular lamina, or plate of uniform thickness. Divide the lamina into a large number of narrow strips parallel to the base, such as *ch*, *ba*, and *de*. The centres of gravity of these strips are at their middle points, *i*, *m*, and *n*, respectively. Similarly, the centres of gravity of all the strips will be at their middle points,

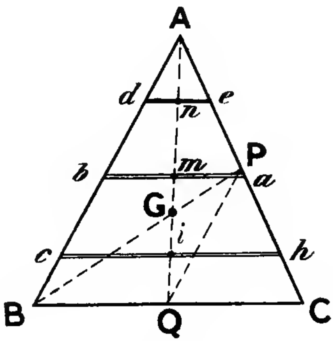

FIG. 318.

which lie on the line AQ joining the vertex to the middle point of the opposite side; and hence the centre of gravity of the whole triangle must be in this line. Similarly, it is in the line BP; hence it must be at G, where these lines intersect. Join PQ. Because QC = $\frac{1}{2}$BC, and PC = $\frac{1}{2}$AC, then PQ = $\frac{1}{2}$AB from the similar triangles CPQ and CAB. Also GPQ and GBA are similar triangles, in which QP = $\frac{1}{2}$BA, and hence QG = $\frac{1}{2}$AG, and consequently QG = $\frac{1}{3}$AQ; therefore—

The centre of gravity of a triangular lamina lies on the line joining the middle point of any side to the opposite angle, and one-third of the line from the side in question.

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THE END.

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